#6.32 Évaluer $I = \int_{0,3}^{1,7} e^{-x^2} dx$ par quadrature gaussienne à 3 points

où $a = 0,3$, $b = 1,7$

Poser $x = \frac{(b-a)T + b + a}{2} = \frac{(1,7-0,3)T + 1,7 + 0,3}{2} = 0,7T + 1$

$dx = \left(\frac{b-a}{2}\right) dT = \left(\frac{1,7-0,3}{2}\right) dt = 0,7\, dt$

$f(t) = e^{-(0,7T+1)^2}$

formule gaussienne à 3 points:

$\int_{-1}^{1} f(x)\,dx = \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$

$\int_{-1}^{1} e^{-(0,7T+1)^2} dt = 0,450520666 + 0,327003947 + 0,0514975$

$= 0,8290221151$

$0.7 \int_{0,3}^{1,7} e^{-(0,7T+1)^2} = 0,8290221151 \times 0,7$

$0,7 \boxed{\int_{0,3}^{1,7} e^{-(0,7T+1)^2} = 0,58031548}$

✓

6,34

Formule de Bonnet : $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$ $n = 1, 2 \ldots$

$n = 3$ Par $P_4(x)$

$$(3+1)P_4(x) = (2(3)+1)x\,P_3(x) - 3P_2(x)$$

$$4P_4(x) = (7)x\,P_3(x) - 3P_2(x)$$

$$P_4(x) = \frac{7x\,P_3(x) - 3P_2(x)}{4}$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x) \qquad P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_4(x) = \frac{7x\left(\frac{1}{2}(5x^3-3x)\right) - 3\left(\frac{1}{2}(3x^2-1)\right)}{4}$$

$$= \frac{7/2\,(5x^4 - 3x^2) - \frac{3}{2}(3x^2-1)}{4}$$

$$= \frac{\frac{35}{2}x^4 - \frac{21}{2}x^2 - \frac{9}{2}x^2 + \frac{3}{2}}{4}$$

$$= \frac{\frac{35}{2}x^4 - \frac{30}{2}x^2 + \frac{3}{2}}{4}$$

$$= \frac{\frac{1}{2}(35x^4 - 30x^2 + 3)}{4}$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

#4.2 Résoudre le système d'équation différentielles $y' = Ay$

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

① $\det \begin{bmatrix} (2-\lambda) & 0 & 4 \\ 0 & (2-\lambda) & 0 \\ -1 & 0 & (2-\lambda) \end{bmatrix} = \det(A - \lambda I) =$

$0 = (2-\lambda)((2-\lambda)(2-\lambda) - 0) + 4(0 - (-1 \cdot (2-\lambda)))$

$0 = (2-\lambda)(2-\lambda)(2-\lambda) + 4(2-\lambda)$

$0 = (2-\lambda)^3 + 4(2-\lambda)$

$0 = (2-\lambda)((2-\lambda)^2 + 4)$ mise en évidence de $(2-\lambda)$

$0 = (2-\lambda)(\lambda^2 - 4\lambda + 8)$

$0 = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2}$

$= \dfrac{4 \pm \sqrt{(-1)(16)}}{2} = \dfrac{4 \pm i \cdot 4}{2} = 2 \pm 2i$

$\lambda_1 = 2 + 2i$, $\lambda_2 = 2 - 2i$, $\lambda_3 = 2$

② $\rightarrow (A - \lambda_1 I)u = \begin{bmatrix} 2-(2+2i) & 0 & 4 \\ 0 & 2-(2+2i) & 0 \\ -1 & 0 & 2-(2+2i) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \vec{0}$

$$\begin{cases} \text{(A)} & -2i \cdot u_1 + \cancel{0u_2} + 4u_3 = 0 \rightarrow u_3 = iu_1/2 \\ \text{(B)} & \cancel{0u_1} - 2iu_2 + \cancel{0u_3} = 0 \rightarrow -2i \cdot u_2 = 0 \quad \boxed{u_2 = 0} \\ \text{(C)} & -u_1 + \cancel{0u_2} - 2iu_3 = 0 \rightarrow u_1 = -2iu_3 \end{cases}$$

(C) dans (A)

$u_3 = \dfrac{i(-2iu_3)}{2} = -\dfrac{\cancel{2}(-1)u_3}{\cancel{2}} = u_3 = u_3$

* indéterminé! Posons $u_3 = 1$

$u_1 = -2i(1) = -2i$

$$u = \begin{bmatrix} -2i \\ 0 \\ 1 \end{bmatrix}$$

suite 4.2

$\rightarrow (A-\lambda_2 I)V = \begin{bmatrix} 2i & 0 & 4 \\ 0 & 2i & 0 \\ -1 & 0 & 2i \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \vec{0}$

$$\begin{matrix} Ⓐ \\ Ⓑ \\ Ⓒ \end{matrix} \left\{ \begin{array}{l} 2iV_1 + 4V_3 = 0 \\ 2iV_2 = 0 \\ -V_1 + 2iV_3 = 0 \end{array} \right\} \begin{array}{l} V_3 = -iV_1/2 \\ V_2 = 0 \\ V_1 = 2iV_3 \end{array}$$

Ⓒ dans Ⓐ

$V_3 = -i(2iV_3)/2$

$= -\cancel{2}i^2 V_3 / \cancel{2}$

$= V_3$

* indéterminé! Posons $V_3 = 1$

$V_1 = 2i$

$V = \begin{bmatrix} 2i \\ 0 \\ 1 \end{bmatrix}$

$\rightarrow (A-\lambda_3 I)w = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \vec{0}$

$$\begin{matrix} Ⓐ \\ Ⓑ \\ Ⓒ \end{matrix} \left\{ \begin{array}{l} 4w_3 = 0 \rightarrow w_3 = 0 \\ 0 = 0 \\ -w_1 = 0 \rightarrow w_1 = 0 \end{array} \right.$$

* $w_2$ indéterminer! Posons $w_2 = 1$ (On ne peut pas poser $w_2 = 0$ car le vecteur $w$ ne peut pas être nul)

$w = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

③ $y_1 = e^{(2+2i)x} \begin{bmatrix} -2i \\ 0 \\ 1 \end{bmatrix}, y_2 = e^{(2-2i)x} \begin{bmatrix} -2i \\ 0 \\ 1 \end{bmatrix}, y_3 = e^{2x} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\boxed{\vec{y}(x) = c_1 e^{2x} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 e^{2x} \begin{bmatrix} -2\sin 2x \\ 0 \\ -\cos 2x \end{bmatrix} + c_3 e^{2x} \begin{bmatrix} 2\cos 2x \\ 0 \\ -\sin 2x \end{bmatrix}}$$

✓

#4.5 Résoudre les systèmes d'équations différentielle $y' = Ay$ pour A donné.

$$A = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$$

① $\det |A - \lambda I| = \begin{vmatrix} (1-\lambda) & 1 \\ -4 & (1-\lambda) \end{vmatrix} = (1-\lambda)(1-\lambda) - 1(-4) = 0$

$= (1-\lambda)(1-\lambda) + 4$

$= 1 - 2\lambda + \lambda^2 + 4$

$= \lambda^2 - 2\lambda + 5$

$\dfrac{2 \pm \sqrt{4 - 4(1)(5)}}{2} = \dfrac{2 \pm 4i}{2} = 1 \pm 2i$

$\lambda_1 = 1 + 2i \qquad \lambda_2 = 1 - 2i$

② Verteurs Propre

$\lambda_1 \rightarrow (A - \lambda_1 I)u = A - (1+2i)I)u = \begin{bmatrix} -2i & 1 \\ -4 & -2i \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \vec{0}$

$\begin{cases} -2iu_1 + u_2 = 0 \rightarrow u_2 = 2iu_1 \\ -4u_1 - 2iu_2 = 0 \rightarrow u_1 = -2iu_2/4 \end{cases}$

$u_1 = \dfrac{-2i(2iu_1)}{4} = \dfrac{-4i^2 u_1}{4} = u_1$

* $u_1$ indéterminé. Posons $u_1 = 1$, Donc $u_2 = 2i$

$\vec{u} = \begin{bmatrix} 1 \\ 2i \end{bmatrix}$

$\lambda_2 \rightarrow (A - \lambda_2 I)v = A - (1-2i)I)v = \begin{bmatrix} 2i & 1 \\ -4 & 2i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \vec{0}$

$\begin{cases} 2iv_1 + v_2 = 0 \rightarrow v_2 = -2iv_1 \\ -4v_1 + 2iv_2 = 0 \rightarrow v_1 = \dfrac{2iv_2}{4} \end{cases}$

$v_1 = \dfrac{2i(-2iv_1)}{4} = \dfrac{-4(-1)v_1}{4} = v_1$

* $v_1$ indéterminé. Posons $v_1 = -1$, Donc $v_2 = +2i$

$\vec{v} = \begin{bmatrix} -1 \\ +2i \end{bmatrix}$

Suite 4.5...

$$u(x) = e^{(1+2i)x}\begin{bmatrix}1\\2i\end{bmatrix} = e^x e^{2ix}\begin{bmatrix}1\\2i\end{bmatrix} = e^x(\cos 2x + i\sin 2x)\begin{bmatrix}1\\2i\end{bmatrix}$$

$$u(x) = \begin{bmatrix}1\\0\end{bmatrix}e^x(\cos 2x + i\sin 2x) + 2i\begin{bmatrix}0\\1\end{bmatrix}e^x(\cos 2x + i\sin 2x)$$

$$= e^x\left(\begin{bmatrix}1\\0\end{bmatrix}\cos 2x - 2\begin{bmatrix}0\\1\end{bmatrix}\sin 2x\right) + ie^x\left(\begin{bmatrix}1\\0\end{bmatrix}\sin 2x + \begin{bmatrix}0\\2\end{bmatrix}\cos 2x\right)$$

$$= e^x\begin{bmatrix}\cos 2x\\-2\sin 2x\end{bmatrix} + ie^x\begin{bmatrix}\sin 2x\\2\cos 2x\end{bmatrix}$$

$$y_1(x) = e^x\begin{bmatrix}\cos 2x\\-2\sin 2x\end{bmatrix},\quad y_2(x) = \begin{bmatrix}\sin 2x\\2\cos 2x\end{bmatrix}e^x$$

$$\boxed{y(x) = c_1 e^x\begin{bmatrix}\cos 2x\\-2\sin 2x\end{bmatrix} + c_2 e^x\begin{bmatrix}\sin 2x\\2\cos 2x\end{bmatrix}}$$

✓

#4.7 Résoudre le système d'équations différentielles

$y' = Ay + f(x)$

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}, \quad f(x) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} (1-\lambda) & 1 \\ 3 & (1-\lambda) \end{vmatrix} = (1-\lambda)(1-\lambda) - 1 \cdot 3$$

$= 1 - 2\lambda + \lambda^2 - 3 = 0$

$= \lambda^2 - 2\lambda - 2$

$\lambda_1 = 1+\sqrt{3} \quad \lambda_2 = 1-\sqrt{3}$

Valeurs propres :

$\underline{\lambda_1}$: $|A - \lambda_1 I| \cdot u = \begin{bmatrix} -\sqrt{3} & 1 \\ 3 & -\sqrt{3} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \vec{0}$

$$\begin{cases} -\sqrt{3} \cdot u_1 + u_2 = 0 \rightarrow u_2 = \sqrt{3}\, u_1 \\ 3u_1 - \sqrt{3}\, u_2 = 0 \rightarrow u_1 = \sqrt{3} u_2 / 3 \end{cases}$$

$$u_1 = \frac{\sqrt{3} \cdot \sqrt{3} u_1}{3} = u_1$$

* $u_1$ indéterminé donc posons $u_1 = 1$

$u_2 = \sqrt{3}$

$\vec{u} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$

$\underline{\lambda_2}$: $|A - \lambda_2 I| V = \begin{bmatrix} \sqrt{3} & 1 \\ 3 & \sqrt{3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \vec{0}$

$$\begin{cases} \sqrt{3} V_1 + V_2 = 0 \rightarrow V_2 = -\sqrt{3} V_1 \\ 3V_1 + \sqrt{3} V_2 = 0 \rightarrow V_1 = -\sqrt{3} V_2 / 3 \end{cases}$$

$$V_1 = \frac{-\sqrt{3} \cdot -\sqrt{3} V_1}{3} = V_1$$

* $V_1$ indéterminé, Posons $V_1 = 1$

$V_2 = -\sqrt{3}$

$\vec{V} = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$

$$y_h(x) = c_1 \vec{u} e^{\lambda_1 x} + c_2 \vec{V} e^{\lambda_2 x}$$

$$= c_1 \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} e^{(1+\sqrt{3})x} + c_2 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{(1-\sqrt{3})x}$$

$$A\begin{bmatrix} a \\ b \end{bmatrix} + f(x) = 0 \qquad \text{et} \quad y_p(x) = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

$$\begin{cases} a+b+1=0 \to a = -b-1 \\ 3a+b+1=0 \to b = -3a-1 \end{cases}$$

$3(-b-1)+b+1=0$

$-3b-3+b+1=0$

$-2b-2=0$

$2b=-2$

$\boxed{b=-1}$ donc $\boxed{a=0}$ $\quad y_p(x) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$y(x) = y_h(x) + y_p(x)$

$$= c_1\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} e^{(1+\sqrt{3})x} + c_2\begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{(1-\sqrt{3})x} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

✓

#4.11 Résoudre les systèmes d'équations différentielles $y' = Ay$ avec $y(0) = y_0$ pour $A$ et $y_0$ données.

$$A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}, \quad y_0 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} (5-\lambda) & -1 \\ 3 & (1-\lambda) \end{vmatrix} = (5-\lambda)(1-\lambda) - (-1)(3) = 5 - 5\lambda - \lambda + \lambda^2 + 3$$

$$= \lambda^2 - 6\lambda + 8$$

$$\frac{6 \pm \sqrt{36 - 4(1)(8)}}{2} = \frac{6 \pm 2}{2} = \frac{3 \pm 1}{1}$$

$\lambda_1 = 4 \quad \lambda_2 = 2$

Vecteurs propres:

$\lambda_1$ $\swarrow 4$

$$(A - \lambda_1 I)u = \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \vec{0}$$

$$\begin{cases} u_1 - u_2 = 0 \rightarrow u_2 = u_1 \\ 3u_1 - 3u_2 = 0 \rightarrow u_1 = u_2 \end{cases}$$

$u_1 = u_1$

* $u_1$ indéterminé, Posons $u_1 = 1$, Donc $u_2 = 1$

$$\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda_2$ $\swarrow 2$

$$(A - \lambda_2 I)v = \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \vec{0}$$

$$\begin{cases} 3v_1 - v_2 = 0 \rightarrow v_2 = 3v_1 \\ 3v_1 - v_2 = 0 \rightarrow v_1 = v_2/3 \end{cases}$$

$$v_1 = \frac{\cancel{3}v_1}{\cancel{3}}$$

* $v_1$ indéterminé, Posons $v_1 = 1$, donc $v_2 = 3$

$$\vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Solution générale:

$$y(x) = c_1 \vec{u} e^{\lambda_1 x} + c_2 \vec{v} e^{\lambda_2 x} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4x} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2x}$$

$\rightarrow$

Suite 4.11...

Condition initiale

$Y_0(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ -1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4 \cdot 0} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2 \cdot 0} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\begin{cases} 2 = c_1 + c_2 \rightarrow c_1 = 2 - c_2 \\ -1 = c_1 + 3c_2 \end{cases}$

$-1 = 2 - c_2 + 3c_2$

$-3 = 2c_2$

$\boxed{\frac{-3}{2} = c_2}$ $\quad c_1 = 2 + \frac{3}{2} = \frac{7}{2}$ $\quad \boxed{c_1 = \frac{7}{2}}$

Solution particulière

$y(x) = \frac{7}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4x} - \frac{3}{2} \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2x}$

✓

$$\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}1&1\\3&1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}$$

$$\begin{bmatrix}-1\\-1\end{bmatrix}=\begin{bmatrix}1&1\\3&1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} \Rightarrow \left.\begin{aligned}a+b&=-1\\3a+b&=-1\end{aligned}\right\} \quad 2a=0 \Rightarrow \boxed{\begin{aligned}a&=0\\b&=-1\end{aligned}}$$

$$y(x)=c_1e^{(1+\sqrt{3})x}\begin{bmatrix}1\\\sqrt{3}\end{bmatrix}+c_2e^{(1-\sqrt{3})}\begin{bmatrix}1\\-\sqrt{3}\end{bmatrix}+\begin{bmatrix}0\\-1\end{bmatrix}$$ ✓

$\boxed{4.11}$ $y'=\begin{bmatrix}5&-1\\3&1\end{bmatrix}y, \quad y_0=\begin{bmatrix}2\\-1\end{bmatrix}$

$$[A-\lambda I]=\begin{bmatrix}5-\lambda&-1\\3&1-\lambda\end{bmatrix}=\lambda^2-6\lambda+8=(\lambda-2)(\lambda-4)=0$$

$\Rightarrow \lambda_1=2, \lambda_2=4$

$$[A-2I]\vec{u}=\begin{bmatrix}3&-1\\3&-1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}=0 \Rightarrow \begin{aligned}3u_1-u_2&=0\\3u_1 \quad u_2&=0\end{aligned}$$

Posons $u_1=1$ : $u_2=3$

$3\cdot1-3=0$ ✓ $\quad \vec{u}=\begin{bmatrix}1\\3\end{bmatrix}$

$$[A-4I]\vec{v}=\begin{bmatrix}1&-1\\3&-3\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix}=0 \Rightarrow \begin{aligned}v_1-v_2&=0\\3v_1-3v_2&=0\end{aligned}$$

Posons $v_1=1$ : $v_2=1$

$3\cdot1-3\cdot1=0$ ✓ $\quad \vec{v}=\begin{bmatrix}1\\1\end{bmatrix}$

$$y(x)=c_1e^{2x}\begin{bmatrix}1\\3\end{bmatrix}+c_2e^{4x}\begin{bmatrix}1\\1\end{bmatrix} \Rightarrow y_0=\begin{bmatrix}c_1\\3c_1\end{bmatrix}+\begin{bmatrix}c_2\\c_2\end{bmatrix}=\begin{bmatrix}2\\-1\end{bmatrix}$$

$$\left.\begin{aligned}c_1+c_2&=2\\3c_1+c_2&=-1\end{aligned}\right\} \quad 2c_1=-3 \Rightarrow \boxed{\begin{aligned}c_1&=-3/2\\c_2&=7/2\end{aligned}}$$

$$y(x)=\frac{-3}{2}e^{2x}\begin{bmatrix}1\\3\end{bmatrix}+\frac{7}{2}e^{4x}\begin{bmatrix}1\\1\end{bmatrix}$$

$\boxed{10.18}$ $y' = x + 2\cos y$ , $y(0) = 0$

→ 4 pas avec la paire ode23 de RK. → $h = 0{,}1$

$0 \leq x \leq 1$ & estimer l'erreur locale de méthode.

$$y_{n+1} = y_n + \frac{2}{9}k_1 + \frac{1}{3}k_2 + \frac{4}{9}k_3$$

$$\to k_1 = 0{,}1(x_n + 2\cos y_n)$$

$$k_2 = 0{,}1((x_n + 1/2\,h) + 2\cos(y_n + 1/2\,k_1))$$

$$k_3 = 0{,}1((x_n + 3/4\,h) + 2\cos(y_n + 3/4\,k_2))$$

$$k_4 = 0{,}1((x_n + h) + 2\cos(y_n + \frac{2}{9}k_1 + \frac{1}{3}k_2 + \frac{4}{9}k_3))$$

$$E = -\frac{5}{72}k_1 + \frac{1}{12}k_2 + \frac{1}{9}k_3 - \frac{1}{8}k_4$$

Résultats numériques :

| $n$ | $x_n$ | $y_n$ | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $E$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0,000000 | 0,200000 | 0,204001 | 0,205164 | 0,205869 | 0,000174 |
| 1 | 0,1 | 0,203629 | 0,205869 | 0,205675 | 0,204828 | 0,203506 | 0,000164 |
| 2 | 0,2 | 0,408970 | 0,203506 | 0,199478 | 0,197102 | 0,194126 | 0,000125 |
| 3 | 0,3 | 0,608287 | 0,194126 | 0,187277 | 0,184009 | 0,179966 | 0,000075 |
| 4 | 0,4 | 0,795634 | 0,179966 | 0,171562 | 1,679778 | 0,163477 | 0,000029 |

$\boxed{10.22}$ $y' = x + \cos y \;.\; y(0) = 0$

→ prédicteur-correcteur d'Adams-Bashforth-Mouton. d'ordre 3. avec $h = 0{,}1$ , $0 \leq x \leq 1$ → 2 pas ☆ valeurs de 10.13.

→ Estimer l'erreur locale en $x = 0{,}5$

$y_0^c = 0 \qquad y_1^c = 0.104821 \qquad y_2^c = 0{,}218475$

$$y_{n+1}^p = y_n^c + \frac{0{,}1}{12}\left(23(x_n + \cos y_n^c) - 16(x_{n-1} + \cos y_{n-1}^c) + 5(x_{n-2} + \cos y_{n-2}^c)\right).$$

$$y_{n+1}^c = y_n^c + \frac{0{,}1}{12}\left(5(x_{n+1} + \cos y_{n+1}^p) + 8(x_n + \cos y_n^c) - (x_{n-1} + \cos y_{n-1}^c)\right)$$

$$E \approx -\frac{1}{10}\left[y_{n+1}^c - y_{n+1}^p\right]$$

Résultats numériques

| $n$ | $x_n$ | $y_n^c$ départ | $y_n^p$ | $y_n^c$ | $E(\times 10^{-6})$ |
|---|---|---|---|---|---|
| 0 | 0 | 0.000000 | | | |
| 1 | 0,1 | 0, 104821 | | | |
| 2 | 0,2 | 0,218475 | | | |
| 3 | 0,3 | | 0,339651 | 0,339556 | 9,5096 |
| 4 | 0,4 | | 0,466553 | 0,466494 | 5.8669 |
| 5 | 0,5 | | 0,597637 | 0,597245 | 1, 2712 |