MAT 2784 - Devoir #3.

$\boxed{3.1}$ $y' - y\tan x = \frac{1}{\cos^3 x}$, $y(0) = 0$

$y' + f(x)y = r(x)$ $\quad f(x) = -\tan x$

$\mu(x) = e^{\int -\tan x\, dx} = \cancel{e}^{-(-\ln|\cos x|)} = \boxed{\cos x}$

$(\cos x\, y)' = \cos x \frac{1}{\cos^3 x} dx$

$\cos x\, y(x) = \int \frac{1}{\cos^2 x} dx + c = \tan x + c$

$y(x) = \frac{\tan x + c}{\cos x}$ $\quad \rightarrow y(0) = 0$

$0 = \frac{\tan(0) + c}{\cos(0)}$

$0 = \frac{0 + c}{1} = \frac{c}{1}$

$\boxed{c = 0}$

Solution implicite $\boxed{y(x) = \frac{\tan x}{\cos x}}$

✓

$\boxed{3.2}$ $y = cx^2$

$u(x,y) = c \Rightarrow \frac{y}{x^2} = c \Rightarrow yx^{-2} = c$

$\frac{d}{dx}(u(x,y)) = \frac{dc}{dx} \Rightarrow y' = -\frac{2y}{x^3} = 0 \Rightarrow \frac{1}{x^3}dx + \frac{1}{2y}dy = 0$

$u_x\,dx + u_y\,dy = 0 \Rightarrow u_y\,dy = -u_x\,dx \Rightarrow \frac{dy}{dx} = -\frac{u_x}{u_y} = \boxed{\frac{2y}{x} = m}$

$m_{ORTH} = \frac{-1}{m} = y'_{ORTH} = \frac{-x}{2y} = \frac{dy_{ORTH}}{dx}$

$\Rightarrow x\,dx + 2y\,dy = 0$

$\int x\,dx + 2\int y\,dy = \int 0 \Rightarrow \frac{x^2}{2} + y^2 = C_1$

Solution générale : $\boxed{x^2 + 2y^2 = \cancel{c}\ k}$

c est déjà employé

$\boxed{c=1}$

$\rightarrow x^2 + 2y^2 = 1$

$0^2 + 2y^2 = 1$

$y = \pm\sqrt{1/2}$

$\rightarrow x^2 + 2y^2 = 1$

$x^2 + 0^2 = 1$

$x = \pm 1$

$\boxed{c=2}$

$\rightarrow x^2 + 2y^2 = 2$

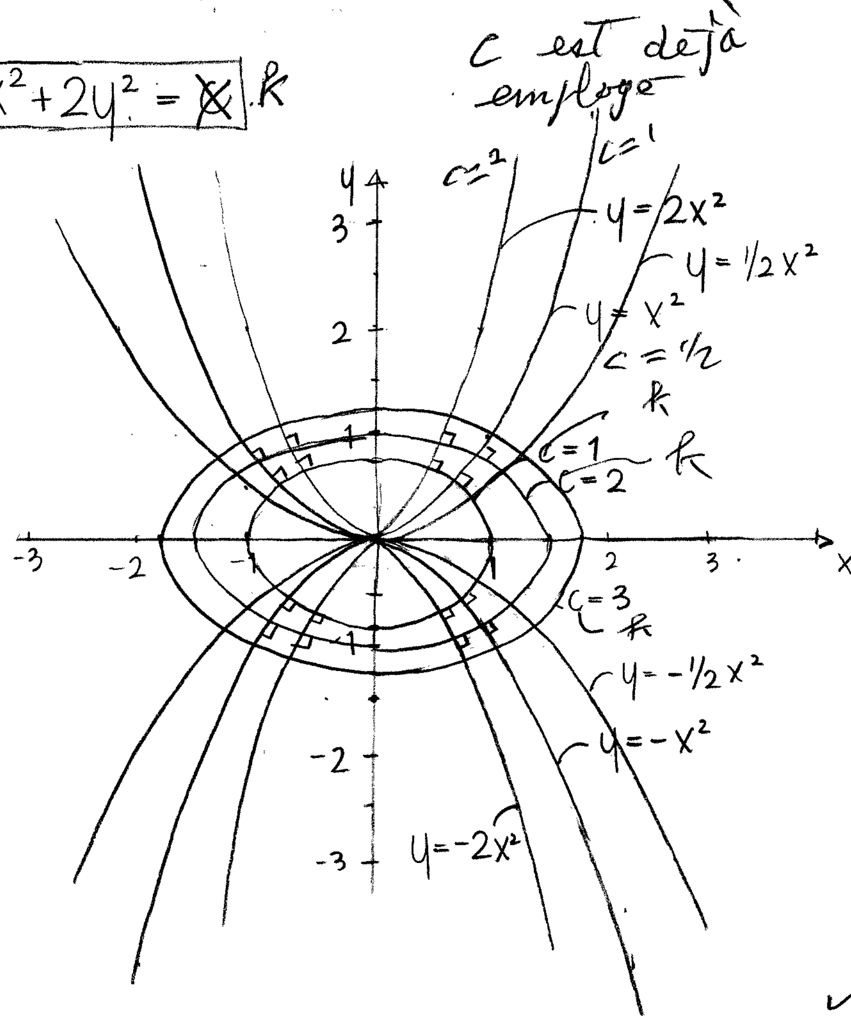
$0^2 + 2y^2 = 2$

$y = \pm 1$

$\rightarrow x^2 + 2y^2 = 2$

$x^2 + 0^2 = 2$

$x = \pm\sqrt{2}$



3.3 Soit $y' = -y + x + 1$ avec $y(0) = 1$. Trouver $y_1(x)$, $y_2(x)$ et $y_3(x)$ par la récurrence de Picard.

$\partial_y f(x,y) = -1$

$$y_n(x) = y_0 + \int_{x_0}^{x} f(t, y_{n-1}(t))\,dt, \quad n = 1, 2, \ldots$$

On prend $x_0 = 0$ et $y_0 = 1$

$$y_1(x) = 1 + \int_0^x (\underbrace{-1 + t + 1}_{-y_0(t) + t + 1})\,dt$$

$$= 1 + \int_0^x t\,dt = 1 + \frac{t^2}{2} = \boxed{1 + \frac{x^2}{2}}$$

$$y_2(x) = 1 + \int_0^x -y_1(t) + t + 1$$

$$= 1 + \int_0^x \left(-1 - \frac{t^2}{2} + t + 1\right)dt$$

$$= 1 + \cancel{-t} - \frac{t^3}{6} + \frac{t^2}{2} + \cancel{t} = 1 - \frac{t^3}{6} + \frac{t^2}{2} = \boxed{1 - \frac{x^3}{6} + \frac{x^2}{2}}$$

$$y_3(x) = 1 + \int_0^x -y_2(t) + t + 1\,dt$$

$$= 1 + \int_0^x \left(-1 + \frac{t^3}{6} - \frac{t^2}{2} + t + 1\right)dt$$

$$= 1 + \cancel{-t} + \frac{t^4}{24} - \frac{t^3}{6} + \frac{t^2}{2} + \cancel{t} = \boxed{1 + \frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{2}}$$

✓

$\boxed{3.4}$ - Exercice 2.2 $y'' + 2y' + y = 0$

Equation charactéristique: $\lambda^2 + 2\lambda + 1 = (\lambda + 1)^2 = 0$

$\lambda_1 = \lambda_2 = -1 \Rightarrow$ cas III

Solution générale $\boxed{y(x) = c_1 e^{-x} + c_2 x e^{-x}}$ ✓

$\boxed{3.5}$ - Exercice 2.8 $y'' + 2y' + 2y = 0$, $y(0) = 2$, $y'(0) = -3$

Equation charactéristique: $\lambda^2 + 2\lambda + 2 = 0$

$$\lambda_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i \quad (\text{cas II}).$$

$y_1(x) = e^{-x} \cos x$ & $y_2(x) = e^{-x} \sin x$

Solution générale: $y(x) = c_1 e^{-x} \cos x + c_2 e^{-x} \sin x$

$$y(0) = c_1 e^0 \cos(0) + \cancel{c_2 e^0 \sin(0)} = \boxed{2 = c_1}$$

$$\Rightarrow y(x) = 2e^{-x} \cos x + c_2 e^{-x} \sin x$$

$$y'(x) = -2e^{-x} \cos x - 2e^{-x} \sin x - c_2 e^{-x} \sin x + c_2 e^{-x} \cos x$$

$$= -2e^{-x}(\cos x + \sin x) - c_2 e^{-x}(\sin x - \cos x)$$

$$y'(0) = -2e^0(\cos(0) + \sin(0)) - c_2 e^0(\sin(0) - \cos(0)) = -3$$

$$= -2 + c_2 = -3$$

$$= \boxed{c_2 = -1}$$

Solution unique: $\boxed{y(x) = 2e^{-x} \cos x - e^{-x} \sin x}$ ✓

$\boxed{3.6}$ Exercice 2.12 $y'' + 6y' + 9y = 0$, $y(0) = 0$
$y'(0) = 2$

Eq. char : $\lambda^2 + 6\lambda + 9\lambda = (\lambda+3)^2 = 0$
$\lambda_{1,2} = -3 \Rightarrow$ Cas III

Solution générale : $y(x) = c_1 e^{-3t} + c_2 x e^{-3t}$

$y(0) = c_1 e^0 + c_2 (0) e^0 = \boxed{0 = c_1}$
$\Rightarrow y(x) = c_2 x e^{-3t}$

$y'(x) = c_2 e^{-3t} - 3c_2 x e^{-3t}$
$y'(0) = c_2 e^0 - 3c_2 (0) e^0 = \boxed{2 = c_2}$

Solution unique : $\boxed{y(\cancel{x}) - 2te^{-3t}}$
t

X

$y(t)$ est max lorsque $y'(t) = 0$ (pente)

$y'(t) = 2e^{-3t} - 6te^{-3t}$
$0 = 2e^{-3t} - 6te^{-3t}$
$2\cancel{e^{-3t}} = 6t\cancel{e^{-3t}}$
$\boxed{t = 1/3}$

Max : $y(1/3) = 2(1/3)e^{-3(1/3)}$
$= \boxed{0,2453}$

✓

3.7 - Exercice 7.16 $\quad x_{n+1} = \sqrt{2x_n^2 + 3}$

n	x_n	Δx_n	$\Delta^2 x_n$
1	$x_1 = 4,000000$		
		1,916080	
2	$x_2 = 5,916080$		0,711844
		2,627924	
3	$x_3 = 8,544004$		

$$a_1 = x_1 - \frac{(\Delta x_1)^2}{\Delta^2 x_1} = 4.000000 - \frac{1,916080^2}{0,711844} = \boxed{-1,157538}$$

3.8 - Exercice 7.17 + 7.8 $\quad f(x) = x^3 + 9x - 9 = 0$

$$x_{n+1} = g(x_n) = \frac{9 - x_n^3}{9}$$

$$g(x) = \frac{9 - x^3}{9} \qquad |g'(x)| = \left| -\frac{3x^2}{9} \right| < \frac{1}{3} \rightarrow \text{converge d'ordre 1.}$$

n	x_n	S_n	$g(x) = x - \frac{x^3+9x-9}{3x+9}$
0	0,500000	0,500000	0,500000
1	0,986111	0,908288	0,948718
2	0,893455	0,914905	0,915183
3	0,920755	$\boxed{0,914905}$	0,914908
4	0,913266		$\boxed{0,914908}$
5	0,915365		

$S_0 = x_0 = 0.500000$

$z_1 = g(S_0) = x_1 = 0,986111 \quad , \quad z_2 = g(z_1) = x_2 = 0,893455$

$$S_1 = S_0 - \frac{(z_1 - S_0)^2}{z_2 - 2z_1 + S_0} = 0,908288$$

$\boxed{n=1}$ $z_1 = g(s_1) = 0{,}916741$

$z_2 = g(z_1) = 0{,}914395$

$$s_2 = s_1 - \frac{(z_1 - s_1)^2}{z_2 - 2z_1 + s_1} = 0{,}914905$$

$\boxed{n=2}$ $z_1 = g(s_2) = 0{,}914909$

$z_2 = g(z_1) = 0{,}914908$

$$s_3 = s_2 - \frac{(z_1 - s_2)^2}{z_2 - 2z_1 + s_2} = 0{,}914905$$

Converge d'ordre $\boxed{2}$ p.c.q. $|g'(0.91)| < 1$ ✓

Par la méthode de NEWTON: $g(x) = x - \dfrac{f(x)}{f'(x)}$

$$g(x) = x - \frac{x^3 + 9x - 9}{3x^2 + 9}$$

$x_0 = 0{,}500000$

$$x_1 = g(x_0) = x_0 - \frac{x_0^3 + 9x_0 - 9}{3x_0^2 + 9} = 0{,}948718$$

$x_2 = g(x_1) = \ldots$

Newton converge <u>aussi</u> d'ordre $\boxed{2}$. p.c.q. $f'(0.91) \neq 0$