MAT 2784 B

MAT2784 - DEVOIR #2 2010.01.26

$\boxed{2.1}$ $-\pi \sin(\pi x)\sinh(y)\,dx + \cos(\pi x)\cosh(y)\,dy = 0$

$M\,dx = -\pi \sin(\pi x)\sinh(y)\,dx$ $N\,dy = \cos(\pi x)\cosh(y)\,dy$
$M_y = -\pi \sin(\pi x)\cosh(y)$ $N_x = -\pi \sin(\pi x)\cosh(y)$
$\Rightarrow \boxed{M_y = N_x}$

$u(x,y) = \int \cos(\pi x)\cosh(y)\,dy + T(x)$
$= \cos(\pi x)\sinh(y) + T(x)$
$u_x(x,y) = -\pi \sin(\pi x)\sinh y + T'(x) = -\pi \sin(\pi x)\sinh y$
$\Rightarrow T'(x) = 0$
$T(x) = \int 0\,dx = \cancel{k}\ 0.$ PAS DE CONST ICI
$u(x,y) = \cos(\pi x)\sinh y = c$ CONST. ICI
Solution générale $\boxed{\cos(\pi x)\sinh(y) = c}$

✓

$\boxed{2.2}$ $(e^y - ye^x)dx + (xe^y - e^x)dy = 0$

$M(x,y)dx = (e^y - ye^x)dx$ $\quad$ $N(x,y)dy = (xe^y - e^x)dy$

$M_y = e^y - e^x$ $\quad$ $N_x = e^y - e^x$

$\Rightarrow \boxed{M_y = N_x}$

$u(x,y) = \int (xe^y - e^x)\,dy + T(x)$

$= xe^y - ye^x + T(x)$

$u_x(x,y) = e^y - ye^x + T'(x) = (e^y - ye^x)$

$\Rightarrow T'(x) = 0$

$T(x) = \int 0\,dx = c$

Solution Générale $\boxed{xe^y - ye^x = c}$

$\underline{y(0) = 1}$ $\Rightarrow$ $(0)e^{(1)} - (1)e^{(0)} = c$

$0 - 1 = c$

$\boxed{-1 = c}$

Solution unique $\boxed{xe^y - ye^x + 1 = 0}$ ✓

$\boxed{2.3}$ $e^x(\cos y\, dx - \sin y\, dy) = 0$

$e^x \cos y\, dx - e^x \sin y\, dy = 0$

$M(x,y)\,dx = e^x \cos y\, dx$ $\quad$ $N(x,y)\,dy = -e^x \sin y\, dy$

$M_y = -e^x \sin y$ $\quad$ $N_x = -e^x \sin y$

$\rightarrow \boxed{M_y = N_x}$

$u(x,y) = \int e^x \cos y\, dx + T(y)$

$= e^x \cos y + T(y)$

$u_y(x,y) = -e^x \sin y + T'(y) = -e^x \sin y$

$\rightarrow T'(y) = 0$

$T(y) = \int 0\, dy = C$

Solution générale $\boxed{e^x \cos y = C}$ ✓

$\boxed{2.4}$ $\left(2x+\frac{1}{y}-\frac{y}{x^2}\right)dx+\left(2y+\frac{1}{x}-\frac{x}{y^2}\right)dy=0$

$M(x,y)\,dx=\left(2x+\frac{1}{y}-\frac{y}{x^2}\right)dx$

$M_y=-\frac{1}{y^2}-\frac{1}{x^2}$

$N(x,y)\,dy=\left(2y+\frac{1}{x}-\frac{x}{y^2}\right)dy$

$N_x=-\frac{1}{x^2}-\frac{1}{y^2}$

$\boxed{M_y=N_x}$

$u(x,y)=\int\left(2y+\frac{1}{x}-\frac{x}{y^2}\right)dy+T(x)$

$=y^2+\frac{y}{x}+\frac{x}{y}+T(x)$ (-1)

$u_x(x,y)=-\frac{y}{x^2}+\frac{1}{y}+T'(x)=2x+\frac{1}{y}-\frac{y}{x^2}$

$T'(x)=2x$

$T(x)=\int 2x\,dx=\boxed{x^2}$

Solution générale $\boxed{y^2+\frac{y}{x}+\frac{x}{y}+x^2=C}$

$\boxed{2.5}$ $(e^{x+y} - y)\,dx + (x e^{x+y} + 1)\,dy = 0$

$M(x,y)\,dx = (e^{x+y} - y)\,dx \qquad N(x,y)\,dy = (x e^{x+y} + 1)\,dy$

$M_y = e^{x+y} - 1 \qquad N_x = e^{x+y}(x+1)$

$$\frac{M_y - N_x}{N} = \frac{(\cancel{e^{x+y}} - 1) - (x e^{x+y} + \cancel{e^{x+y}})}{x e^{x+y} + 1} = -\frac{\cancel{(1 + x e^{x+y})}}{\cancel{x e^{x+y} + 1}} = -1 = f(x)$$

$\mu(x) = e^{\int -1\,dx} = \boxed{e^{-x}}$

$$\mu M\,dx + \mu N\,dy = e^{-x}(e^{x+y} - y)\,dx + e^{-x}(x e^{x+y} + 1)\,dy = 0$$
$$= (e^{y} - y e^{-x})\,dx + (x e^{y} + e^{-x})\,dy = 0$$

$u(x,y) = \int (x e^{y} + e^{-x})\,dy + T(x)$

$= x e^{y} + y e^{-x} + T(x)$

$u_x(x,y) = e^{y} - y e^{-x} + T'(x) = e^{y} - y e^{-x}$

$T'(x) = 0$

$T(x) = \int 0\,dx = c$

Solution générale $\boxed{x e^{y} + y e^{-x} = c}$

$\boxed{2.6}$ $(x^4+y^2)dx - xy\,dy = 0 \quad , \; y(2) = 1$

$M(x,y)dx = x^4+y^2\,dx \qquad N(x,y)dy = -xy\,dy$

$M_y = 2y \qquad\qquad N_x = -y$

$$\frac{M_y - N_x}{N} = \frac{(2y-(-y))}{-xy} = \frac{3\cancel{y}}{-x\cancel{y}} = -\frac{3}{x} = f(x).$$

$$\mu(x) = e^{-3\int \frac{1}{x}dx} = e^{-3\ln x} = \boxed{x^{-3}}$$

$$\mu M dx + \mu N dy = x^{-3}(x^4+y^2)dx - x^{-3}(xy)\,dy = 0$$
$$= \left(x + \frac{y^2}{x^3}\right)dx - \frac{y}{x^2}\,dy = 0$$

$$u(x,y) - \int -\frac{y}{x^2}\,dy + T(x) = -\frac{y^2}{2x^2} + T(x)$$

$$u_x(x,y) = \frac{y^2}{x^3} + T'(x) = x + \frac{y^2}{x^3}$$

$$T'(x) = x$$
$$T(x) = \int x\,dx = \frac{x^2}{2}$$

Solution générale : $\dfrac{-y^2}{2x^2} + \dfrac{x^2}{2} = C$

$$\Rightarrow \frac{-(1)^2}{2(2)^2} + \frac{(2)^2}{2} = C$$
$$-\frac{1}{8} + 2 = \frac{15}{8}$$

Solution implicite : $\boxed{\dfrac{-y^2}{2x^2} + \dfrac{x^2}{2} = \dfrac{15}{8}}$ ✓

$$-y^2 + x^4 = \frac{15x^2}{4}$$
$$-4y^2 + 4x^4 = 15x^2$$

$\boxed{2.7}$ $f(x) = x^4 - x + 0{,}2 = 0$, voisin de 1 , $x_0 = 1$

$$
\begin{aligned}
g_1(x) &= x - f(x) \\
&= x - (x^4 - x + 0{,}2) \\
&= 2x - x^4 - 0{,}2
\end{aligned}
$$

$|g'(x)| = |2 - 4x^3| > 1$: Répulsif.

$$
\begin{aligned}
g_2(x) \Rightarrow \quad x^4 &= x - 0{,}2 \\
x &= \sqrt[4]{x - 0{,}2} \quad \Rightarrow g_2(x) = \sqrt[4]{x-0{,}2} = (x-0{,}2)^{1/4}
\end{aligned}
$$

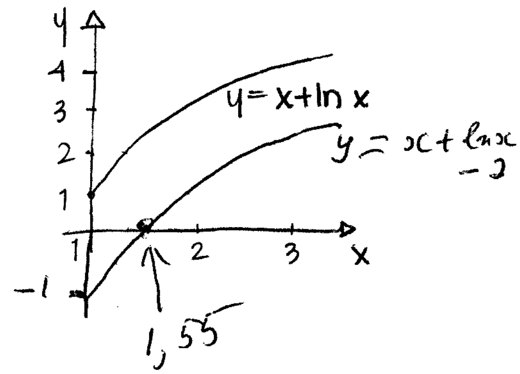
$|g'(x)| = \left|\frac{1}{4}(x - 0{,}2)^{-3/4}\right| < 1$: attractif.

$$
\begin{aligned}
x_0 &= 1{,}000000 \\
x_1 &= (x_0 - 0{,}2)^{1/4} = 0{,}945742 \\
x_2 &= (x_1 - 0{,}2)^{1/4} = 0{,}929281 \\
x_3 &= \ldots = 0{,}924110 \\
x_4 &= \ldots = 0{,}922468 \\
x_5 &= \ldots = 0{,}921944 \\
&\vdots \\
x_{11} &= 0.921698\,978 \\
x_{12} &= 0.921698\,801
\end{aligned}
$$

Solution exacte $\boxed{0{,}921699}$

~~Il fa~~ donner les résultats jusqu'à 6 décimales.

$\boxed{2.8}$ $y = x + \ln x$



$\rightarrow x + \ln x = 2 \quad , \quad x_0 = 2$

$f(x) = x + \ln x - 2 = 0$

☆ AVEC NEWTON.

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{(x + \ln x - 2)}{(1 + \frac{1}{x})}$$

$x_0 = 2{,}000\,000$

$$x_1 = x_0 - \frac{x_0 + \ln x_0 - 2}{1 + \frac{1}{x_0}} = 1{,}537902$$

$x_2 = \ldots = 1{,}557099$

$x_3 = \ldots = 1{,}557146$

$x_4 = \ldots = 1{,}557146$

Solution exacte $\boxed{1{,}557146}$