MAT2784: Assignment #1.

$\boxed{1.1}$ $y' = 2xy^2$

$$\frac{dy}{dx} = 2xy^2 \Rightarrow \int \frac{dy}{y^2} = \int 2x \, dx$$

$$-\frac{1}{y} = x^2 + C \Rightarrow \boxed{y = \frac{-1}{x^2 + C}}$$

$\boxed{1.16}$ $(x^2+y^2)\,dx - 2xy\,dy = 0 \qquad y(1) = 2$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$
$$M(\lambda x, \lambda y) = (\lambda x)^2 + (\lambda y)^2 = \lambda^2(x^2+y^2) \Rightarrow \lambda^2 M(x,y)$$
$$N(\lambda x, \lambda y) = 2(\lambda x)(\lambda y) = \lambda^2(2xy) \Rightarrow \lambda^2 N(x,y)$$

☆ $y = ux \qquad dy = x\,du + u\,dx \qquad \Rightarrow u = \frac{y}{x}$

$$(x^2 + u^2x^2)\,dx - 2x^2u\,[x\,du + u\,dx] = 0$$
$$\cancel{x^2}(1+u^2)\,dx - 2\cancel{x^2}u\,[x\,du + u\,dx] = 0$$
$$(1+u^2)\,dx - 2xu\,du - 2u^2\,dx = 0$$
$$(1-u^2)\,dx - 2xu\,du = 0$$
$$2xu\,du = (1-u^2)\,dx$$
$$\int \frac{2u\,du}{(1-u^2)} = \int \frac{dx}{x} \qquad \Rightarrow \begin{array}{l} 1-u^2 = v \\ 2u\,du = -dv \end{array} \qquad \Rightarrow \int \frac{-dv}{v} = \int \frac{dx}{x}$$
$$-\ln(1-u^2) = \ln|x| + c_1$$
$$-\ln\left(x(1-u^2)\right) + c_1$$

$$e^{\ln\left(x - \frac{y^2}{x}\right)} + e^{c} = 0 \Rightarrow x - \frac{y^2}{x} + e^c = 0 \Rightarrow y^2 = x^2 + xe^c$$
$$(2)^2 = (1)^2 + (1)e^c$$
$$4 = 1 + e^c$$
$$\boxed{e^c = 3}$$

$$\boxed{y^2 = x^2 + 3x}$$

$\boxed{1.17}$ $x(2x^2+y^2)+y(x^2+2y^2)y'=0$

$x(2x^2+y^2)\,dx+y(x^2+2y^2)\,dy=0$
$M(x,y)\,dx+N(x,y)\,dy=0$
$M(\lambda x,\lambda y)\Rightarrow \lambda^{3}M(x,y)$
$N(\lambda x,\lambda y)\Rightarrow \lambda^{3}N(x,y).$

☆ $y=ux \qquad dy=x\,du+u\,dx$

$\cancel{x}(2x^2+(ux)^2)\,dx+u\cancel{x}(x^2+2(ux)^2)[x\,du+u\,dx]=0$
$(2\cancel{x^2}+u^2\cancel{x^2})\,dx+(u\cancel{x^2}+2u^3\cancel{x^2})[x\,du+u\,dx]=0$
$(2+u^2)\,dx+(ux+2u^3x)\,du+(u^2+2u^4)\,dx=0$
$2(1+u^2+u^4)\,dx+x(u+2u^3)\,du=0$
$2(1+u^2+u^4)\,dx=-x(u+2u^3)\,du$
$-\dfrac{2\,dx}{x}=\dfrac{(u+2u^3)\,du}{(1+u^2+u^4)} \qquad \Rightarrow 1+u^2+u^4=v$
$\qquad\qquad \cancel{2}u+\cancel{4}u^3=dv \rightarrow u+2u^3=\dfrac{dv}{2}$

$\Rightarrow \displaystyle\int -\frac{2\,dx}{x}=\int\frac{dv}{2v} \quad\rightarrow -2\int\frac{dx}{x}=\frac{1}{2}\int\frac{dv}{v}$
$-2\ln|x|=\dfrac{\ln(1+u^2+u^4)}{2} \quad\rightarrow \ln(1+u^2+u^4)+\ln|x^4|=C_1$
$\ln((1+u^2+u^4)x^4)=C_1$
$e^{\ln\left(\left(1+\left(\frac{y}{x}\right)^2+\left(\frac{y}{x}\right)^4\right)x^4\right)}=e^{C_1}\rightarrow \boxed{x^4+x^2y^2+y^4=C}$

✓

$\boxed{\#1.26}$ $(x^2 - 2y)\,dx + x\,dy = 0$

$M = x^2 - 2y \qquad N = x$

$\frac{\partial M}{\partial y} = -2 \qquad \frac{\partial N}{\partial x} = 1 \qquad \Rightarrow -2 \neq 1$

Facteur d'intégration :

$\frac{M_y - N_x}{N} = \frac{(-2-1)}{x} = \frac{-3}{x} = f(x)$

$\mu(x) = e^{\int -\frac{3}{x} dx} = e^{-3 \ln x} = x^{-3} = \boxed{\frac{1}{x^3}}$

$\mu M\,dx + \mu N\,dy = \frac{x^2 - 2y}{x^3}\,dx + \frac{x}{x^3}\,dy = 0$

$= \frac{1 - 2y}{x}\,dx + \frac{1}{x^2}\,dy = 0$

$u(x,y) = \int \frac{1}{x^2}\,dy + T(x) = \frac{y}{x^2} + T(x)$

$u_x(x,y) = -\frac{2y}{x} + T'(x) = \frac{1}{x} - \frac{2y}{x}$

$\int T'(x) = \int \frac{1}{x} \qquad T(x) = \ln|x|$

$u(x,y)\ \frac{y}{x^2} + \ln|x| = c$?

$\boxed{\cancel{e^{\frac{y}{x^2}} + x = c}}$

(−2)

$\boxed{1.32}$ $(x+\sin x+\sin y)dx + \cos y\, dy = 0$

$M = x+\sin x+\sin y$ $\quad N = \cos y$

$M_y = \cos y$ $\quad N_x = 0$

$\frac{M_y - N_x}{N} = \frac{(\cos y - 0)}{\cos y} = 1 = f(x)$

$\mu(x) = e^{\int 1 dx} = \boxed{e^x}$

$\mu M dx + \mu N dy = e^x(x+\sin x+\sin y)dx + e^x \cos y\, dy = 0$

$u(x,y) = \int e^x \cos y\, dy + T(x) = e^x \sin y + T(x)$

$u_x(x,y) = e^x \sin y + T'(x) = e^x x + e^x \sin x + e^x \sin y$

$\int T'(x) = \int \underset{①}{e^x x} + \int \underset{②}{e^x \sin x}$ $\quad$ *$\int uv' = uv - \int u'v$

① $u = x \quad dv = e^x dx \Rightarrow \int xe^x dx = xe^x - \int 1 \cdot e^x dx$

$du = 1dx \quad v = e^x \quad\quad = xe^x - e^x = e^x(x-1)$

② $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b\cos bx) + C$

$\Rightarrow \int e^x \sin(x)\, dx = \frac{e^x}{2}(\sin x - \cos x) + c$

$$\boxed{e^x \sin y + e^x(x-1) + \frac{e^x}{2}(\sin x - \cos x) = C}$$

✓

D 1.5

$\boxed{1.33}$ $y' + \frac{2}{x} y = 12$

$y' + f(x) y = r(x) \Rightarrow f(x) = \frac{2}{x}$

$\mu(x) = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = \boxed{x^2}$

$(\mu y)' = \mu r(x) \Rightarrow (x^2 y)' = 12x^2 + c$

$x^2 y = \int 12 x^2 dx + c$

$\frac{x^2 y(x)}{x^2} = \frac{4x^3}{x^2} + c$

$\boxed{y(x) = 4x + c} / x^2$

-2

$\boxed{7.8}$ $x_{n+1} = g(x)$, $g(x) = \frac{\cos^2 x}{2}$, $x_0 = 2$

$|g'(x)| = |-\cos x \sin x|$

$- \left|\frac{-1}{2} \sin 2x\right| \leq \frac{1}{2}$

$x_0 = 2{,}000\,000$

$x_1 = \frac{\cos^2(x_0)}{2} = 0{,}086589$

$x_2 = \frac{\cos^2(x_1)}{2} = 0{,}496261$

$x_3 = \ldots = 0{,}386645$

$x_4 = \ldots = 0{,}428904$

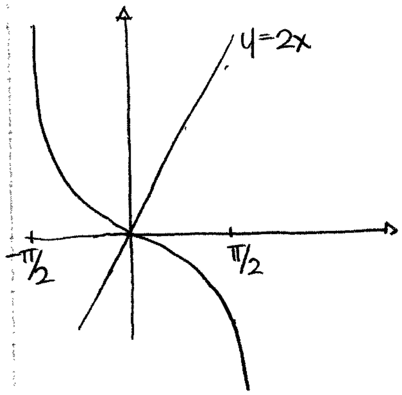
$x_5 = \ldots = 0{,}413524$

La solution exacte $\boxed{0{,}417715}$

$g'(p) = \frac{-1}{2} \sin 2(p) = -0{,}370793 \neq 0$

L'ordre de convergence $\boxed{1}$

$\boxed{7.10}$ $f(x) = 2x - \tan(x)$, $f(x) = 0$, $x_0 = 1$



$f'(x) = 2 - \sec^2 x$

$$x_{n+1} = x_n - \frac{2x_n - \tan x_n}{2 - \sec^2 x_n} = g(x)$$

n	x_n	$\lvert x_n - x_{n-1} \rvert$
	$x_0 = 1.000000$	
	$x_1 = x_0 - \frac{2x_0 - \tan x_0}{2 - \sec^2 x_0} = 1{,}310478$	0,310478
	$x_2 = 1{,}223929$	0,086549
	$x_3 = 1{,}176051$	0,047878
	$x_4 = 1{,}165927$	0,010124
	$x_5 = 1{,}165562$	0,000365
	$x_6 = 1{,}165561$	0,000001
	$x_7 = 1{,}165561$	0,000000

Solution exacte $\boxed{p = 1{,}165561}$

$f'(p) = 2 - \sec^2(p) = -4{,}434126 \neq 0$

Ordre de convergence $\boxed{2}$