[redacted]

MAT2784 - Devoir 6

10 Mars 2010

6,8) $f(t) = 2e^{-2t}\sin t$

$$\mathcal{L}(\sin t)(s) = \frac{1}{s^2+1}$$

Donc, $\boxed{F(s) = \dfrac{2}{(s+2)^2+1}}$

6,20) $F(s) = \dfrac{3s-5}{s^2+4}$

$$F(s) = \frac{3s}{s^2+4} - \frac{5}{s^2+4}$$

$$= 3\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - 5\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}$$

$$= 3(\cos 2t) - 5\left(\tfrac{1}{2}\sin 2t\right)$$

$$= 3\cos 2t - \tfrac{5}{2}\sin 2t$$

Donc, $\boxed{f(t) = 3\cos 2t - \frac{5}{2}\sin 2t}$

D 6.2

MAT2784 - Devoir 6

[redacted]

10 Mars 2010

6.28) $F(s) = \frac{1 + 2e^{-2s} + e^{-4s}}{s+2}$

$= \frac{1}{s+2} + \frac{2e^{-2s}}{s+2} + \frac{e^{-4s}}{s+2}$

$\therefore \quad f(t) = L^{-1}\left(\frac{1}{s+2}\right)(t) + 2 \cdot L^{-1}\left(\frac{e^{-2s}}{s+2}\right) + L^{-1}\left(\frac{e^{-4s}}{s+2}\right)$

Donc,

$$\boxed{f(t) = e^{-2t} + 2u(t-2)e^{-2(t-2)} + u(t-4)e^{-4(t-4)}}$$

[redacted]

D6.3

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10 Mars 20

6.41) $y'' - 6y' + 13y = 0$, $y(0) = 0$
$y'(0) = -3$

On pose $\mathcal{L}(y)(s) = Y(s)$

On a donc,

$$s^2 Y(s) - s y(0) - y'(0) - 6[sY(s) - y(0)] + 13 Y(s) = 0$$
$$(s^2 - 6s + 13) Y(s) = s y(0) - y'(0) - 6 y(0)$$
$$= s(0) - (-3) - 6(0)$$
$$= 3$$

On isole pour $Y(s)$:

$$Y(s) = \frac{3}{s^2 - 6s + 13}$$

$$Y(s) = \frac{3}{(s-3)^2 + 4}$$

Donc, $\boxed{y(t) = \frac{3}{2} e^{3t} \sin 2t}$

D6.4

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[redacted]

10 Mars 201

<u>6.46)</u> $y'' + 2y' + 5y = 4t$, $y(0)=0$
$y'(0)=0$

On pose $\mathcal{L}(y)(s) = Y(s)$

$$s^2Y(s) - sy'(0) - y(0) + 2[sY(s) - y(0)] + 5Y(s) = 4/s^2$$
$$(s^2+2s+5)Y(s) - \cancel{sy'(0)}^{=0} - y(0)^{=0} - 2y(0)^{=0} = 4/s^2$$

$$\therefore \quad (s^2+2s+5)Y(s) = \frac{4}{s^2}$$

On isole pour $Y(s)$:

$$Y(s) = \frac{4}{s^2(s^2+2s+5)}$$
$$= \frac{4}{s^2((s+1)^2+4)}$$

$$Y(s) = \frac{A+Bs}{s^2} + \frac{C+Ds}{s^2+2s+5}$$

$$Y(s) = As^2 + 2sA + 5A + s^3B + 2s^2B + 5sB + s^2C + s^3D$$

$$4 = s^3(B+D) + s^2(A+2B+C) + s(2A+5B) + 5A$$

Donc, on obtient $A = 4/5$

$2A+5B=0 \Rightarrow B = (-8/5)1/5 = -8/25$
$B+D=0 \Rightarrow D = -B = 8/25$
$A+2B+C=0 \Rightarrow C = -A-2B = -4/5 + 16/25$
$= -4/25$

D6.5

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[redacted]

10 Mars 2010

6.46) $Y(s) = \dfrac{4/5 - 8s/25}{s^2} + \dfrac{-4/25 + 8s/25}{s^2+2s+5}$

$= 4/5s^2 - 8/25s - 4/25((s+1)^2+4) + 8s/25((s+1)^2+4)$

$= 4/5s^2 - 8/25s - 4/25((s+1)^2+4) + \dfrac{8(s+1)}{25((s+1)^2+4)} - \dfrac{8}{25((s+1)^2+4)}$

$= \frac{4}{5}t - 8/25 - 4/25 \cdot (1/2)(e^{-t}\sin 2t) + (8/25)e^{-t}\cos 2t$
$- (8/25)(1/2)\, e^{-t}\sin 2t$

$$\boxed{y(t) = \frac{-6}{25}e^{-t}\sin 2t + \frac{8}{25}e^{-t}\cos 2t + \frac{4t}{5} - \frac{8}{25}}$$

D6.6 [redacted]

MAT2784 - Devoir 6

6,48) $y'' + 4y = \begin{cases} 1 & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$ $\quad y(0) = 0$, $y'(0) = -1$ 10 mars 2016

$$\mathcal{L}(y) = Y(s)$$
$$\mathcal{L}(y'' + 4y) = s^2 Y(s) - sY(0) - Y'(0) + 4Y(s)$$
$$G(s) = (s^2+4)Y_s + 1$$

$$Y(s) = \frac{G(s)-1}{s^2+4}$$

$$g(t) = 1 - u(t-1)(1) + u(t-1)(0)$$
$$= 1 - u(t-1)$$

$$G(s) = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$Y(s) = \frac{\left(1/s - \frac{e^{-s}}{s}\right) - 1}{s^2+4}$$
$$= \frac{1}{s(s^2+4)} - \frac{e^{-s}}{s(s^2+4)} - \frac{1}{s(s^2+4)}$$

$$\frac{1}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} = \frac{As^2 + 4A + Bs^2 + Cs}{s(s^2+4)}$$

On trouve donc que $A = 1/4$, $B = -1/4$ et $C = 0$

$$Y(s) = \frac{1}{4s} - \frac{s}{4(s^2+4)} - e^{-s}\left[\frac{1}{4s} - \frac{s}{4(s^2+4)}\right] - \frac{1}{s^2+4}$$

$$y(t) = 1/4 - (1/4)\cos 2t - u(t-1)\left[1/4 - 1/4(\cos(2(t-1))\right] - 1/2(\sin 2t)$$

$$\therefore y(t) = \begin{cases} 1/4 - (1/4)\cos 2t - (1/2)\sin 2t & \text{si } 0 \le t < 1 \\ -(1/4)\cos 2t + (1/4)\cos(2t-2) - 1/2 \sin 2t & \text{si } t \ge 1 \end{cases}$$

Graphique sur prochaine page $\Rightarrow$

D6.7

[redacted]

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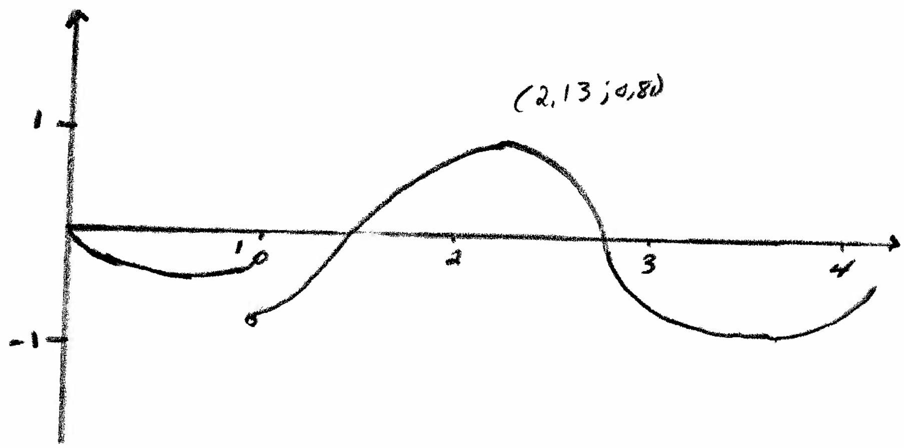
6,48 (Graphique)



[redacted]

D6.8

MAT2784- Devoir 6

10,2) $f(x) = x^2 e^{-x}$ $\quad h=0{,}4,\ h/2=0{,}2$ et $h/4=0{,}1$ $\quad$ 10 Mars 20

$$N_1(h) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right]$$

$$N(0{,}4) = \frac{1}{0{,}8}\left[f(1{,}8) - f(1{,}0)\right] = 0{,}209611$$

$$N(0{,}2) = \frac{1}{0{,}4}\left[f(1{,}6) - f(1{,}2)\right] = 0{,}207839$$

$$N(0{,}1) = \frac{1}{0{,}2}\left[f(1{,}5) - f(1{,}3)\right] = 0{,}207321$$

$$N_2(h) = N_1\left(\frac{h}{2}\right) + \frac{N_1(h/2) - N_1(h)}{3}$$

$$N_2(0{,}4) = N_1(0{,}2) + \frac{N_1(0{,}2) - N_1(0{,}4)}{3} = 0{,}207248$$

$$N_2(0{,}2) = N_1(0{,}1) + \frac{N_1(0{,}1) - N_1(0{,}2)}{3} = 0{,}207148$$

$$N_3(h) = N_2\left(\frac{h}{2}\right) + \frac{N_2(h/2) - N_2(h)}{15}$$

Donc, $N_3(0{,}4) = N_2(0{,}2) + \dfrac{N_2(0{,}2) - N_2(0{,}4)}{15}$

$$\boxed{= 0{,}207141}$$

D6.8 [redacted]

MAT2784 - Devoir 6

10 Mars 2010

10,9) $\int_1^3 \ln x \, dx \qquad a=1,\ b=3$

Alors $f(x) = \ln x$ $\longrightarrow$ $|f''(1)| = 1$

$f'(x) = 1/x$ $\qquad |f''(3)| = 1/9$

$f''(x) = -1/x^2$

On voit donc que $M = \left|\max_{1\leq x\leq 3} f''(x)\right| = |f''(1)| = 1$

On commence par la méthode des points milieux (composés):

$$|E_m| \leq \frac{M}{24}(b-a)h^2 = 1/12\ h^2 \leq 10^{-3}$$

$h^2/12 \leq 10^{-3}$

$h^2 \leq 0{,}012$

$h \leq 0{,}1095$

Aussi, $nh = (b-a)$

$n = 2/h$

$\therefore n = 18{,}26$

Pour assurer une approximation à 10^{-3}, on arrondit à la hausse.

Donc, $n = 19$.

$\therefore h = 2/19 = \boxed{0{,}105}$

Ensuite, la méthode des trapèzes:

$$\left|\frac{(b-a)h^2}{12} f''(\xi)\right| \leq \frac{2h^2}{12}M = \frac{h^2}{6}(1)$$

$h^2/6 \leq 10^{-3}$

$h^2 \leq 0{,}006$

$h \leq 0{,}0775$

Pour la même raison que tout à l'heure, $n = 26$.

Aussi $nh = (b-a)$

$n = 2/h$

$\therefore n = 25{,}82$

$\therefore h = \frac{2}{n} = \boxed{0{,}0769}$

Suite sur la prochaine page ⇒

10.9 (Suite)

Finalement, la méthode de Simpson (composé):

$f'''(x) = 2/x^3 \qquad f''''(x) = -6/x^4$

Donc, on a $\quad M = \left|\max_{1\le x\le 3} f''''(x)\right| = |f''''(1)| = 6$

$$\left|\frac{(b-a)h^4}{180} f''''(\varepsilon)\right| \le \frac{2h^4}{180}(6) = \frac{h^4}{15}$$

$$\therefore h^4/15 \le 10^{-3}$$

$$h^4 \le 0{,}015$$

$$h \le 0{,}34996$$

Aussi, $nh = (b-a)$

$n = 2/h$

$n = 5{,}71$ Encore pour la même raison, $n=6$.

On a donc, $h = \dfrac{2}{n} = \boxed{\dfrac{1}{3}}$