D2.1

MAT2784B
27/01/10

DEVOIR II

1.27 $(x^2-y^2+x)dx + 2xydy = 0$

$M(x,y) = x^2 - y^2 + x$

$N(x,y) = 2xy$

$M_y = -2y \neq N_x = 2y$ $\therefore$ equation pas exact

$$\frac{M_y - N_x}{N} = \frac{-2y-2y}{2xy} = -\frac{2}{x}$$

$$\begin{aligned}\mu(x) &= e^{\int -\frac{2}{x}dx} \\ &= e^{-2\int \frac{1}{x}dx} \\ &= e^{-2\ln|x|} \\ &= x^{-2} \Rightarrow 1/x^2\end{aligned}$$

$$x^{-2}\left((x^2-y^2+x)dx + (2xy)dy\right) = 0$$

$$(1 - y^2x^{-2} + x^{-1})dx + (2x^{-1}y)dy = 0$$

$M_y = -2yx^{-2} = N_x = -2yx^{-2}$ Equation exacte

$$\begin{aligned}u(x,y) &= \int N(x,y)dy + T(x) \\ &= \int 2x^{-1}ydy + T(x) \\ &= 2x^{-1}\frac{y^2}{2} + T(x) \\ &= x^{-1}y^2 + T(x)\end{aligned}$$

$$u_x(x,y) = -x^{-2}y^2 + T'(x) = -y^2x^{-2} + x^{-1} + 1$$

$$T'(x) = x^{-1} + 1$$

$$T(x) = \ln|x| + x$$

$\therefore$ $\frac{y^2}{x} + \ln|x| + x - C = 0$

1.31 $(2x^2y - 2y + 5)\,dx + (2x^3 + 2x)\,dy = 0$

$\underset{M}{\shortparallel}$ $\qquad\qquad$ $\underset{N}{\shortparallel}$

$M_y = 2x^2 - 2 \qquad N_x = 6x^2 + 2 \qquad M_y \neq N_x \Rightarrow$ Pas exacte

Facteur d'intégration

$$\frac{M_y - N_x}{N} = \frac{(2x^2-2)-(6x^2+2)}{2x^3+2x} = \frac{-4(\cancel{x^2+1})}{2x(\cancel{x^2+1})} = \frac{-2}{x} = f(x)$$

$$e^{\int f(x)\,dx} = e^{\int -\frac{2}{x}\,dx} = e^{-2\ln|x|} = x^{-2}$$

L'équation initiale devient:

$$x^{-2}\left[(2x^2y - 2y + 5)\,dx + (2x^3 + 2x)\,dy = 0\right]$$

$$\underbrace{\left(2y - \frac{2y}{x^2} + \frac{5}{x^2}\right)}_{P(x,y)}dx + \underbrace{\left(2x + \frac{2}{x}\right)}_{Q(x,y)}dy = 0$$

$P_y = 2 - \frac{2}{x^2} \qquad Q_x = 2 - \frac{2}{x^2} \qquad P_y = Q_x \Rightarrow$ équation exacte

$$\mathcal{U}(x,y) = \int\left(2y - \frac{2y}{x^2} + \frac{5}{x^2}\right)dx = 2xy + \frac{2y}{x} - \frac{5}{x} + T(y)$$

$$\mathcal{U}_y(x,y) = \cancel{2x} + \cancel{\frac{2}{x}} + T'(y) = Q(x,y) = 2x + \frac{2}{x}$$

$$\Rightarrow T'(y) = 0 \qquad \Rightarrow T(y) = c_1$$

$$\mathcal{U}(x,y) = 2xy + \frac{2y}{x} - \frac{5}{x} = C$$

1.3A $y' + \frac{2x}{x^2+1} y = x$ Nonhomogène

$f(x) = \frac{2x}{x^2+1}$ ; $r(x) = x$

$$\mu(x) = e^{\int \frac{2x}{x^2+1} dx}$$
$$= e^{\int \frac{du}{u}}$$
$$= e^{\ln u}$$
$$= u$$

$u = x^2+1$
$du = 2x\,dx$

$$\boxed{\mu(x) = x^2+1}$$

$$(\mu(x)y)' = \mu(x)r(x)$$
$$(\mu(x)y(x))' = \mu(x)r(x)$$
$$(\mu(x)y(x))' = (x^2+1)(x)$$
$$\int (\mu(x)y(x))'dx = \int (x^3+x)dx + C$$
$$\mu(x)y(x) = \int (x^3+x)dx + C$$
$$y(x) = \frac{1}{x^2+1} \int (x^3+x)dx + C$$
$$y(x) = \frac{1}{x^2+1}\left(\frac{x^4}{4} + \frac{x^2}{2} + C\right)$$
$$= \frac{x^4}{4x^2+4} + \frac{x^2}{2x^2+2} + \frac{C}{x^2+1}$$
$$= \frac{x^4 + 2x^2 + 4C}{4x^2+4}$$

✓

1.38 $xy' - 2y = 2x^4$ condition $y(2) = 8$

$$y' - \frac{2y}{x} = 2x^3$$

$$f(x) = \frac{-2}{x} \quad ; \quad r(x) = 2x^3$$

$$\mu(x) = e^{\int \frac{-2}{x} dx}$$
$$= e^{-2\int \frac{1}{x} dx}$$
$$= e^{-2\ln|x|}$$
$$= e^{\ln|x|^{-2}}$$
$$\boxed{\mu(x) = |x|^{-2}}$$

$$(\mu(x)y)' = \mu(x)r(x)$$
$$(\mu(x)y(x))' = \mu(x)r(x)$$
$$\int (\mu(x)y(x))' dx = \int \mu(x)r(x) + C$$
$$y(x) = \frac{1}{\mu(x)} \int \mu(x)r(x) + C$$
$$= \frac{1}{x^{-2}} \left( (x^{-2})(2x^3) + C \right)$$
$$y(x) = x^4 + x^2 C$$

Avec condition

$$8 = 2^4 + 2^2 C$$
$$C = -2$$

$$\therefore \; y(x) = x^4 - 2x^2$$

✓

1.40 $y' - y\tan x = \frac{1}{\cos^3 x}$ condition $y(0)=0$

$f(x) = -\tan x$ ; $r(x) = \frac{1}{\cos^3 x}$

$$\mu(x) = e^{\int -\tan x\,dx}$$
$$= e^{\ln|\cos x|}$$
$$\mu(x) = \cos x$$

$$(\mu(x)y)' = \mu(x)r(x)$$
$$(\mu(x)y(x))' = \mu(x)r(x)$$
$$\int (\mu(x)y(x))'dx = \int \mu(x)r(x)\,dx + c$$
$$y(x) = \frac{1}{\mu(x)}\int \mu(x)r(x)dx + c$$
$$= \frac{1}{\cos x}\int \frac{\cos x}{\cos^3 x}\,dx + C$$
$$= \frac{1}{\cos x}\int \frac{1}{\cos^2 x}\,dx + C$$
$$y(x) = \frac{\tan x}{\cos x} + \frac{C}{\cos x}$$

Condition $y(0)=0$ * en RAD

$$0 = \frac{\tan(0)}{\cos(0)} + \frac{C}{\cos(0)}$$
$$C = 0$$

$$\therefore\ y(x) = \frac{\tan x}{\cos x}$$

n

1.42 $y = e^x + c$

$y - e^x = c$ $\qquad$ c = constante

$dy - e^x dx = 0$

$\frac{dy}{dx} = e^x$ $\qquad$ $\therefore\ m = e^x$

$\therefore\ m_{ortho} = -1/e^x$

$e^x dx = dy$

$\int e^x dx + c = \int dy$

$y = e^x + c$

$M_y = N_x$ équation exacte

$\frac{dy_{ortho}}{dx} = \frac{-1}{e^x}$

$dy_{ortho} = \frac{-dx}{e^x}$

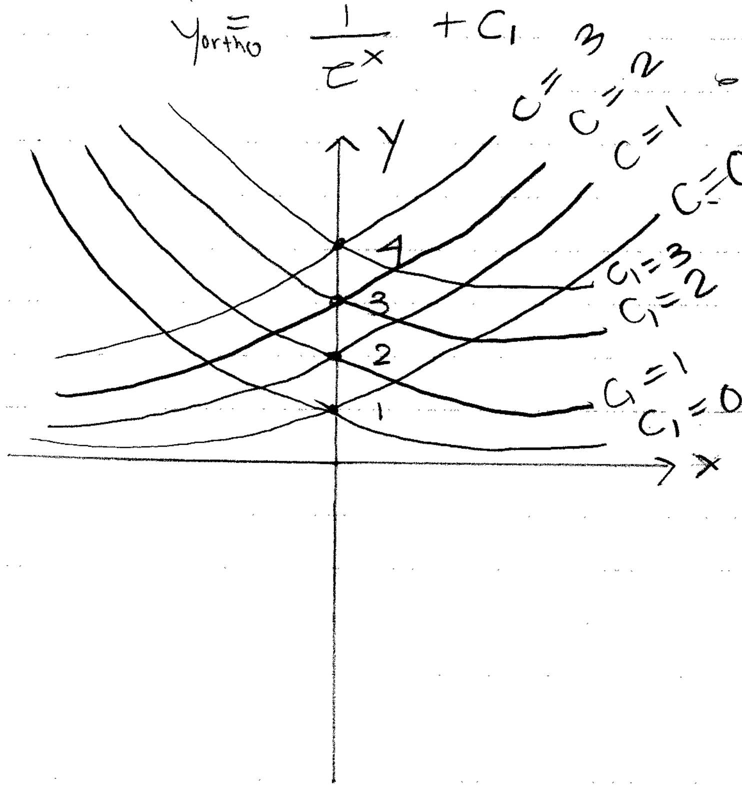
$\int dy_{ortho} = \int \frac{-1}{e^x}\, dx + c_1$

$y_{ortho} = e^{-x} + c_1$

$y_{ortho} = \frac{1}{e^x} + c_1$

$\therefore\ y = e^x + c$

$y_{ortho} = \frac{1}{e^x} + c$



8.10. $f(x) = 2x - \tan x$
$f(x) = 0$ à 6 décimales près
Démarrer en $x_0 = 1$ * angle en radian

| $n$ | $x_n$ | $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ | $\lvert x_n - x_{n-1}\rvert$ |
|---|---|---|---|
| 0 | 1 | 1,310 478 | |
| 1 | 1,31047803 | 1,223 929 | 0,310478 |
| 2 | 1,22392909 | 1,176 050 | 0,086549 |
| 3 | 1,176 0509 | 1,165 926 | 0,047878 |
| 4 | 1,165 92651 | 1,165 561 | 0,0101244 |
| 5 | 1,165 56164 | 1,165 561 | $3,64872 \cdot 10^{-4}$ |
| 6 | 1,165 56119 | 1,165 561 | $4,5079 \cdot 10^{-7}$ |

$(\tan x)^2$

$f(x_n) = 2x_n - \tan x_n$ $\qquad f''(p) = -2\sec^3 x_n \cdot \tan x_n$

$f'(x_n) = 2 - \sec^2 x_n$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \left(\frac{2x - \tan x}{2 - \sec^2 x}\right)$$

$p = 1,165 561$

$$f(p) = 2p - \tan p = 0$$
$$= 2(1,165561) - \tan(1,165561)$$
$$= 8,21 \cdot 10^{-7}$$
$$\simeq 0$$

$$f'(p) = 2 - \sec^2(1,165561)$$
$$= -4,434126$$

$$f''(p) = -2\sec^3(1,165561)\tan(1,165561)$$
$$= -76,090280$$

∴ Converge au moins d'ordre 2 $\Rightarrow$

8.12 $\quad f(x) = (e^{-x}) - 1$

$$x_{n+1} = x_n - \left(\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right) f(x_n)$$

$$x_2 = 0{,}5 - \left(\frac{0{,}5 - 1}{f(0{,}5) - f(1)}\right) f(0{,}5)$$

$=$

| n | $x_{n+1}$ | $\lvert x_n - x_{n+1}\rvert$ |
|---|---|---|
| 0 | 1 | / |
| 1 | 0,5 | 0,5 |
| 2 | 0,737117 | 0,237117 |
| 3 | -0,4657816 | 1,202903 |
| 4 | 0,1743806 | 0,113291 |
| 5 | 0,038388 | 0,135992 |
| 6 | 0,004502 | 0,033886 |
| 7 | 0,000087 | 0,004415 |
| 8 | 0,000000195 | 0,0000868 |

$f'(x) = -e^{-x}$
$f'(0) = -1$
$\neq 0$
Donc zéro simple
Donc Newton converge d'ordre 2.

La fonction exponentielle converge vers 0 lorsque $x=0$
ordre ~~2~~ 2

MAIS SI NOUS reprenons le 8.12 avec la fonction
$f(x) = 2x - \tan(x) \quad x_0 = 0 \quad x_1 = 0{,}5$

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

| n | $x_{n+1}$ | $\lvert x_n - x_{n+1}\rvert$ |
|---|---|---|
| 0 | 1 | / |
| 1 | 0,5 | / |
| 2 | 20,927192 | 20,427192 |
| 3 | 0,285427 | 20,641714 |
| 4 | 0,153446 | 0,132031 |
| 5 | 0,000705 | 0,160497 |
| 6 | 0,000053 | 0,007105 |
| 7 | $-8{,}834 \cdot 10^{-8}$ | $5{,}338 \cdot 10^{-6}$ |
| 8 | 0,000000 | $1 \cdot 10^{-8}$ |

$\therefore x = 0{,}000\,000$

$f'(x) = 2 - \sec^2(x)$
$f'(0) = 1 \neq 0$

Tel que vue en classe convergence devrait être 1,618.

RACINE SIMPLE
convergence ordre 1.