D 1.1

MATH 2784 B
2010/01/20

Devoir I

$\frac{10}{10}$

1.2 $y' = \frac{xy}{x^2-1}$

$$\frac{dy}{dx} = \frac{xy}{x^2-1}$$

$$\frac{dy}{y} = \frac{x\,dx}{x^2-1}$$

$$\int \frac{dy}{y} = \int \frac{x\,dx}{x^2-1}$$

$$\int \frac{dy}{y} = \int \frac{1}{x^2-1} \cdot x\,dx \quad + C$$

$$\mathrm{Ln}|y| = \frac{1}{2}\mathrm{Ln}|x^2-1| + C$$

$$e^{\mathrm{Ln}|y|} = e^{1/2\,\mathrm{Ln}\,|x^2-1|\,+C}$$

$$y = e^{1/2\,\mathrm{Ln}(x^2-1)}\,e^{C} \qquad e^{c} = K$$

$$y = e^{1/2\,\mathrm{Ln}\,|x^2-1|} \cdot K$$

$$y = e^{\mathrm{Ln}\,|x^2-1|^{1/2}} \cdot K$$

$$y = \sqrt{(x^2-1)} \cdot K$$

$\therefore$ La solution générale : $y = K \cdot \sqrt{(x^2-1)}$

D1.2

1.9 $(x^2-3y^2)dx + 2xy\,dy = 0$ eq. pas exacte

$M_y = -6y \neq N_x = 2y$

$\frac{M_y - N_x}{N} = \frac{-8y}{2xy} = -4/x = f(x)$

$\mu(x) = e^{\int -4/x \, dx} = e^{-4\ln x} = e^{\ln x^{-4}} = x^{-4}$

$\mu M dx + \mu N dx = x^{-4}(x^2-3y^2)dx + x^{-4}(2xy)dy = 0$ (eq. exacte)

$u(x,y) = \int (2x^{-3}y)\,dy + T(x)$

$= x^{-3}y^2 + T(x)$ $T'(x) = x^{-2}$

$u_x(x,y) = -3x^{-4}y^2 + T'(x) = \mu M$ $T(x) = -x^{-1} = -1/x$

$= -3x^{-4}y^2 + x^{-2}$

$u(x,y) = x^{-3}y^2 - 1/x$

sol. générale : $\boxed{\frac{y^2-x^2}{x^3} = C}$

✓

1.15 $yy' = -(x+2y)$, $y(1) = 1$

$$y\frac{dy}{dx} + (x+2y) = 0 \quad \therefore \quad y\,dy + (x+2y)\,dx = 0$$

Par la substitution : $y = xu$, $dy = x\,du + u\,dx$

$$\therefore xu[x\,du + u\,dx] + (x + 2xu)\,dx = 0$$

$$\therefore \cancel{x}[ux\,du + u^2dx] + \cancel{x}(1+2u)\,dx = 0$$

$$\therefore ux\,du + (u^2+2u+1)\,dx = 0$$

$$\therefore \frac{u}{u^2+2u+1}\,du + \frac{dx}{x} = 0$$

$$\therefore \int \frac{u}{(u+1)^2}\,du = -\int \frac{dx}{x} + c_1$$

Par le formulaire $\int \frac{u}{(u+1)^2}\,du = \frac{1}{u+1} + \ln(u+1)$

$$\therefore \frac{1}{u+1} + \ln(u+1) = -\ln(x) - c_1$$

$$\therefore e^{\frac{1}{y/x+1} + \ln(y/x+1)} = e^{\ln(x)^{-1} - c_1}$$

$$\therefore e^{\frac{1}{y/x+1}}\, e^{\ln(y/x+1)} = e^{\ln(x)^{-1}}\, e^{-c_1}$$

$$e^{\frac{1}{y/x+1}}\left(\frac{y}{x}+1\right) = \frac{C}{x} \quad \text{où } C = e^{-c_1}$$

avec $y(1) = 1$ on a que $\boxed{2e^{1/2} = C}$

$$e^{\frac{1}{y/x+1}}\left(y/x+1\right) = \frac{2e^{1/2}}{x}$$

✓

1.18 $(3x^2y^2-4xy)y' + 2xy^3 - 2y^2 = 0$

$(2xy^3-2y^2)dx + (3x^2y^2-4xy)dy = 0$

$M_y = 6xy^2-4y \quad = \quad N_x = 6xy^2-4y$ eq. exacte

$u(x,y) = \int M(x,y)\,dx + T(y)$

$= \int 2xy^3 - 2y^2\,dx + T(y)$

$= x^2y^3 - 2xy^2 + T(y)$

$u_y(x,y) = 3x^2y^2 - 4xy + T'(y) = N \qquad T'(y) = 0$

$= 3x^2y^2 - 4xy + 0 \qquad T(y) = 0$

$u(x,y) = \boxed{x^2y^3 - 2xy^2 = C}$ ⇐ Sol. gén.

1.20 $\left(\frac{\sin 2x}{y}+x\right)dx + \left(y-\frac{\sin^2 x}{y^2}\right)dy = 0$

$M_y = -\frac{\sin 2x}{y^2} = N_x = -\frac{2\sin x \cos x}{y^2} = -\frac{\sin 2x}{y^2}$ éq. exacte

$u(x,y) = \int M(x,y)dx + T(y)$

$= \int \frac{\sin 2x}{y} + x \, dx + T(y)$ $\quad u = 2x$, $du = 2$

$= \frac{\sin^2 x}{y} + \frac{1}{2}x^2 + T(y)$

$u_y(x,y) = -\frac{\sin^2 x}{y^2} + T'(y) = N$ $\quad T'(y) = y$, $T(y) = \frac{1}{2}y^2$

$= -\frac{\sin^2 x}{y^2} + y$

$u(x,y) = \frac{\sin^2 x}{y} + \frac{1}{2}x^2 + \frac{1}{2}y^2$

$\boxed{\frac{\sin^2 x}{y} + \frac{1}{2}(x^2+y^2) = C}$ sol. gén

1.22 $\frac{2x}{y^3}dx + \frac{(y^2-3x^2)}{y^4}dy = 0$, $y(1) = 1$

$M_y = -\frac{6x}{y^4} = N_x = -\frac{6x}{y^4}$ éq. exacte

$u(x,y) = \int M(x,y)dx + T(y)$

$= \int \frac{2x}{y^3}dx + T(y)$

$= \frac{x^2}{y^3} + T'(y)$

$u_x(x,y) = -\frac{3x^2}{y^4} + T'(y)$

$= -\frac{3x^2}{y^4} + \frac{1}{y^2}$

$T'(y) = \frac{1}{y^2} \Rightarrow T(y) = -\frac{1}{y}$

$u(x,y) = \frac{x^2}{y^3} - \frac{1}{y} = C$

SOL. GÉN.

8.4 $\sqrt{3}$ $\cdot 10^{-4}$

$f(x) = x^2 - 3$

n	x_n	a_n	b_n	$\lvert x_{n-1} - x_n \rvert$	$f(x_n)$	$f(a_n)$
0		1	2			−
1	1,5	1,5	2	0,5	−	−
2	1,75	1,5	1,75	0,25	+	−
3	1,625	1,625	1,75	0,125	−	−
4	1,6875	1,6875	1,75	0,0625	−	−
5	1,71875	1,71875	1,75	0,03125	−	−
6	1,734375	1,71875	1,734375	0,01563	+	−
7	1,7265625	1,7265625	1,734375	0,00781	−	−
8	1,73046875	1,73046875	1,734375	0,00391	−	−
9	1,732421875	1,73046875	1,732421875	0,00195	+	−
10	1,731445313	1,731445313	1,732421875	0,00098	−	−
11	1,731932503	1,731932503	1,732421875	0,00019	−	−
12	1,732177189	1,731932503	1,732177189	0,00024	−	−
13	1,732054846	1,731932503	1,732054846	0,00012	+	−
14	1,731993675	1,731993675	1,732054846	0,00006	−	−

La réponse est $\sqrt{3} \approx 1,731993675$ à 10^{-4} près
On remarque que la racine es dans l'intervalle

$[1,731993675 \; ; \; 1,732054846]$

✓

8.6 $f(x) = x^3 - x - 1 = 0$ $[1,2]$ $x_0 = 1$

$x^3 = x+1$ $g(x) = \sqrt{\frac{x+1}{x}}$

$x^2 = \frac{x+1}{x}$

$x = \sqrt{\frac{x+1}{x}}$ $g(1) = \sqrt{\frac{2}{1}} = \sqrt{2}$ $g(2) = \sqrt{\frac{3}{2}}$ et donc $g(1)$ et $g(2)$ sont dans $[1,2]$

$$g'(x) = \frac{1}{2}\left(\frac{x+1}{x}\right)^{-1/2} \cdot \left(\frac{1}{x} + \frac{-(x+1)}{\cancel{x^2}}\right)$$

$$= \frac{1}{2}\left(\frac{x+1}{x}\right)^{-1/2} \cdot \frac{-1}{x^2}$$

$$= \frac{1}{2}\sqrt{\frac{1}{\left(\frac{x+1}{x}\right)}} \cdot \frac{-1}{x^2}$$

$$= \frac{-1}{2x^2}\sqrt{\frac{1}{\left(\frac{x+1}{x}\right)}}$$

$$\left.\begin{array}{l} g'(1) = \left|\frac{-1}{2} \cdot \sqrt{\frac{1}{2}}\right| = 0,353553391 \\ g'(2) = \left|\frac{-1}{8} \cdot \sqrt{\frac{1}{3/2}}\right| = 0,102 \end{array}\right\} 0 < K < 1$$

$g(x) = \sqrt{\frac{x+1}{x}}$

$g(x_0) = g(1) = \sqrt{\frac{2}{1}} = 1,414213562$

$g(x_1) = 1,306562965 = x_2$

$g(x_2) = 1,3286710 9 = x_3$

$g(x_3) = 1,323869976 = x_4$

$g(x_4) = 1,3249 0045 = x_5$

$\boxed{1,32 \text{ est la réponse}}$

✓