MAT 2784 A
DEVOIR 3

D3.1

Exercice #2.2 p 289

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$$y'' + 2y' + y = 0 \quad (1)$$

posons $y = e^{\lambda x}$

$y' = \lambda e^{\lambda x}$

$y'' = \lambda^2 e^{\lambda x}$

donc (1) Devient

$$e^{\lambda x}(\lambda^2 + 2\lambda + 1) = 0$$

$e^{\lambda x} \neq 0$ donc $\lambda^2 + 2\lambda + 1 = 0$

$$(\lambda+1)(\lambda+1) = 0$$

$\lambda_{1,2} = -1$ et on $\Delta = b^2 - 4ac = 2^2 - 4 \times 1 = 0$

alors $y_1(x) = e^{-x}$

$y_2(x) = x e^{-x}$

Donc la solution générale est

$$y(x) = c_1 e^{-x} + c_2 x e^{x}.$$

Exercice # 28 p 283

① $y'' + 2y' + 2y = 0$ avec $y(0) = 2$, $y'(0) = -3$

Posons $y = e^{\lambda x}$
$y' = \lambda e^{\lambda x}$
$y'' = \lambda^2 e^{\lambda x}$

donc ① devient $e^{\lambda x}(\lambda^2 + 2\lambda + 2) = 0$
$\lambda^2 + 2\lambda + 2 = 0$

$$\lambda_{1,2} = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i \quad (\Delta < 0)$$

donc $\left.\begin{array}{l} \lambda_1 = -1 + i = \alpha + i\beta \\ \lambda_2 = -1 - i = \alpha - i\beta \end{array}\right\}$ avec $\begin{array}{l} \alpha = -1 \\ \beta = 1 \end{array}$

alors
$$y_1 = e^{\alpha x} \cos \beta x = e^{-x} \cos x$$
$$y_2 = e^{\alpha x} \sin \beta x = e^{-x} \sin x$$

donc la solution générale est

$$y(x) = C_1 e^{-x} \cos x + C_2 e^{-x} \sin x$$

$$y'(x) = -C_1 e^{-x} \cos x - C_1 e^{-x} \sin x - C_2 e^{-x} \sin x + C_2 e^{-x} \cos x$$

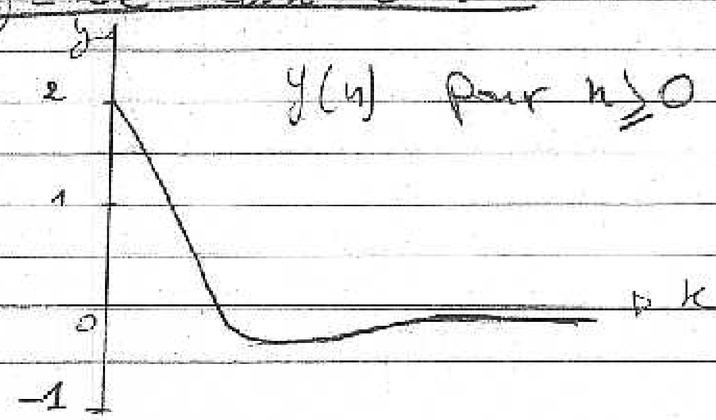
or $y(0) = C_1 = 2$.

$y'(0) = -2 + C_2 = -3 \Rightarrow C_2 = -1$

Donc la solution générale unique

$$\underline{y(x) = 2e^{-x} \cos x - e^{-x} \sin x}$$

Le graphique



Exercice #2.10 p 289.

⊙ $y'' + 16y = 0$ avec $y(0) = 0$ $y'(0) = 1$

posons $y = e^{dx}$

$y'' = d^2 e^{dx}$

⊙ devient $e^{dx}(d^2 + 16) = 0$

$d_{1,2} = \pm 4i$

donc avec $w = 4$

$y_1 = \cos wt = \cos 4t$

$y_2 = \sin wt = \sin 4t$

donc la solution generale est

$y(t) = C_1 \cos 4t + C_2 \sin 4t$

$y'(t) = -4C_1 \sin 4t + 4C_2 \cos 4t$

or

$y(0) = C_1 = 0$

$y'(0) = 4C_2 = 1 \Rightarrow C_2 = \frac{1}{4}$

donc la solution generale unique est

$$\boxed{y(t) = \frac{1}{4} \sin 4t}$$

Amplitude

$$A = \sqrt{C_1^2 + C_2^2} = \boxed{\frac{1}{4}}$$

periode $P = \frac{2\pi}{w} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

Exercice #2 13 p 289

(1) $x^2 y'' + 3xy' - 3y = 0$

posons

$$y = x^m$$
$$y' = m x^{m-1}$$
$$y'' = m(m-1) x^{m-2}$$

(1) devient

$$x^m \left(m(m-1) + 3m - 3 \right) = 0$$
$$m^2 + 2m - 3 = 0$$
$$(m-1)(m+3) = 0$$
$$m_1 = 1$$
$$m_2 = 3$$

solution generale.

$$y(x) = C_1 x^{m_1} + C_2 x^{m_2} = C_1 x^1 + C_2 x^{-3}$$

Alors $\boxed{y(x) = C_1 x + C_2 x^{-3}}$

Exercice 2016 p 290

① $x^2y''+xy'+4y=0$

posons $y=x^m$

$y'=mx^{m-1}$

$y''=m(m-1)x^{m-2}$

① devient $x^m\left(m(m-1)+m+4\right)=0$

$m^2+4=0$

$m_{1,2}=\pm 2i \implies \beta=2$ et $\alpha=0$

$y_1(x)=x^{\alpha}\cos(\beta \ln x)$ et $y_2=x^{\alpha}\sin(\beta \ln x)$

$y_1(x)=\cos(2\ln x)$ et $y_2=\sin(2\ln x)$

donc la solution generale

$$\boxed{y(x)=C_1\cos(2\ln x)+C_2\sin(2\ln x)}$$

Exercice # 2.17 p

① $x^2y'' + 4xy' + 2y = 0$ $\quad y(1)=1$ et $y'(1)=2$

posons $y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1)x^{m-2}$

① devient $x^m\left(m(m-1)+4m+2\right)=0$

$$m^2+3m+2=0$$

$$m_{1,2}=\frac{-3\pm\sqrt{9-8}}{2} \Rightarrow \begin{cases} m_1=-1 \\ m_2=-2 \end{cases}$$

Donc

$y_1(x) = x^{m_1} = x^{-1}$

$y_2(x) = x^{m_2} = x^{-2}$

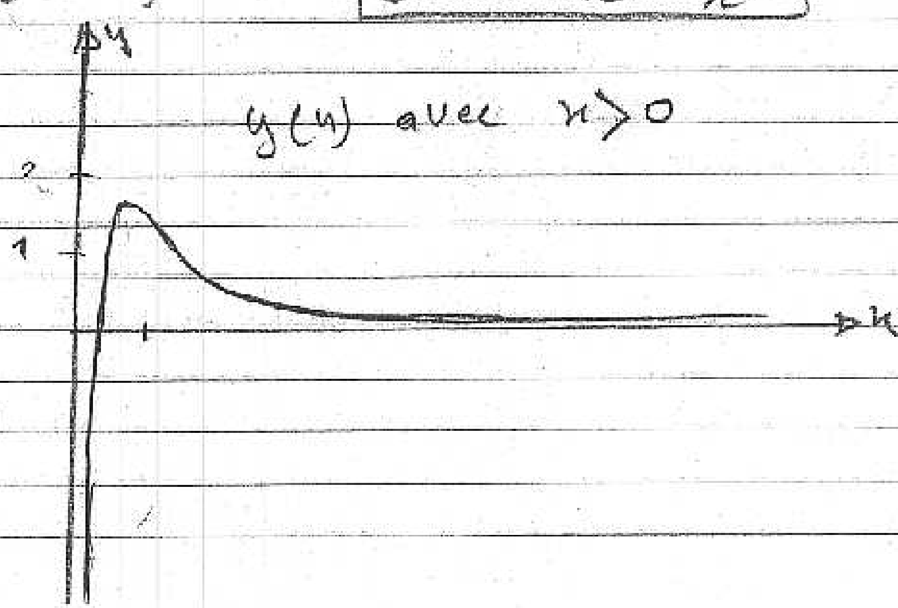
La solution générale est $y(x) = \dfrac{c_1}{x} + \dfrac{c_2}{x^2}$

$y'(x) = -c_1x^{-2} - 2c_2x^{-3}$

$y(1) = c_1 + c_2 = 1$ et $y'(1) = -c_1 - 2c_2 = 2 \Leftrightarrow c_1 = -2 - 2c_2$

on $-2-2c_2+c_2 = 1 \Leftrightarrow \boxed{c_2=-3}$ et $\boxed{c_1=4}$

Solu. unique est $\boxed{y(x) = \dfrac{4}{x} - \dfrac{3}{x^2}}$



Exercice # 8.16 p 300

n	x_n	Δx_n	$\Delta^2 x_n$
1	$x_1 = 4.000$		
		0,6834	
2	$x_2 = 3.3166$		0,4705
		0,2129	
3	$x_3 = 3.1037$		

on a $x_{n+1} = g(x_n)$ et $g(x_n) = \sqrt{2x_n + 3}$

$$x_2 = \sqrt{2x_1+3} = \sqrt{2 \times 4 + 3} = 3.3166$$
$$x_3 = \sqrt{2x_2+3} = \sqrt{2 \times 3,3166 + 3} = 3,1037.$$

$$\Delta x_1 = 4.000 - 3,3166 = 0,6834$$
$$\Delta x_2 = 3,3166 - 3,1037 = 0,2129.$$
$$\Delta^2 x_1 = 0,6834 - 0,2129 = 0,4705$$

$$a_1 = x_1 - \frac{(\Delta x_1)^2}{\Delta^2 x_1}$$

$$= 4.000 - \frac{(0,6834)^2}{0,4705}$$

$$\boxed{a_1 = 3,0074}$$

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Exercice # 8.17 p 300

on a $g(x) = 1 + \sin^2 x$ avec $x_0 = 1$

utilisons la méthode de steffensen.

$$a_n = x_n - \frac{(x_{n+1} - x_n)^2}{x_{n+2} - 2x_{n+1} + x_n} \quad \text{et} \quad S_{n+1} = S_n - \frac{(Z_1 - S_n)^2}{Z_2 - 2Z_1 + S_n}$$

$$a_0 = x_0 - \frac{(x_1 - x_0)^2}{x_2 - 2x_1 + x_0} = 1 - \frac{(0{,}708073)^2}{1{,}981273 - 2(1{,}708073) + 1} = 2{,}152905$$

on a $S_0 = x_0 = 1$

$S_1 = a_0 = 2{,}152905$

$Z_1 = g(S_1) = 1{,}697735$

$Z_2 = g(Z_1) = 1{,}983973$

$$S_2 = S_1 - \frac{(Z_1 - S_1)^2}{Z_2 - 2Z_1 + S_1} = 1{,}873473$$

n	x_n	a_n	S_n
0	1	2.152905	1
1	1.708073	1.888486	2.152905
2	1.981273	1.894514	1.873473
3	1.840762	1.896288	1.897027
4	1.928872		1.897133
5	1.877169		1.897193

Donc la racine suivant la méthode de steffensen est $p = 1.897193$

on $g'(1.877169) = \sin(2 \times 1{,}877169)$
$= -0{,}575 \neq 0$ alors $x_{n+1} = g(x_n)$ Converge

d'ordre 1 ; donc la méthode de steffensen Converge

d'ordre 2