MAT 2784 A

D2.1

$\boxed{1\text{-}24}$ DEVOIR 2

Rémi Vaillancourt

$$(2x\cos y + 3x^2 y)dx + (x^3 - x^2\sin y - y)dy = 0 \; ; \quad y(0) = 2$$

$$Mdx + Ndy = 0$$

$$\left.\begin{aligned} M_y &= -2x\sin y + 3x^2 \\ N_x &= 3x^2 - 2x\sin y \end{aligned}\right\} M_y = N_x \Rightarrow \text{L'éq. exacte.}$$

$$u(x,y) = \int u_y(x,y)\,dy + T(x)$$

$$= \int (x^3 - x^2\sin y - y)\,dy + T(x)$$

$$= x^3 y + x^2\cos y - \frac{y^2}{2} + T(x)$$

$$u(x,y) = \cancel{3x^2y} - 2x\sin y + T'(x) = 2x\cos y + \cancel{3x^2y}$$

$$T'(x) = 0 \Rightarrow T(x) = c$$

$$u(x,y) = x^3 y + x^2\cos y - \frac{y^2}{2} = c$$

D'où la solut° générale : $\boxed{x^3 y + x^2\cos y - \frac{y^2}{2} = c}$

* Avec $y(0) = 2$

$$\Rightarrow 0^3(y) + 0^2\cos 2 - \frac{2^2}{2} = -\frac{4}{2}$$

$$\Rightarrow \boxed{c = -2}$$

Donc la solution unique $\boxed{x^3 y + x^2\cos y - \frac{y^2}{2} = -2}$ ✓

$\boxed{1.28}$

$(1-x^2y)\,dx + x^2(y-x)\,dy = 0$

$M_y = -x^2$

$N_x = 2xy - 3x^2$ } $M_y \neq N_x$ L'éq. n'est pas exacte.

$$\frac{M_y - N_x}{N} = \frac{-x^2-2xy+3x^2}{x^2y-x^3} = \frac{2x^2-2xy}{x^2y-x^3} = \frac{-2\cancel{(-x^2+xy)}}{x\cancel{(-x^2+xy)}}$$

$$= -\frac{2}{x} = f(x)$$

$$\mu(x) = e^{-\int \frac{2}{x}dx} = e^{-2\int\frac{1}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2}$$

$$\mu(x) = \frac{1}{x^2}$$

$$\Rightarrow \left(\frac{1}{x^2} - y\right)dy + (y-x)\,dy = 0$$

$$u(x,y) = \int (y-x)\,dy + T(x)$$

$$= \frac{y^2}{2} - xy + T(x)$$

$$u_x(x,y) = -\cancel{y} + T'(x) = \frac{1}{x^2} - \cancel{y}$$

$$T'(x) = \frac{1}{x^2} \Rightarrow T(x) = -x^{-1} \Rightarrow T(x) = -\frac{1}{x}$$

$$u(x,y) = \boxed{\frac{y^2}{2} - xy - \frac{1}{x} = C}$$ solut° générale

$\boxed{1.36}$ $xy' + 6y = 3x + 1$

$$x\frac{dy}{dx} + 6y = 3x + 1$$

$$x\,dy + 6y = (3x + 1 - 6y)\,dx$$

$$-(3x + 1 - 6y)\,dx + x\,dy = 0$$

$M(x,y) = -(3x + 1 - 6y)$

$N(x,y) = x$

$M_y = 6$ $\quad M_y \neq N_x$ (Équation non exacte)

$N_x = 1$

$$\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0$$

$$\frac{M_y - N_x}{N} = \frac{6-1}{x} = \frac{5}{x}$$

$\mu(x) = e^{\int f(x)dx}$

$\mu(x) = e^{5\ln|x|} = e^{\ln|x^5|} = x^5$

$\mu M = x^5(-3x - 1 + 6y) = -3x^6 - x^5 + 6x^5y$

$\mu N = x^5(x) = x^6$

$\mu M_y = 6x^5$ $\quad$ Equation Exacte

$\mu N_x = 6x^5$

2ᵉ SOLUTION

Linéaire en y !!

$y' + \frac{6}{x}y = 3 + \frac{1}{x}$

$\mu(x) = e^{6\int \frac{1}{x}dx}$

$= e^{6\ln x}$

$= e^{\ln x^6}$

$= x^6$

$(x^6 y)' = 3x + x$

$x^6 y = \frac{3}{7}x^7 + \frac{1}{6}x^6 + c$

$y = \frac{3}{7}x + \frac{1}{6} + cx^{-6}$

D2.4

$$du = \mu M dx + \mu N dy$$

$$u = \int \mu N dy + T(x)$$

$$u = \int x^6 dy + T(x)$$

$$u = \frac{x^7}{7} + T(x)$$

$$u_x = \frac{7x^6}{7} + T'(x) = x^6 + T'(x)$$

$$u_x = \mu M$$

$$x^6 + T'(x) = -3x^6 - x^5 + 6x^5 y$$

$$T'(x) = -4x^6 - x^5 + 6x^5 y$$

$$T(x) = \int (-4x^6 - x^5 + 6x^5 y)\,dx$$

$$T(x) = -\frac{4x^7}{7} - \frac{x^6}{6} + \frac{\cancel{6}x^6 y}{\cancel{6}}$$

$$T(x) = -\frac{4x^7}{7} - \frac{x^6}{6} + x^6 y$$

$$u(x,y) = \frac{x^7}{7} - \frac{4x^7}{7} - \frac{x^6}{6} + x^6 y$$

Sol Générale: $u(x,y) = -\frac{3x^7}{7} - \frac{x^6}{6} + x^6 y = C$

D25

1.37 $y' + 3x^2 y = x^2 \qquad y(0) = 2$

$(3x^2 y - x^2)dx + dy = 0$

$M = 3x^2 y - x^2 \qquad N = 1$

$M_y = 3x^2 \qquad N_x = 0$

$M_y \neq N_x$

$\frac{M_y - N_x}{N} = \frac{3x^2}{1}$

$f(x) = 3x^2$

$u(x) = e^{3\int x^2 dx}$

$= e^{x^3}$

$(3x^2 y e^{x^3} - x^2 e^{x^3})dx + e^{x^3} dy = 0 = du$

$u(x,y) = \int u_y \, dy + T(x)$

$= e^{x^3} y + T(x)$

$u(x) = 3x^2 e^{x^3} y + T'(x)$

$M = u(x)$

$T'(x) = -x^2 e^{x^3}$

$u = x^3, \ \frac{du}{dx} = 3x^2$

$\frac{du}{3} = x^2 dx$

$T(x) = -\int x^2 e^{x^3} dx$

$= \frac{1}{3} \int e^u du$

$= \frac{-e^u}{3}$

$= \frac{-e^{x^3}}{3}$

$u(x,y) = e^{x^3} y - \frac{e^{x^3}}{3} = C$

$y(0) = 2$

$C = e^{0^3} \cdot 2 - \frac{e^{0^3}}{3}$

$C = \frac{5}{3}$

$\boxed{\frac{5}{3} = e^{x^3} y - \frac{e^{x^3}}{3}}$

2e sol.

linéaire en y !!

$u(x) = e^{3\int x^2 dx}$

$= e^{x^3}$

$(e^{x^3} y)' = x^2 e^{x^3}$

$e^{x^3} y = \int x^2 e^{x^3} dx + c$

$= \frac{e^{x^3}}{3} + c$

$y = \frac{1}{3} + c e^{-x^3}$

$\updownarrow$

$\boxed{1.42}$ $y = e^x + C$

$C' = [y - e^x]'$

$C' = y' - e^{x'}$

$C' = \frac{dy}{dx} - e^x$

$0 = \frac{dy}{dx} - e^x$

$\frac{dy}{dx} = e^x$ ((la pente d'une courbe orthogonale est $-\frac{1}{m} = \frac{dx}{dy}$

donc,

$-\frac{dx}{dy} = e^x$

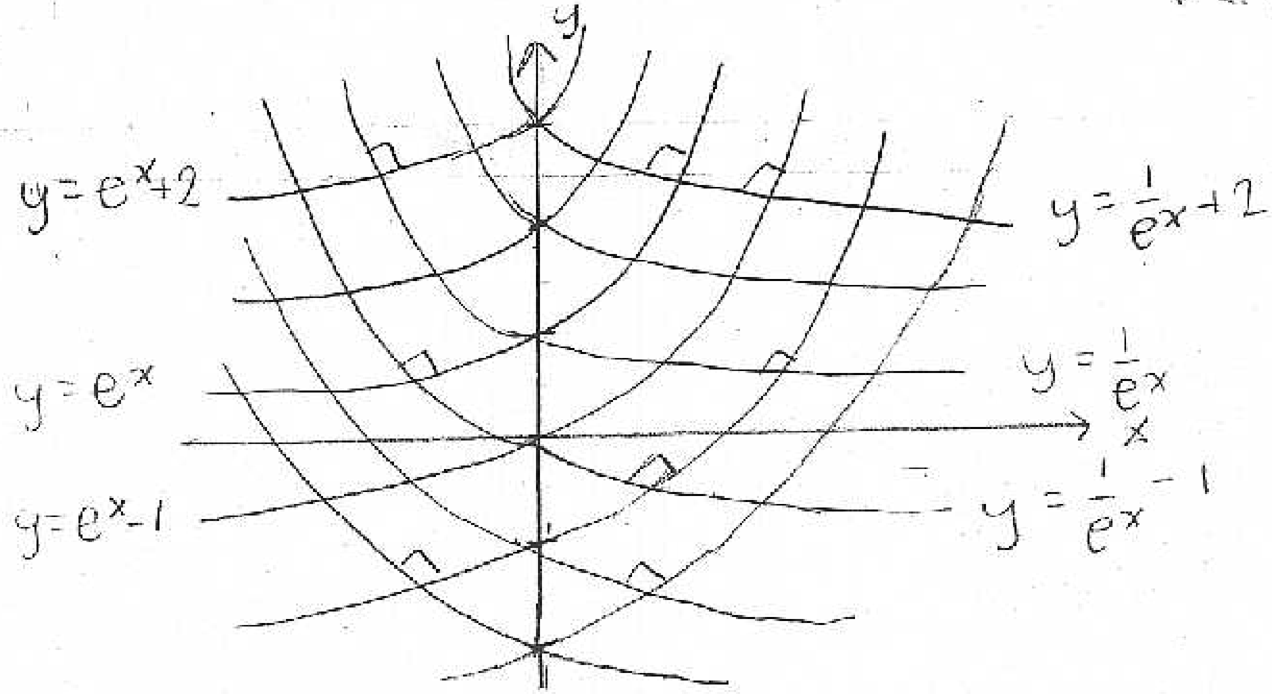
$\frac{dx}{e^x} = dy\,(-1)$

$\int \frac{dx}{e^x} = -\int dy \rightarrow \int e^{-x}dx = -\int dy$

$-\frac{1}{e^x} + C_1 = -y$

$\frac{1}{e^x} + C_1 = y$

Il est mieux d'écrire e^{-x} au lieu de $\frac{1}{e^x}$!



$\boxed{26}$ $y'' - 4y' + 3y = 0$, $y(0) = 6$, $y'(0) = 0$

Posons $y = e^{\lambda x}$
$y' = \lambda e^{\lambda x}$
$y'' = \lambda^2 e^{\lambda x}$

$\lambda^2 e^{\lambda x} - 4\lambda e^{\lambda x} + 3e^{\lambda x} = 0$
$e^{\lambda x}(\lambda^2 - 4\lambda + 3) = 0$
$P_2(x) = \lambda^2 - 4\lambda + 3$ (polynôme caractéristique)

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \lambda_{1,2}$

$\frac{4 \pm \sqrt{16 - 4(1)(3)}}{2} = \lambda_{1,2}$

$\frac{4 \pm 2}{2} = \lambda_{1,2} \Rightarrow \left.\begin{matrix}\lambda_1 = 3 \\ \lambda_2 = 1\end{matrix}\right\}$ valeurs propres

$\lambda_1 \neq \lambda_2$
$y_1(x) = e^{3x}$
$y_2(x) = e^x$

$\frac{y_1(x)}{y_2(x)} = e^{2x}$ ce n'est pas une constante $\Rightarrow$ f.n indépendantes

Solution générale :

$$y = C_1 e^{3x} + C_2 e^{x}$$

$y(x) = C_1 e^{3x} + C_2 e^{x}$
$y(0) = C_1 + C_2 = 6$
$y'(x) = 3C_1 e^{3x} + C_2 e^{x}$
$y'(0) = 3C_1 + C_2 = 0$

① $C_1 = 6 - C_2$
② $C_2 = -3C_1$

Équation 1 dans l'équation 2

$C_2 = -3(6 - C_2)$
$C_2 = -18 + 3C_2$
$18 = 2C_2$
→ $9 = C_2$

→ $C_1 = 6 - 9 = -3$

→ Solution unique :

$$y = -3e^{3x} + 9e^{x}$$

p.299 $\boxed{\#8.5}$ $x_{n+1} = \sqrt{2x_n + 3} = g(x)$

$f(x) = x^2 - 2x - 3 = 0$ converge sur [2,4]
$f(x) = (x+1)(x-3) = 0$
Racines : $x = -1$ et $x = 3$

Intervalle [2,4]
$x_n = 2 \rightarrow \sqrt{2(2)+3} = \sqrt{7} > 2$
$x_n = 4 \rightarrow \sqrt{2(4)+3} = \sqrt{11} < 4$

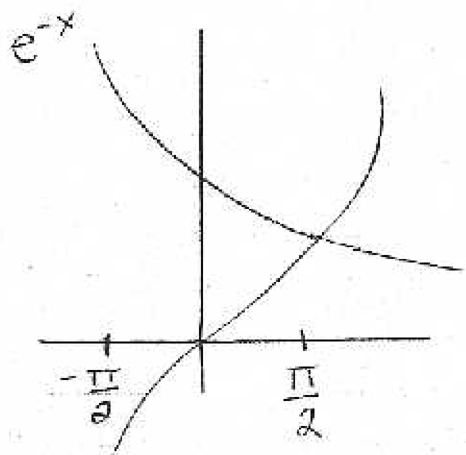
D 7.9

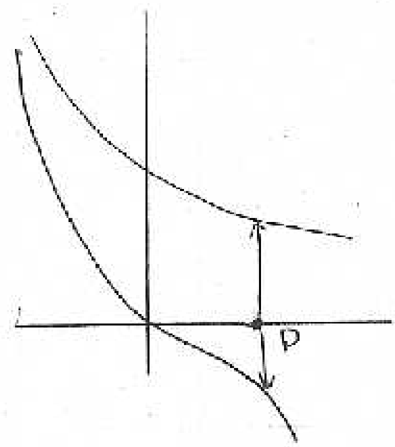
$g'(x) = \frac{1}{\sqrt{(2x_n+3)}}$ $\exists \ \forall x \in [2,4]$

$K = \frac{1}{\sqrt{7}}$ (la valeur maximale)

$0 < K < 1$ donc converge

$\boxed{8.11}$ $f(x) = e^{-x} - \tan x$





$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(x) = -e^{-x} - \sec^2 x$ $f'(p) \neq 0$ convergence d'ordre

$x_{n+1} = x_n - \frac{e^{-x_n} - \tan x_n}{-e^{-x_n} - \sec^2 x_n}$ (x est en rad)

n	x_n	$x_{n+1} - x_n$
0	1	
1	0,686421	-0,313579
2	0,541130	-0,145291
3	0,531416	-0,009714
4	0,531391	-0,000025
5	0,531391	0
6	0,531391	0

converge vers p

$p \cong 0{,}531391$