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SOLUTIONS

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MAT 2784 A

Devoir #1

#1.5

$y' \sin x = y \ln|y|$

$dy \sin x = y \ln|y| \, dx$

$\int \frac{dy}{y \ln|y|} = \int \csc x \, dx + C_1$

$\ln|\ln|y|| = \ln|\csc x - \cot x| + C_1$

$\ln|y| = C(\csc x - \cot x)$

$y = e^{C(\csc x - \cot x)}$

$y = e^{C\left(\frac{1}{\sin x} - \cos x/\sin x\right)}$

$$\boxed{y = e^{C\left(\frac{1-\cos x}{\sin x}\right)}}$$ //

#1.7

$y' \sin x - y \cos x = 0, \quad y(\pi/2) = 1$

$dy \sin x - y \cos x \, dx = 0$

$\int \frac{dy}{y} = \int \cot x \, dx + C_1$

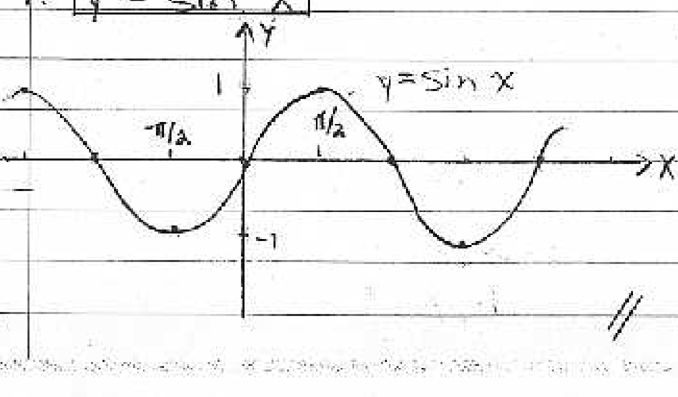
$\ln|y| = \ln|\sin x| + C_1$

$y = C \sin x$

Mais $x = \pi/2$; $y = 1$

$\Rightarrow C = 1$

$\therefore \boxed{y = \sin x}$



//

#1.12 $xy' = y + x\cos^2(y/x)$

$x\,dy - (y + x\cos^2(y/x))dx = 0$

$M(\lambda x,\lambda y) = \lambda^1 M(x,y) \quad ; \quad N(\lambda x,\lambda y) = \lambda^1 N(x,y)$

Voici une équation à coefficients homogènes de degré 1

posons $y = xu \Rightarrow dy = u\,dx + x\,du$

$x[u\,dx + x\,du] - (xu + x\cos^2(u))dx = 0$

$(xu - xu - x\cos^2(u))dx + x^2 du = 0$

$x^2 du = x\cos^2(u)\,dx$

$$\int \frac{du}{\cos^2(u)} + C_1 = \int \frac{dx}{x}$$

$\ln|x| = \tan(u) + C_1$

$\ln|x| = \tan(y/x) + C_1$

$$\frac{y}{x} = \arctan(\ln|x| + C)$$

$$\boxed{y = x\arctan(\ln|x| + C)}$$ //

#1.16 $(x^2 + y^2)dx - 2xy\,dy = 0 \quad , \quad y(1) = 2$

$M(\lambda x,\lambda y) = \lambda^2 M(x,y)$ et $N(\lambda x,\lambda y) = \lambda^2 N(x,y)$

Voici une équation à coefficients homogènes de degré 2.

posons $y = xu \rightarrow dy = u\,dx + x\,du$

$(x^2 + x^2u^2)dx - 2x^2u[u\,dx + x\,du] = 0$

$(1 + u^2)dx - 2u[u\,dx + x\,du] = 0$

$(1 - u^2)dx = 2ux\,du$

$$\int \frac{1}{x}dx = -\int \frac{2u}{u^2-1}du + C_1$$

$\ln|x| = -\ln|u^2 - 1| + C_1$

$$x = \frac{C}{u^2 - 1} = \frac{C}{y^2/x^2 - 1}$$

$x(y^2/x^2 - 1) = C$ ①

$y^2 - x^2 = Cx = -2rx$ ②

$y^2 - (x^2 - 2rx + r^2) = -r^2$

$(x - r)^2 - y^2 = r^2$

Mais $y = 2, x = 1$ (ds ①)

$\Rightarrow C = 3 = -2r \Rightarrow r = -3/2$

$$\boxed{(x + 3/2)^2 - y^2 = 9/4}$$ //

une hyperbole.

Devoir #1 (suite)

#1.17 $x(2x^2+y^2) + y(x^2+2y^2)y' = 0$

$x(2x^2+y^2)dx + y(x^2+2y^2)dy = 0$

$M(\lambda x,\lambda y)=\lambda^3 M(x,y)$ et $N(\lambda x,\lambda y)=\lambda^3 N(x,y)$

Voici une équation à coefficients homogènes de degré 3

posons $y = xu \rightarrow dy = u\,dx + x\,du$

$x(2x^2+u^2x^2)dx + xu(x^2+2x^2u^2)[u\,dx + x\,du] = 0$

$(2+u^2)dx + u(1+2u^2)[u\,dx + x\,du] = 0$

$(2+u^2+u^2+2u^4)dx + x(u+2u^3)du = 0$

$$\int \frac{dx}{x} = -\frac{1}{2}\int \frac{u+2u^3}{(u^4+u^2+1)}\,du + C_1$$

sub $w = u^4+u^2+1 \rightarrow \frac{dw}{2} = (2u^3+u)\,du$

$$\int \frac{dx}{x} = -\frac{1}{4}\int \frac{1}{w}\,dw + C_1$$

$$\ln|x| = -\frac{1}{4}\ln|w| + C_1 = -\frac{1}{4}\ln|u^4+u^2+1| + C_1$$

$$x = \frac{C_2}{(u^4+u^2+1)^{1/4}}$$

$$x^4(u^4+u^2+1) = C$$

$$x^4\left(\frac{y^4}{x^4}+\frac{y^2}{x^2}+1\right) = C$$

$$\boxed{y^4 + x^2y^2 + x^4 = C}$$

#1.21 $(2xy-3)dx + (x^2+4y)dy = 0 \quad , \quad y(1) = 2$

$M_y = 2x = N_x$

$u(x,y) = \int (2xy-3)\,dx + T(y)$

$= x^2y - 3x + T(y)$

$u_y(x,y) = x^2 + T'(y)$

$= x^2 + 4y$

$\Rightarrow T'(y) = 4y$

$\therefore T(y) = 2y^2$

$u(x,y) = x^2y - 3x + 2y^2 = C$

Mais $y = 2 \; ; \; x = 1$

$(1)^2(2) - 3(1) + 2(2)^2 = C$

$C = 7$

$\boxed{x^2y - 3x + 2y^2 = 7}$ //

8.9

$g(x) = 1 + \sin^2 x$

$g'(x) = 2\sin x \cos x$

$= \sin(2x)$

X_{n+1}	$g(X_n)$
0	1,000000000
1	1,708073418
2	1,981273081
3	1,840761872
4	1,928872054
5	1,877168913

$X_{40} = 1{,}847194306 \approx p$

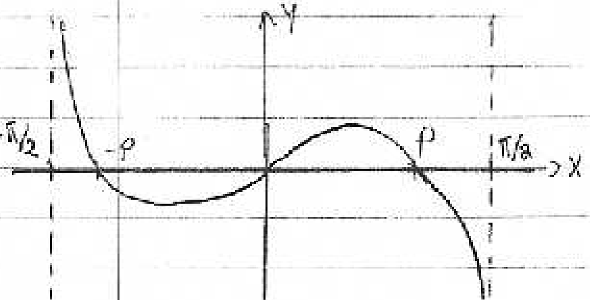
$g'(p) = \sin(2p)$

$= -0{,}607 \neq 0$

– L'ordre de convergence de cette méthode est 1 //

Devoir #1 (suite)

\# 8.10 $f(x) = 2x - \tan x$ $\quad f'(x) = 2 - \frac{1}{\cos^2 x}$ ⟹ pas de double 0



$$f''(x) = \frac{2\sin x}{\cos^3 x}$$

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{2x - \tan x}{2 - \sec^2 x}$$

x_{n+1}	$g(x_n)$
0	1,00000000
1	1,31047803
2	1,2239290947
3	1,1760509
4	1,1659286508
5	1,1655811636
6	1,165561185

$x_6 = 1{,}165561185 \approx p$

$$g'(x) = 1 - \frac{(2-\sec^2 x)^2 - (-2\sec x \sec x \sin x)(2x - \tan x)}{(2-\sec^2 x)^2}$$

$$= \frac{2\sin x\,(2x - \tan x)}{(2-\sec^2 x)^2}$$

$$g'(p) = 0$$

pour $x = p$, $g''(x) = \frac{-f''(p)}{f'(p)} = \frac{-\cos^3 x}{2\sin x} \cdot \frac{(2\cos^2 x - 1)}{\cos^2 x}$

$$g''(x) = -\frac{\cos x\,(2\cos^2 x - 1)}{2\sin x}$$

$$g''(x) = -\frac{\cos x \cos(2x)}{2\sin x}$$

$$g''(p) = 0{,}148 \neq 0$$

- L'ordre de convergence de cette méthode est 2 //