SOLUTIONS

MAT 2784 A

RÉMI VAILLANCOURT

DEVOIR N° 8

Exercice n° 6.24

Transformée de Laplace de F(s)

$$F(s) = \frac{s^2 - 5}{s^3 + s^2 + 9s + 9}$$

$$\begin{array}{r|l} s^3 + s^2 + 9s + 9 & s+1 \\ \hline -s^3 - s^2 & s^2 + 9 \\ = 0 \quad 9s + 9 & \\ -9s - 9 & \\ \hline 0 & \end{array}$$

$$s^3 + s^2 + 9s + 9 = (s+1)(s^2+9)$$

$$F(s) = \frac{s^2 - 5}{s^3 + s^2 + 9s + 9} = \frac{s^2 - 5}{(s+1)(s^2+9)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + 9}$$

$$A(s^2 + 9) + (Bs + C)(s+1) = s^2 - 5$$

$$(A+B)s^2 + (B+C)s + 9A + C = s^2 - 5$$

$$\begin{cases} A + B = 1 \\ B + C = 0 \\ 9A + C = -5 \end{cases} \Rightarrow \begin{cases} A = 1 - B \Rightarrow A = 1 + C \\ B = -C \\ 9A + C = -5 \Rightarrow 9(1+C) + C = -5 \end{cases}$$

$$\Rightarrow 9C + C = -14$$

$$\Rightarrow C = \frac{-14}{10} = \frac{-7}{5}$$

$$\begin{cases} A = 1 - \frac{7}{5} \\ B = \frac{7}{5} \\ C = -\frac{7}{5} \end{cases} \Rightarrow \begin{cases} A = \frac{-2}{5} \\ B = \frac{7}{5} \\ C = \frac{-7}{5} \end{cases}$$

$$F(s) = \frac{-2/5}{s+1} + \frac{7/5\, s}{s^2 + 9} + \frac{-7/5}{s^2 + 9} \Rightarrow$$

$$f(t) = -\frac{2}{5} e^{-t} + \frac{7}{5} \cos 3t - \frac{7}{5} \sin 3t$$

Exercice nº 6.36

$f(t) = e^{-t}\, t \cos t$

Nous savons que : si $F(s) = \mathcal{L}(f(t))(s)$ alors $\mathcal{L}(t f(t))(s) = -F'(s)$ (1

On a donc $e^{-t}\, t \cos t = e^{at} f(t)$ avec $f(t) = t \cos t$

$a = -1$

D'où $e^{at} f(t) \Rightarrow F(s-a)$

D'où la transformée de Laplace de $f(t)$ devient :

$$F(s) = \frac{s+1}{s^2+2s+2}$$

$$F'(s) = \frac{(s^2+2s+2) - (s+1)(2s+2)}{(s^2+2s+2)^2}$$

$$= \frac{-(s^2+2s)}{(s^2+2s+2)^2}$$

Selon le théorème énoncé en (1) on a :

$$\mathcal{L}(e^{-t}\, t \cos t) = -F'(s) = \frac{s^2+2s}{(s^2+2s+2)^2}$$

Exercice nº 6.57

$$y(t) = \cos 3t + 2\int_0^t y(\tau)\cos 3(t-\tau)\,d\tau$$

Posons $\sigma = t + \tau$

$$\frac{d\sigma}{d\tau} = +1$$

$$d\sigma = +d\tau$$

$$2\int_0^t y(\tau)\cos 3(t-\tau)\,d\tau = 2\int y(t-\sigma)\cos 3(\sigma)\,d\sigma$$

$$y(t) = \cos 3t + 2\,y \cos 3t$$

$$Y(s) = \frac{s}{s^2+9} + 2\,Y(s)\,\frac{s}{s^2+9}$$

$$Y(s)\left[1 - \frac{2s}{s^2+9}\right] = \frac{s}{s^2+9}$$

$$Y(s)\left[\frac{s^2+9-2s}{s^2+9}\right] = \frac{s}{s^2+9}$$

$$Y(s) = \frac{s}{s^2+9} \times \frac{s^2+9}{s^2-2s+9}$$

$$Y(s) = \frac{s}{s^2-2s+9} = \frac{(s-1)+1}{(s-1)^2+8} = \frac{(s-1)}{(s-1)^2+8} + \frac{1}{(s-1)^2+8}$$

$$Y(s) = \frac{(s-1)}{(s-1)^2+8} + \frac{1}{\sqrt{8}} \times \frac{\sqrt{8}}{(s-1)^2+(\sqrt{8})^2}$$

$$Y(s) = \frac{(s-1)}{(s-1)^2+(\sqrt{8})^2} + \frac{1}{\sqrt{8}} \times \frac{\sqrt{8}}{(s-1)^2+(\sqrt{8})^2}$$ Alors

$$y(t) = e^t\cos(\sqrt{8})t + \frac{1}{\sqrt{8}}e^t\sin(\sqrt{8})t$$

t	y(t)
0	1
0.5	0.8328
1	-2,28799
1.5	-3,441

$$\boxed{y(t) = e^t\cos(\sqrt{8})t + \frac{1}{\sqrt{8}}e^t\sin(\sqrt{8})t}$$

Exercício nº 6·49

$$y'' - 5y' + 6y = \begin{cases} t & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases}$$

Condições : $g(t) = t - u(t-1)t$

$\mathcal{L}(g) = G(s)$

$$g(t) = t - u(t-1)t$$

$$= t - u(t-1)(t-1) + u(t-1)$$

$$\mathcal{L}(g) = \mathcal{L}(t) - \mathcal{L}(u(t-1)(t-1)) + \mathcal{L}(u(t-1))$$

$$= \frac{1}{s^2} - e^{-(1)s}\frac{1}{s^2} - \frac{1}{s}e^{-(1)s}$$

$$= \frac{1}{s^2} - \frac{1}{s^2}e^{-s} - \frac{1}{s}e^{-s}$$

$$= \frac{1}{s^2} - e^{-s}\left[\frac{1}{s^2} + \frac{1}{s}\right]$$

$$\mathcal{L}(y''-5y'+6y) = \mathcal{L}(y'') - 5\mathcal{L}(y') + 6\mathcal{L}(y)$$

$$= s^2Y(s) - sy(0) - y'(0) - 5\left[sY(s) - y(0)\right] + 6Y(s)$$

$$= s^2Y(s) - 1 - 5sY(s) + 6Y(s)$$

$$= Y(s)(s^2-5s+6) - 1$$

$$Y(s)(s^2-5s+6) - 1 = \frac{1}{s^2} - e^{-s}\left[\frac{1}{s^2} + \frac{1}{s}\right]$$

$$Y(s) = \frac{\frac{1}{s^2} - e^{-s}\left[\frac{1}{s^2}+\frac{1}{s}\right] + 1}{s^2-5s+6} = \frac{\frac{1}{s^2} - e^{-s}\left[\frac{1}{s^2}+\frac{1}{s}\right] + 1}{(s-2)(s-3)}$$

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$$Y(s) = \frac{1}{s^2(s^2-5s+6)} - \frac{e^{-s}}{s^2(s^2-5s+6)} - \frac{e^{-s}}{s(s^2-5s+6)} + \frac{1}{s^2-5s+6}$$

$$Y(s) = \frac{1}{(s-2)(s-3)} + \frac{1}{s^2(s-2)(s-3)} - \frac{e^{-s}}{s^2(s-2)(s-3)} - \frac{e^{-s}}{s(s-2)(s-3)}$$

Pour $\frac{1}{(s-2)(s-3)} = \frac{A}{(s-2)} + \frac{B}{(s-3)}$

$$A(s-3) + B(s-2) = 1$$

$$As - 3A + Bs - 2B = 1$$

$$(A+B)s - 3A - 2B = 1$$

$$\begin{cases} A+B=0 \\ -3A-2B=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ 3B-2B=1 \end{cases} \Rightarrow B=1 \Rightarrow A=-1$$

$$\frac{1}{(s-2)(s-3)} = \frac{-1}{s-2} + \frac{1}{s-3}$$

$$\frac{1}{s^2(s-2)(s-3)} = \frac{A}{s^2} + \frac{B}{s-2} + \frac{C}{s-3} + \frac{D}{s}$$

$$D\,s(s-2)(s-3) + A(s-2)(s-3) + B\,s^2(s-3) + C\,s^2(s-2) = 1$$

$$D(s^3-5s^2+6s) + A(s^2-3s-2s+6) + B(s^3-3s^2) + C(s^3-2s^2) = 1$$

$$Ds^3 - 5s^2 + 6s + As^2 - 5sA + 6A + Bs^3 - 3s^2B + Cs^3 - 2s^2C = 1$$

$$(D+B+C)s^3 + (A-3B-2C \underset{-5D}{}) s^2 + (6D-5A)s + 6A = 1$$

$$(B+C+D)s^2 + (A-3B-2C-5D) + (6D-5A)s + 6A = 1$$

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$$\begin{cases} B + C + D = 0 \\ A - 3B - 2C - 5D = 0 \\ 6D - 5A = 0 \\ 6A = 1 \\ B + C + D = 0 \end{cases} \Rightarrow \begin{cases} A = 1/6 \\ 6D = 5A \Rightarrow D = \frac{5}{6}\left(\frac{1}{6}\right) = \frac{5}{36} \\ \frac{1}{6} - 3B - 2C - 5\left(\frac{5}{36}\right) = 0 \\ B + C + \frac{5}{36} = 0 \Rightarrow B = -C - \frac{5}{36} \end{cases}$$

$$\frac{1}{6} - 3\left(-C - \frac{5}{36}\right) - \frac{25}{36} = 0$$

$$\frac{1}{6} + 3C + \frac{15}{36} - \frac{25}{36} = 0$$

$$\frac{1}{6} - \frac{10}{36} + 3C = 0$$

$$C = \frac{-1}{6} + \frac{10}{36} = \frac{-6 + 10}{36} = \frac{+4}{36} =$$

$$B = -\frac{1}{9} - \frac{5}{36} = \frac{-9}{36} = \frac{-1}{4}$$

$$\begin{cases} A = 1/6 \\ B = -\frac{1}{4} \\ C = \frac{1}{9} \\ D = \frac{5}{36} \end{cases}$$

$$\frac{1}{s^2(s-2)(s-3)} = \frac{5/36}{s} + \frac{1/6}{s^2} + \frac{-1/4}{s-2} + \frac{1/9}{s-3}$$

$$\frac{1}{s(s-2)(s-3)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$A(s-2)(s-3) + B\,s(s-3) + C(s-2)s = 1$$

$$A(s^2 - 5s + 6) + B(s^2 - 3s) + C(s^2 - 2s) = 1$$

$$(A + B + C)s^2 + (-5A - 3B - 2C)s^2 + 6A = 1$$

$$\begin{cases} A = 1/6 \\ A + B + C = 0 \\ -5A - 3B - 2C = 0 \end{cases} \Rightarrow \begin{cases} A = 1/6 \\ B + C = -1/6 \Rightarrow B = -1/6 - C \\ -\frac{5}{6} - 3B + 2C = 0 \Rightarrow -3\left(\frac{-1}{6} - C\right) - 2C = \frac{5}{6} \end{cases}$$

$$B = \frac{-3}{6} = \frac{-1}{2} \qquad C = \frac{5}{6} - \frac{3}{6} \qquad C = \frac{2}{6} =$$

$$\begin{cases} A = 1/6 \\ B = -1/2 \\ C = 1/3 \end{cases}$$

$$\frac{1}{s(s-2)(s-3} = \frac{1/6}{s} + \frac{-1/2}{s-2} + \frac{1/3}{s-3}$$

$$Y(s) = \left[\frac{-1}{s-2} + \frac{1}{s-3}\right] + \left[\frac{5/36}{s} + \frac{1/6}{s^2} + \frac{-1/4}{s-2} + \frac{1/9}{s-3}\right] +$$

$$- e^{-s}\left[\frac{5/36}{s} + \frac{1/6}{s^2} + \frac{-1/4}{s-2} + \frac{1/9}{s-3}\right] +$$

$$- e^{-s}\left[\frac{1/6}{s} + \frac{-1/2}{s-2} + \frac{1/3}{s-3}\right]$$

$$y(t) = -e^{2t} + e^{3t} + \frac{5}{36} + \frac{1}{6}t - \frac{1}{4}e^{2t} + \frac{1}{9}e^{3t} - \frac{5}{36}u(t-1)$$

$$- \frac{1}{6}u(t-1)(t-1) + \frac{1}{4}u(t-1)e^{2(t-1)} + \frac{1}{9}u(t-1)e^{3(t-1)}$$

$$- \frac{1}{6}u(t-1) + \frac{1}{2}u(t-1)e^{2(t-1)} - \frac{1}{3}u(t-1)e^{3(t-1)}$$

$$y(t) = -\frac{5}{4}e^{2t} + \frac{10}{9}e^{3t} + \frac{1}{6}t + \frac{5}{36} - \frac{11}{36}u(t-1) - u(t-1)\left[\frac{1}{6}(t-1) - \frac{1}{4}e^{2(t-1)} - \frac{1}{9}e^{3(t-1)} - \frac{1}{2}e^{2(t-1)} + \frac{1}{3}e^{3(t-1)}\right]$$

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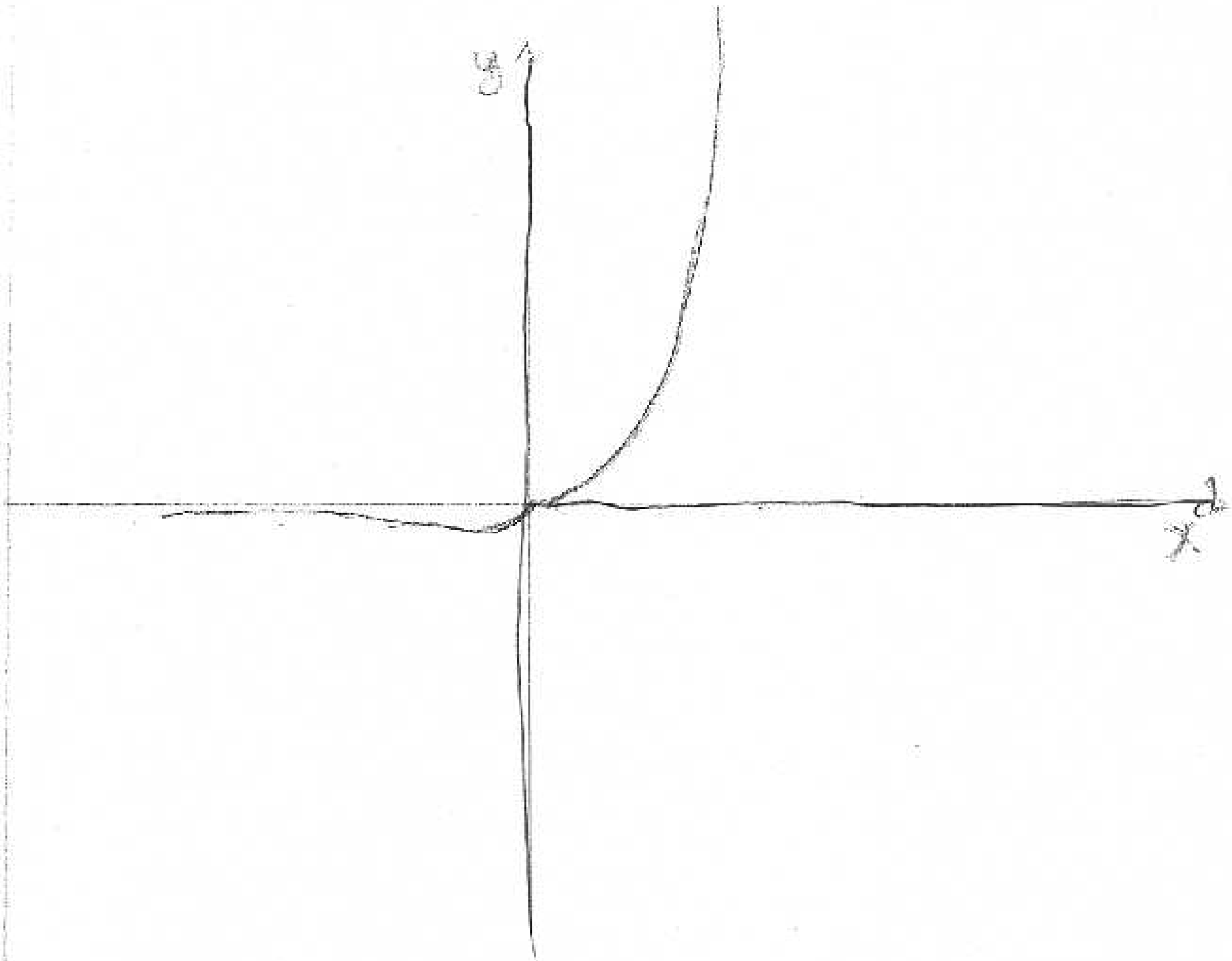
$$Y(t) = -\frac{5}{4}e^{2t} + \frac{10}{9}e^{3t} + \frac{1}{6}t + \frac{5}{36} - \frac{11}{36}u(t-1) - u(t-1)\left[\frac{1}{6}(t-1) + \frac{4}{9}e^{3(t-1)} - \frac{3}{4}e^{2(t-1)}\right]$$

$$Y(t) = -\frac{5}{4}e^{2t} + \frac{10}{9}e^{3t} + \frac{1}{6}t + \frac{5}{36} - u(t-1)\left[\frac{11}{36} + \frac{1}{6}(t-1) + \frac{4}{9}e^{3(t-1)} - \frac{3}{4}e^{2(t-1)}\right]$$



Exercice nº 6.54

$$\mathcal{L}y: \quad y'' + 5y' + 6y = u(t-1) + \delta(t-2) \qquad y(0)=0 \quad y'(0)=1$$

$$\mathcal{L}(y'') = s^2 Y(s) - s\,y(0) - y'(0)$$
$$= s^2 Y(s) - 1$$

$$5\,\mathcal{L}(y') = 5\left[sY(s) - y(0)\right]$$
$$= 5s\,Y(s)$$

$$\mathcal{L}(u(t-1)) = \frac{e^{-s}}{s}$$
$$\mathcal{L}(\delta(t-2)) = e^{-2s}$$

$$6\,\mathcal{L}(y) = 6Y(s)$$

$$\mathcal{L}(y'' + 5y' + 6y) = s^2 Y(s) - 1 + 5\left[sY(s)\right] + 6Y(s)$$
$$= Y(s)\left[s^2 + 5s + 6\right] - 1$$

$$\frac{e^{-s}}{s} + \frac{e^{-2s}}{s} = Y(s)\,(s^2+5s+6) - 1$$

$$Y(s) = \frac{\frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + 1}{s^2+5s+6} = \frac{e^{-s}}{s(s+2)(s+3)} + \frac{e^{-2s}}{(s+2)(s+3)} + \frac{1}{(s+2)(s+3)}$$

$$\frac{1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A(s+3) + B(s+2) = 1$$
$$As + 3A + Bs + 2B = 1$$
$$(A+B)s + 2B + 3A = 1$$

$$\begin{cases} A + B = 0 \Rightarrow A = -B \\ 3A + 2B = 1 \end{cases}$$
$$-B = 1 \Rightarrow B = -1 \quad A = 1$$

$$\frac{1}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{-1}{s+3}$$

08.10

$$\frac{1}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A(s+2)(s+3) + B(s(s+3)) + C(s(s+2)) = 1$$

$$A(s^2+5s+6) + B(s^2+3s) + C(s^2+2s) = 1$$

$$(A+B+C)s^2 + (5A+3B+2C)s + 6A = 0$$

$$\begin{cases} A+B+C=0 \\ 5A+3B+2C=0 \\ 6A=1 \end{cases} \Rightarrow \begin{cases} B+C = -1/6 \\ 3B+2C = -\frac{5}{6} \\ A = \frac{1}{6} \end{cases}$$

$\Rightarrow B = -\frac{1}{6} - C$

$-3/6 + 2C - 3C = \frac{-5}{6}$

$-C = \frac{-5}{6} + \frac{3}{6}$

$-C = \frac{-2}{6}$

$C = \frac{1}{3} \quad B = \frac{-1}{2}$

$$\begin{cases} A = 1/6 \\ B = -1/2 \\ C = \frac{1}{3} \end{cases}$$

$$\frac{1}{s(s+2)(s+3)} = \frac{1/6}{s} + \frac{-1/2}{s+2} + \frac{\frac{1}{3}}{s+3}$$

$$Y(s) = \frac{1}{s+2} - \frac{1}{s+3} + e^{-s}\left[\frac{1/6}{s} - \frac{1/2}{s+2} + \frac{1/3}{s+3}\right] + e^{-2s}\left[\frac{1}{s+2} - \frac{1}{s+3}\right]$$

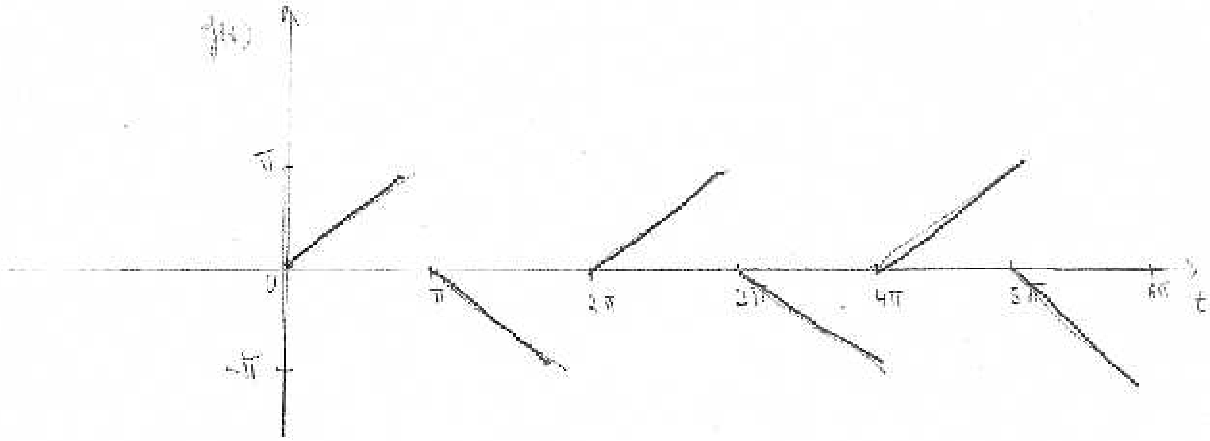
$$y(t) = e^{-2t} - e^{-3t} + \frac{1}{6}u(t-1) - \frac{1}{2}u(t-1)e^{-2(t-1)} + \frac{1}{3}e^{-3(t-1)}u(t-1) +$$

$$\cancel{\frac{1}{6}u(t-2)} + u(t-2)e^{-2(t-2)} - u(t-2)e^{-3(t-2)}$$

Exercice n° 6.63

08.11

$$f(t) = \begin{cases} t & 0 < t < \pi \\ \pi - t & \pi < t < 2\pi \end{cases}$$



$$\mathcal{L}(f) = \frac{1}{1-e^{-2\pi s}} \int_0^{\pi} e^{-st} t \, dt + \frac{1}{1-e^{-2\pi s}} \int_{\pi}^{2\pi} (\pi - t) e^{-st} dt$$

$$\frac{1}{1-e^{-2\pi s}} \int_0^{\pi} \underbrace{e^{-st}}_{dv} \underbrace{t\,dt}_{u} = \frac{1}{1-e^{-2\pi s}} \left[-\frac{t}{s} e^{-st} \Big|_0^{\pi} + \frac{1}{s} \int_0^{\pi} e^{-st} dt \right] =$$

$u = t \Rightarrow du = 1$

$dv = e^{-st} dt \Rightarrow v = -\frac{1}{s} e^{-st}$

$$\frac{1}{1-e^{-2\pi s}} \left[-\frac{\pi}{s} e^{-\pi s} + \frac{1}{s} \left[-\frac{1}{s} e^{-st} \right]_0^{\pi} \right] =$$

$$\frac{1}{1-e^{-2\pi s}} \left[\frac{-\pi}{s} e^{-\pi s} + \frac{1}{s} \left[-\frac{1}{s} e^{-\pi s} + \frac{1}{s} \right] \right] =$$

$$\frac{1}{1-e^{-2\pi s}} \left[\frac{-\pi}{s} e^{-\pi s} + \frac{1}{s^2} - \frac{1}{s^2} e^{-\pi s} \right]$$

$$\frac{1}{1-e^{-2\pi s}}\left[\int_{\pi}^{2\pi} \pi e^{-st}\,dt - \int_{\pi}^{2\pi} t\, e^{-st}\,dt\right] = \frac{1}{1-e^{-2\pi s}}\left[\frac{-\pi}{s} e^{-st}\Big|_{\pi}^{2\pi} + \frac{t}{s} e^{-st}\Big|_{\pi}^{2\pi} + \frac{1}{s^2} e^{-st}\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{1}{1-e^{-2\pi s}}\left[-\frac{\pi}{s} e^{-2\pi s} + \frac{\pi}{s} e^{-\pi s} + \frac{2\pi}{s} e^{-2\pi s} - \frac{\pi}{s} e^{-\pi s} + \frac{1}{s^2} e^{-2\pi s} - \frac{1}{s^2} e^{-\pi s}\right]$$

$$= \frac{1}{1-e^{-2\pi s}}\left[\frac{\pi}{s} e^{-2\pi s} + \frac{1}{s^2} e^{-2\pi s} - \frac{1}{s^2} e^{-\pi s}\right]$$

Donc :

$$\mathcal{L}(f)(s) = \frac{1}{1-e^{-2\pi s}}\left[\frac{\pi}{s} e^{-2\pi s} + \frac{1}{s^2} e^{-2\pi s} - \frac{2}{s} e^{-\pi s} + \frac{1}{s^2} e^{-\pi s} + \frac{1}{s^2}\right]$$

$$= \frac{1}{1-e^{-2\pi s}}\left[\frac{1}{s^2} e^{-2\pi s} + \frac{\pi}{s} e^{-2\pi s} - \frac{2}{s^2} e^{-\pi s} - \frac{\pi}{s} e^{-\pi s} + \frac{1}{s^2}\right]$$

$$= \frac{1}{1-e^{-2\pi s}}\left[e^{-2\pi s}\left(\frac{1}{s^2} + \frac{\pi}{s}\right) + e^{-\pi s}\left(\frac{-2}{s^2} - \frac{\pi}{s}\right) + \frac{1}{s^2}\right]$$

$$\mathcal{L}(f)(s) = \frac{1}{1-e^{-2\pi s}}\left[e^{-2\pi s}\left(\frac{1}{s^2} + \frac{\pi}{s}\right) + e^{-\pi s}\left(\frac{-2}{s^2} - \frac{\pi}{s}\right) + \frac{1}{s^2}\right]$$

7\. $y' = x + \sin y$, $y(0) = 0$ $\quad 0 \leq x \leq 1$, $h = 0.1$

(12.12)
et 12.21

↳ 3 pas avec RK4 → calcule x_4 et x_5 avec ABM3 et estimer l'erreur locale en x_4 et x_5

$f(x, y) = x + \sin y$

$x_n = 0.0 + 0.1n \qquad n = 0, 1, 2$

$y_0 = 0$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = 0.1 \cdot (0.1n + \sin y_n)$
$k_2 = 0.1 \cdot ((0.1n + 0.05) + \sin(y_n + \frac{k_1}{2}))$
$k_3 = 0.1 \cdot ((0.1n + 0.05) + \sin(y_n + \frac{k_2}{2}))$
$k_4 = 0.1 \cdot ((0.1n + 0.1) + \sin(y_n + k_3))$

→ je mets tes k_i dans ma mémoire de calculatrice

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
$= 0.005170833$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
$= 0.02140252$

$y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
$= 0.049857605$

D 8.14

ABM 3:

(utilise les y_3, y_2 et y_1 de page précédente)

$f_1^c = f(x_1, y_1^c)$
$= 0.10512081$

$f_2^c = 0.221400888$

$f_3^c = 0.349936952$

$y_4^p = y_3^c + \frac{0.1}{12}(23 f_3^c - 16 f_2^c + 5 f_1^c)$
$= 0.091771686$

$f_4^p = f(x_4, y_4^p)$
$= 0.491642923$

$y_4^c = y_3^c + \frac{0.1}{12}(5 f_4^p + 8 f_3^c - f_2^c)$
$= 0.091820183$

$Err = -\frac{1}{10}[y_4^c - y_4^p]$
$= -0.00000485$

$f_4^c = f(x_4, y_4^c)$
$= 0.491691216$

$y_5^p = y_4^c + \frac{0.1}{12}(23 f_4^c - 16 f_3^c + 5 f_2^c)$
$= 0.148641109$

$f_5^p = f(x_5, y_5^p)$
$= 0.648094363$

$y_5^c = y_4^c + \frac{0.1}{12}(5 f_5^p + 8 f_4^c - f_3^c)$
$= 0.148688221$

$Err = -\frac{1}{10}[y_5^c - y_5^p] = -0.00000471$

Exercice n° 12.20

$$y' = y^2 + 2y - x \quad , \quad y(0) = 0$$

$x_0 = 0 \qquad y_0 = 0 \qquad f(x,y) = y^2 + 2y - x \qquad h = 0.1$

n	x_n	Y_n
0	0	0
1	0.1	-0.005333
2	0.2	-0.022893
3	0.3	-0.055266

$n=0$

$$k_1 = 0.1\left[0^2 + 2\times 0 - 0\right] = 0$$

$$k_2 = 0.1 f(0.05, 0) = 0.1\left[0^2 + 2\times 0 - 0.05\right] = -0.005$$

$$k_3 = 0.1 f(0.075, -0.00375) = 0.1\left[(-0.00375)^2 + 2\times(-0.00375) - (0.075)\right]$$
$$= -0.002246$$

$$k_4 = 0.1 f\left[0.1, -0.0053327\right] = -0.0110637$$

$$Y_1 = -0{,}0055327$$

$$E_0 = -\frac{5}{72}\, 0.0 + \frac{1}{12}(-0.005) + \frac{1}{9}(-0.002246) - \frac{1}{8}(-0.0110637)$$
$$= 0.0000501$$

$$\boxed{E_0 = 0.0000501}$$

$n=1$

$$K_1 = k_4^{[0]} = -0.0110637$$
$$k_2 = 0.1 f(0.15, -0.0102(456)) = 0.017611$$
$$k_3 = 0.1 f(0.175, -0.0122035) = -0.0211036$$
$$K_4 = 0.1 f(0.2, -0.0228728) = -0.0245262$$

$$Y_2 = -0.022893$$

$$E_2 = -\frac{5}{72}(-0.0110651) + \frac{0.01716}{12} - \frac{-0.024078}{9} + \frac{0.0245268}{8}$$

$$E_2 = 0.0000587$$

$n = 2$

$$K_1 = K_4^{[1]} = -0.0245262$$
$$K_2 = 0.1\, f(0.25,\ 0.0357531) = -0.0317078$$
$$K_3 = 0.1\, f(0.275,\ -0.0452237) = -0.0366455$$
$$K_4 = 0.1\, f(0.3,\ -0.0552660) = -0.0467648$$

$$Y_3 = -0.0552660$$

$$E_3 = \frac{-5}{72}[-0.0245262] - \frac{-0.0319078}{12} + \frac{0.0366456}{9} + \frac{1}{8}(-0.0403747)$$

$$E_3 = 0.000066$$