SOLUTIONS

D5.1

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# MAT 2784 - Devoir 5

p. 294

#3.37) $y'' - 2y' + y = e^x/x \quad , \; y(1) = e \;, \; y'(1) = 0$

$r(x) = e^x/x$ , $r(x)$ est de dim indéfinie

Faisons la méthode de variation des paramètres.

$y_g(x) = y_h(x) + y_p(x)$

1. Trouvons $y_h(x)$

posons $y = e^{\lambda x} \Rightarrow y_h(x) = e^{\lambda x}(\lambda^2 - 2\lambda + 1) = 0$

$\lambda_{1,2} = \underline{1}$

$y_h = y_1 + y_2$

$y_h(x) = c_1 e^x + c_2 x e^x \Rightarrow y_1 = e^x \;, \; y_2 = xe^x$

$y_p(x) = c_1(x)\, e^x + c_2(x)\, xe^x$

$$\begin{bmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{bmatrix} \begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ e^x/x \end{bmatrix}$$

$$\sim \ell_1 - \ell_2 \rightarrow \begin{bmatrix} e^x & xe^x \\ 0 & e^x \end{bmatrix} \begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ e^x/x \end{bmatrix}$$

$e^x \cdot c_2'(x) = e^x/x \quad \therefore \; c_2'(x) = 1/x$

$c_2(x) = \ln|x|$

$e^x \cdot c_1'(x) + xe^x \cdot c_2'(x) = 0$

$e^x \cdot c_1'(x) = -e^x$

$c_1'(x) = -1$

$c_1(x) = -x$

$\therefore y_p(x) = -xe^x + xe^x \ln(x)$

$\therefore$ alors, $y_g(x) = c_1 e^x + c_2 x e^x - x e^x + \ln(x) x e^x$ est la solution general

Considérons maintenant les conditions initiales.

$y(1) = e$ , $y'(1) = 0$

$y_g(1) = c_1 e + c_2 e - e = e$ (divison par e)

$= c_1 + c_2 = 1$

$c_1 = 1 - c_2$ (*)

$y_g'(x) = c_1 e^x + c_2 e^x + c_2 x e^x + e^x \ln(x) + x e^x \ln(x) + e^x$

$y_g'(1) = c_1 e + c_2 e + c_2 e + e = 0$

$= c_1 e + 2c_2 e = -e$

$c_1 + 2c_2 = -1$ (♡)

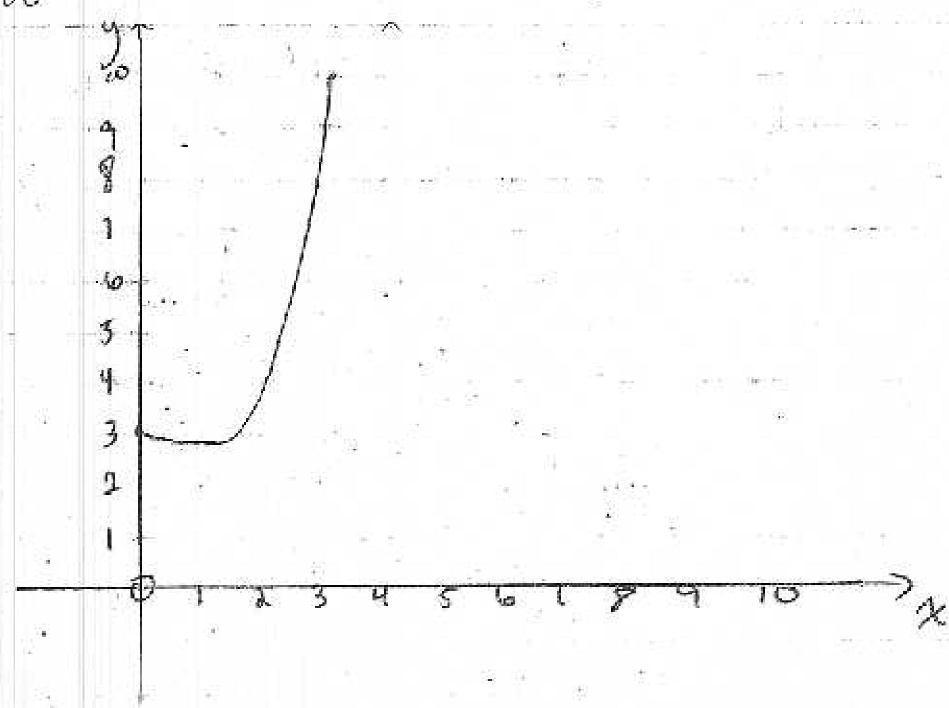
Substituons (*) dans (♡)

$(1 - c_2) + 2c_2 = -1$

$c_2 = -2$

$\Rightarrow c_1 = 3$

$\therefore$ $y_g(x) = 3e^x - 2xe^x + x e^x \ln(x)$ est la sol. unique



D5.3

Exercício nº 3.34

$$Ly = 2x^2y'' + xy' - 3y = 2x^{-3} \qquad y(1)=0, \; y'(1)=3$$

$$y = y_h + y_p$$

$y_h(x)$ tal que $Ly = 0$

Pomos $y = x^m$

$$P(m) = 2m(m-1) + m - 3 = 0$$
$$= 2m^2 - 2m + m - 3 = 0$$
$$= 2m^2 - m - 3 = 0$$

$$m_{1,2} = \frac{1 \pm \sqrt{1+24}}{4}$$

$$m_1 = \frac{1+5}{4} = \frac{3}{2}$$

$$m_2 = \frac{1-5}{4} = \frac{-4}{4} = -1$$

$$y_h(x) = A x^{3/2} + B x^{-1}$$

$$y_p(x) = C_1(x)\, x^{3/2} + C_2(x)\, x^{-1}$$

$$y_p'(x) = \frac{3}{2} C_1(x)\, x^{1/2} - C_2(x)\, x^{-2}$$

$$\begin{bmatrix} x^{3/2} & x^{-1} \\ \frac{3}{2}x^{1/2} & -x^{-2} \end{bmatrix} \begin{bmatrix} C_1' \\ C_2' \end{bmatrix} = \begin{bmatrix} 0 \\ x^{-5} \end{bmatrix}$$

$$x^{3/2}C_1' + C_2'(x^{-1}) = 0 \Rightarrow C_2' = -C_1'(x^{5/2}) \Rightarrow C_2' = -x^{5/2}C_1'$$

$$\frac{3}{2}x^{\frac{1}{2}}C_1' + x^{-2}C_2' = x^{-5} \Rightarrow \frac{3}{2}x^{1/2}C_1' + x^{1/2}C_1' = x^{-5}$$

$$\Rightarrow \frac{5}{2}x^{1/2}C_1' = x^{-5} \rightarrow C_1' = \frac{2}{5}x^{-11/2}$$

$$C_2' = \frac{-2}{5} x^{-11/2} \left(x^{5/2}\right) = \frac{-2}{5} x^{-3}$$

$$C_1(x) = \int \frac{2}{5} x^{-11/2} = \frac{2}{5} \left(x^{-9/2}\right)\left(\frac{-2}{9}\right) = \frac{-4}{45} x^{-9/2}$$

$$C_2(x) = \int \frac{-2}{5} x^{-3} = \frac{-2}{5} \left(x^{-2}\right)\left(\frac{-1}{2}\right) = \frac{x^{-2}}{5}$$

$$y_p(x) = \frac{-4}{45} x^{-9/2}\left(x^{3/2}\right) + \frac{x^{-2}}{5}\left(x^{-1}\right)$$

$$= \frac{-4}{45} x^{-3} + \frac{1}{5} x^{-3}$$

$$= \frac{1}{9} x^{-3}$$

$$y = y_h + y_p = a x^{3/2} + b x^{-1} + \frac{1}{9} x^{-3}$$

Conditions initiales

$$y(1) = 0 \implies a + b + \frac{1}{9} = 0 \qquad a = -b - \frac{1}{9}$$

$$y'(x) = \frac{3}{2} a x^{1/2} - b x^{-2} - \frac{1}{3} x^{-4}$$

$$y'(1) = 3 \implies \frac{3}{2} a - b - \frac{1}{3} = 3 \implies b = \frac{-7}{5}$$

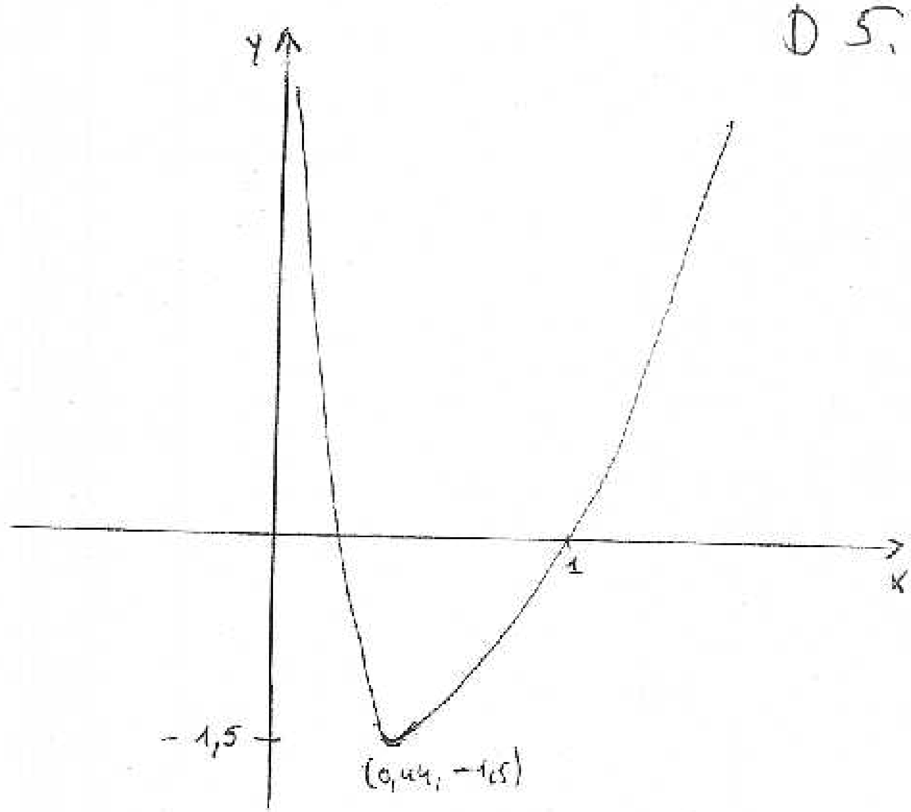
$$a = 58/45$$

Solution particulière

$$y(x) = \frac{58}{45} x^{3/2} + \frac{-7}{5} x^{-1} + \frac{1}{9} x^{-3}$$

$$= \frac{58}{45} x^{3/2} - \frac{7}{5} x^{-1} + \frac{1}{9} x^{-3}$$

D 5.5




La solution générale est :

$$y(x) = c_1 e^{-x}\begin{bmatrix}1\\-1\end{bmatrix} + c_2 e^{-2x}\begin{bmatrix}\frac{1}{2}\\1\end{bmatrix}$$

→ back D5.6

Exercice n° 4.1

$$y' = A\,y$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{vmatrix} = \lambda(-3-\lambda)+2$$

$$= \lambda^2 + 3\lambda + 2$$

$$= (\lambda+1)(\lambda+2)$$

$$\det(A - \lambda I) = 0 \Rightarrow \lambda_1 = -1 \;,\; \lambda_2 = -2$$

Premier vecteur propre : $\lambda = -1$

$$(A - \lambda I)\vec{u} = 0 \Rightarrow \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} u_1 + u_2 &= 0 \\ -2u_1 - 2u_2 &= 0 \end{aligned}\right\} \Rightarrow u_1 = -u_2$$

$$\vec{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Deuxième vecteur propre : $\lambda = -2$ $\qquad (A - \lambda I)\vec{v} = \vec{0}$

$$\begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} 2v_1 + v_2 &= 0 \\ -2v_1 - v_2 &= 0 \end{aligned}\right\} \Rightarrow v_1 = \tfrac{1}{2} v_2$$

$$\vec{v} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

Donc on a deux solutions indépendantes :

$$y_1(x) = e^{-x}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{et} \quad y_2(x) = e^{-2x}\begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

suite sur D5.5

Exercice n° 4.5

$$A = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 1 \\ -4 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(1-\lambda) + 4 = 0$$

$$\Rightarrow \lambda^2 - 2\lambda + 5 = 0$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$$

Pour $\lambda = 1 + 2i$

$$(A - \lambda I)\vec{u} = \vec{0} \Rightarrow \begin{bmatrix} -2i & 1 \\ -4 & -2i \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{aligned} -2i\,u_1 + u_2 &= 0 \\ -4\,u_1 - 2i\,u_2 &= 0 \end{aligned} \right\} \Rightarrow \begin{aligned} u_1 &= 1 \\ u_2 &= 2i \end{aligned}$$

$$\vec{u} = \begin{bmatrix} 1 \\ 2i \end{bmatrix}$$

Pour $\lambda = 1 - 2i$

$$(A - \lambda I)\vec{v} = \vec{0} \Rightarrow \begin{bmatrix} 2i & 1 \\ -4 & 2i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{aligned} 2i\,v_1 + v_2 &= 0 \\ -4\,v_1 + 2i\,v_2 &= 0 \end{aligned} \right\} \Rightarrow \begin{aligned} v_1 &= 1 \\ v_2 &= -2i \end{aligned}$$

$$\vec{v} = \begin{bmatrix} 1 \\ -2i \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} 1 \\ 2i \end{bmatrix} e^{(1+2i)x} = \begin{bmatrix} 1 \\ 2i \end{bmatrix} e^{x} e^{2ix} = \begin{bmatrix} 1 \\ 2i \end{bmatrix} e^{x} (\cos 2x + i \sin 2x)$$

D 5,8

$$\vec{u} = \begin{bmatrix}1\\0\end{bmatrix} e^{x}(\cos 2x + i\sin 2x) + 2i\begin{bmatrix}0\\1\end{bmatrix} e^{x}(\cos 2x + i\sin 2x)$$

$$= e^{x}\left[\begin{bmatrix}1\\0\end{bmatrix}\cos 2x - 2\begin{bmatrix}0\\1\end{bmatrix}\sin 2x\right] + ie^{x}\left[\begin{bmatrix}1\\0\end{bmatrix}\sin 2x + \begin{bmatrix}0\\2\end{bmatrix}\cos 2x\right]$$

$$= e^{x}\begin{bmatrix}\cos 2x\\-2\sin 2x\end{bmatrix} + i e^{x}\begin{bmatrix}\sin 2x\\2\cos 2x\end{bmatrix}$$

$$\vec{u} = \vec{y}_1(x) + i\vec{y}_2(x)$$

$$y_1(x) = e^{x}\begin{bmatrix}\cos 2x\\-2\sin 2x\end{bmatrix}$$

$$y_2(x) = e^{x}\begin{bmatrix}\sin 2x\\2\cos 2x\end{bmatrix}$$

Solution générale

$$y(x) = C_1 e^{x}\begin{bmatrix}\cos 2x\\-2\sin 2x\end{bmatrix} + C_2 e^{x}\begin{bmatrix}\sin 2x\\2\cos 2x\end{bmatrix}$$

Exercice n° 4.8

$$y' = Ay + f(x)$$

$$A = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \qquad f(x) = \begin{bmatrix} 2e^{-x} \\ 3x \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{vmatrix} = (-2-\lambda)(-2-\lambda) - 1$$
$$= \lambda^2 + 4\lambda + 3$$
$$= (\lambda+1)(\lambda+3)$$

$$\det(A - \lambda I) = 0 \quad \Rightarrow \quad \lambda_1 = -1, \quad \lambda_2 = -3$$

Premier vecteur propre : $\lambda = -1$

$$(A - \lambda I)\vec{u} = \vec{0}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \left.\begin{matrix} -u_1 + u_2 = 0 \\ u_1 - u_2 = 0 \end{matrix}\right\} \Rightarrow u_1 = u_2 \quad \vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Deuxième vecteur propre : $\lambda = -3$

$$(A - \lambda I)\vec{v} = \vec{0}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \left.\begin{matrix} v_1 + v_2 = 0 \\ v_1 + v_2 = 0 \end{matrix}\right\} \Rightarrow v_1 = -v_2 \quad \vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\lambda_1 \neq \lambda_2$ deux solutions indépendantes

$$y_1(x) = e^{-x}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{et} \quad y_2(x) = e^{-3x}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

La solution du système homogène est :

$$y_h(x) = C_1 e^{-x}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{-3x}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

D 5.10

$$F(x) = 2e^{-x}\begin{bmatrix}1\\0\end{bmatrix} + 3x\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}e^{-x} + \begin{bmatrix}0\\3\end{bmatrix}x$$

$$y_p(x) = \begin{bmatrix}a_1\\a_2\end{bmatrix}e^{-x} + \begin{bmatrix}b_1\\b_2\end{bmatrix}xe^{-x} + \begin{bmatrix}c_1\\c_1\end{bmatrix} + \begin{bmatrix}d_1\\d_2\end{bmatrix}x$$

$y' = Ay + f(x)$

$$y_p'(x) = -\begin{bmatrix}a_1\\a_2\end{bmatrix}e^{-x} + \begin{bmatrix}b_1\\b_2\end{bmatrix}e^{-x} - \begin{bmatrix}b_1\\b_2\end{bmatrix}xe^{-x} + \begin{bmatrix}d_1\\d_2\end{bmatrix}$$

$$= \begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}\begin{bmatrix}a_1\\a_2\end{bmatrix}e^{-x} + \begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}\begin{bmatrix}b_1\\b_2\end{bmatrix}xe^{-x} + \begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}\begin{bmatrix}c_1\\c_1\end{bmatrix} + \begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}\begin{bmatrix}d_1\\d_2\end{bmatrix}$$

$$+ \begin{bmatrix}2\\0\end{bmatrix}e^{-x} + \begin{bmatrix}0\\3\end{bmatrix}x$$

$$\begin{bmatrix}b_1 - a_1 + 2\\b_2 - a_2\end{bmatrix}e^{-x} = \begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}\begin{bmatrix}a_1\\a_2\end{bmatrix}e^{-x}$$

$$\left.\begin{matrix}-2a_1 + a_2 = b_1 - a_1 + 2\\ b_2 - a_2 = a_1 - 2a_2\end{matrix}\right\} \Rightarrow \left.\begin{matrix}-a_1 + a_2 + 2 = b\\ a_1 - a_2 = b\end{matrix}\right\} \Rightarrow a_1 = a_2 + 1$$

Posons: $a_1 = 2$ ; $a_2 = 1$

$$-\begin{bmatrix}b_1\\b_2\end{bmatrix}xe^{-x} = \begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}\begin{bmatrix}b_1\\b_2\end{bmatrix}xe^{-x}$$

$$\left.\begin{matrix}-2b_1 + b_2 = -b_1\\ b_1 - 2b_2 = -b_2\end{matrix}\right\} \Rightarrow b_1 = b_2$$ Posons $b_1 = b_2 = 1$

$$\begin{bmatrix}d_1\\d_2\end{bmatrix} = \begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix}$$

$$\left.\begin{matrix}-2c_1 + c_2 = d_1\\ c_1 - 2c_2 = d_2\end{matrix}\right\} \Rightarrow c_1 = \frac{-4}{3} \text{ et } c_2 = \frac{-5}{3}$$

D5.11

$$-\begin{bmatrix}0\\3\end{bmatrix}x = \begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}\begin{bmatrix}d_1\\d_2\end{bmatrix}x$$

$$\left.\begin{array}{l}-2d_1 + d_2 = 0\\ d_1 - 2d_2 = -3\end{array}\right\} \Rightarrow \begin{array}{l}d_1 = 1\\ d_2 = 2\end{array}$$

$$y_p(x) = \begin{bmatrix}2\\1\end{bmatrix}e^{-x} + \begin{bmatrix}1\\1\end{bmatrix}xe^{-x} + \begin{bmatrix}-4/3\\-5/3\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix}x$$

La solution générale est :

$$y(x) = y_h(x) + y_p(x)$$

$$y(x) = c_1 e^{-x}\begin{bmatrix}1\\1\end{bmatrix} + c_2 e^{-3x}\begin{bmatrix}1\\-1\end{bmatrix} + e^{-x}\begin{bmatrix}2\\1\end{bmatrix} + xe^{-x}\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}-4/3\\-5/3\end{bmatrix}$$
$$\begin{bmatrix}1\\2\end{bmatrix}x$$

$$\boxed{y(x) = c_1 e^{-x}\begin{bmatrix}1\\1\end{bmatrix} + c_2 e^{-3x}\begin{bmatrix}1\\-1\end{bmatrix} + e^{-x}\begin{bmatrix}2\\1\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}xe^{-x} + \begin{bmatrix}-4/3\\-5/3\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix}x}$$

D 5.12

Exercice n° 4. 12

$$y' = Ay \qquad y(0) = y_0$$

$$A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} \qquad y_0 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} -3-\lambda & 2 \\ -1 & -1-\lambda \end{vmatrix} = (-3-\lambda)(-1-\lambda) + 2$$
$$= \lambda^2 + 4\lambda + 5$$

$$\det(A - \lambda I) = 0 \Rightarrow \lambda^2 + 4\lambda + 5 = 0$$

$$\lambda_{1,2} = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i \qquad \begin{matrix} \alpha = -2 \\ \beta = 1 \end{matrix}$$

Premier vecteur propre: $\lambda = -2+i$

$$(A - (-2+i) I)\vec{u} = 0$$

$$\begin{bmatrix} -1-i & 2 \\ -1 & 1-i \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} -u_1 - iu_1 + 2u_2 = 0 \\ -u_1 + u_2 - iu_2 = 0 \end{matrix}\right\} \begin{matrix} \text{Posons } u_1 = 2 \\ u_2 = 1+i \end{matrix}$$

$$\vec{u} = \begin{bmatrix} 2 \\ 1+i \end{bmatrix}$$

Deuxième vecteur propre $\lambda_2 = -2-i$

$$(A - (-2-i)I)\vec{v} = 0$$

$$\begin{bmatrix} -1+i & 2 \\ -1 & 1+i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} -v_1 + iv_1 + 2v_2 = 0 \\ -v_1 + iv_2 + v_2 = 0 \end{matrix}\right\} \begin{matrix} \text{Posons } v_1 = 2 \Rightarrow \\ v_2 = 1-i \end{matrix}$$

$$\vec{v} = \begin{bmatrix} 2 \\ 1-i \end{bmatrix}$$

D5.13

$$\vec{y}(x) = \begin{bmatrix} 2 \\ 1+i \end{bmatrix} e^{(-2+i)x} = \begin{bmatrix} 2 \\ 1+i \end{bmatrix} e^{-2x} \cdot e^{ix}$$

$$= \begin{bmatrix} 2 \\ 1+i \end{bmatrix} e^{-2x} \left( \cos x + i \sin x \right)$$

$$= \left( 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) e^{-2x} \left( \cos x + i \sin x \right)$$

$$= \left[ 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cos x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cos x - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sin x \right] e^{-2x} + i e^{-2x} \left[ 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sin x + \begin{bmatrix} \ \\ 1 \end{bmatrix} \right.$$

$$= e^{-2x} \begin{bmatrix} 2\cos x \\ \cos x - \sin x \end{bmatrix} + i\, e^{-2x} \begin{bmatrix} 2 \sin x \\ \sin x + \cos x \end{bmatrix}$$

$$= y_1(x) + i\, y_2(x)$$

Solution générale

$$y(x) = C_1 e^{-2x} \begin{bmatrix} 2\cos x \\ \cos x - \sin x \end{bmatrix} + C_2 e^{-2x} \begin{bmatrix} 2 \sin x \\ \cos x + \sin x \end{bmatrix}$$

Conditions initiales :

$$y(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \Rightarrow C_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\Rightarrow \quad 2C_1 = 1 \Rightarrow C_1 = \frac{1}{2}$$

$$C_1 + C_2 = -2 \Rightarrow C_2 = -2 - \frac{1}{2} = \frac{-5}{2}$$

Solution Particulière :

$$\boxed{y(x) = \frac{1}{2} e^{-2x} \begin{bmatrix} 2\cos x \\ \cos x - \sin x \end{bmatrix} - \frac{5}{2} e^{-2x} \begin{bmatrix} 2\sin x \\ \cos x + \sin x \end{bmatrix}}$$

9.11)

| $i$ | $x_i$ | $y_i$ | $\Delta y_i$ | $\Delta^2 y_i$ | $\Delta^3 y_i$ | $\Delta^4 y_i$ |
|---|---|---|---|---|---|---|
| 0 | 0.0 | $\boxed{1.0}$ | | | | |
| | | | $\boxed{0.22140}$ | | | |
| 1 | 0.2 | 1.22140 | | $\boxed{0.04902}$ | | |
| | | | 0.27042 | | $\boxed{0.01086}$ | |
| 2 | 0.4 | 1.49182 | | 0.05988 | | $\boxed{0.00238}$ |
| | | | 0.3303 | | 0.01324 | |
| 3 | 0.6 | 1.82212 | | 0.07312 | | |
| | | | 0.40342 | | | |
| 4 | 0.8 | 2.22554 | | | | |

$$r = (x-0)/0.2 = x/0.2 = 0.05/0.2 = 0.25$$

$$p_4(r) = 1 + r(0.22140) + \frac{r(r-1)}{2}(0.04902) + \frac{r(r-1)(r-2)}{6}(0.01086) + \frac{r(r-1)(r-2)(r-3)}{24}(0.00238)$$

$$= 1 + 0.25 \times 0.22140 - 0.09375\,(0.04902) + 0.05469 \times 0.01086 - 0.03760 \times 0.00238$$

$$= 1 + 0.05535 - 0.004596 + 0.0005939 - 0.00008949$$

$$= 1.05126$$

$$\boxed{p(0.25) = 1.05126}$$

$P(0.05) = 1 + 0.01107 - 0.001164225 + 0.00016765125 - 0{,}0000270967$

$= 1.010046329$

D5.15
MAT2784A.

$f(0.05) = 1.010046329$

Exercice n° 9.13

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 8.3 | 17.56492 | 3.116256 |
| 8.6 | 18.50515 | 3.151762 |

| $z$ | $f(z)$ | $1^{ère}$ D.D | $2^{ème}$ D.D | $3^{ème}$ D.D |
|---|---|---|---|---|
| 8.3 | $\boxed{17.56492}$ | | | |
| | | $\boxed{3.116256}$ | | |
| 8.3 | 17.56492 | | $\boxed{0.05948}$ | $\boxed{-0{,}002022}$ |
| | | 3.1341 | | |
| 8.6 | 18.50515 | | 0.0587333 | |
| | | 3.151762 | | |
| 8.6 | 18.50515 | | | |

$$P_3(x) = 17.56492 + (x-8.3)(3.116256) + (x-8.3)^2(0.05948) + (x-8.3)^2(x-8.6)(-0.002022)$$

$$P_3(x) = 17.56492 + 3.116256\,(x-8.3) + 0.05948(x-8.3)^2 - 0.002022(x-8.3) \; (x-8.$$

$$\boxed{P_3(x) = 17.56492 + 3.116256(x-8.3) + 0.05948(x-8.3)^2 - 0.002022(x-8.3)^2(x-8.6)}$$