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MAT2784 A

Devoir 8

4.4. $A = \begin{bmatrix} -1 & 1 & 4 \\ -2 & 2 & 4 \\ -1 & 0 & 4 \end{bmatrix}$ $|A-\lambda I| = \begin{vmatrix} -1-\lambda & 1 & 4 \\ -2 & 2-\lambda & 4 \\ -1 & 0 & 4-\lambda \end{vmatrix}$

$-1(4-4(2-\lambda)) + (4-\lambda)[(-1-\lambda)(2-\lambda)+2]$

$-(4-8+4\lambda) + (4-\lambda)[(\cancel{-2}+\lambda-2\lambda+\lambda^2+\cancel{2}]$

$+4-4\lambda + (4-\lambda)(\lambda^2-\lambda) = 4-4\lambda+4\lambda^2-4\lambda-\lambda^3+\lambda^2 = 0$

$-\lambda^3+5\lambda^2-8\lambda+4=0$ $\lambda_1 = 1$ (valeur évidente)

$$\begin{array}{l|l} -\lambda^3+5\lambda^2-8\lambda+4 & (\lambda-1) \\ \hline -\;(-\lambda^3+\lambda^2) & -\lambda^2+4\lambda-4 \\ \quad 0+4\lambda^2-8\lambda & \\ \quad -\;(4\lambda^2-4\lambda) & \\ \quad\quad 0-4\lambda+4 & \\ \quad\quad -\;(4\lambda+4) & \\ \quad\quad\quad 0 & \end{array}$$

On obtient :
$(\lambda-1)(-\lambda^2+4\lambda-4)=0$
$(\lambda-1)(\lambda-2)(\lambda-2)=0$

valeurs propres $\rightarrow$ $\lambda_1 = 1$ $\lambda_2 = \lambda_3 = 2$

$\lambda = 1 \rightarrow$
$|A-\lambda_1 I| = \begin{vmatrix} -2 & 1 & 4 \\ -2 & 1 & 4 \\ -1 & 0 & 3 \end{vmatrix}$ $\begin{matrix} 2L_3 - L_1 \rightarrow L_3 \\ L_2 - L_1 \rightarrow L_2 \end{matrix}$ $\begin{vmatrix} -2 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & -1 & 2 \end{vmatrix}$ $\begin{matrix} L_3 + L \rightarrow L_1 \\ L_1/2 \rightarrow L_1 \end{matrix}$

$\begin{bmatrix} -1 & 0 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow Rg(A-\lambda_1 I) = 2$
$N(A) = 3$ $3-2=1 \rightarrow$ on admet un vecteur propre $\vec{u}$.

$\begin{bmatrix} -1 & 0 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $-u_1+3u_3=0$
$-u_2+2u_3=0.$

posons $u_3 = 1 \rightarrow u_1 = 3$ $u_2 = 2$

$\vec{u} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ donc la solution : $y_1(x) = c_1 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} e^x$

$\lambda = 2 \rightarrow$ (2 solutions)

$|A-\lambda_2 I| = \begin{vmatrix} -3 & 1 & 4 \\ -2 & 0 & 4 \\ -1 & 0 & 2 \end{vmatrix}$ $\begin{matrix} 2L_3 - L_2 \rightarrow L_3 \\ 3L_2 - 2L_1 \rightarrow L_2 \end{matrix}$ $\begin{vmatrix} -3 & 1 & 4 \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{vmatrix}$ $\begin{matrix} L_2 + 2L_1 \rightarrow L_1 \\ L_2/2 \rightarrow L_2 \end{matrix}$

$$\begin{vmatrix} -6 & 0 & 12 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{vmatrix} \quad L_1/6 \to L_1 \quad \begin{vmatrix} -1 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{vmatrix}$$

$Rg(A-\lambda_2 I) = 2$
$N(A) = 3$
$3-2=1$ un vecteur propre $\vec{v}$

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-v_1 + 2v_3 = 0$
$-v_2 + 2v_3 = 0$
on pose $v_3 = 1$ $v_1 = 2$ $v_2 = 2$.

donc $\vec{v} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ $\quad y_2(x) = C_2 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} e^{2x}$

Il faut trouver une 3ème solution : $\lambda_3 = 2$

$\vec{v} \times \vec{w} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ $\quad \vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. $\quad y_3(x) = C_3 \left(x \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) e^{2x}$

Solution générale :

$$\boxed{y(x) = C_1 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} e^{x} + C_2 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} e^{2x} + C_3 \left(x \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) e^{2x}}$$

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4.10. $A = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$ $\quad f(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad$ On cherche $y' = Ay + f(x)$

$(A - \lambda I) = \begin{bmatrix} 1-\lambda & \sqrt{3} \\ \sqrt{3} & -1-\lambda \end{bmatrix}$ $\quad (-1-\lambda)(1-\lambda) - (\sqrt{3})(\sqrt{3}) = 0$

$-1 + \lambda - \lambda + \lambda^2 - 3 = 0$

$\lambda^2 - 4 = 0 \quad \lambda^2 = 4 \quad \lambda_1 = 2 \; \lambda_2 = -2.$

$\lambda_1 = 2 \to (A - \lambda_1 I) =$

$$\begin{bmatrix} -1 & \sqrt{3} \\ \sqrt{3} & -3 \end{bmatrix} \quad L_2 + \sqrt{3} L_1 \quad \begin{bmatrix} -1 & \sqrt{3} \\ 0 & 0 \end{bmatrix}$$

$Rg = 1$ $\quad N(A) = 2$
$2-1=1$ (un vecteur propre $\vec{u}$)

$$\begin{bmatrix} -1 & \sqrt{3} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad -u_1 + \sqrt{3} u_2 = 0$$

Soit $u_2 = C_1$ et $u_1 = \sqrt{3} C_1$ $\quad \vec{u} = C_1 \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$

$\lambda_2 = -2 \to (A - \lambda_2 I) =$

$$\begin{bmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} \quad 3L_2 - \sqrt{3} L_1 \to L_2 \quad \begin{bmatrix} 3 & \sqrt{3} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3v_1 + \sqrt{3} v_2 = 0$. $\quad$ Soit $v_2 = C_2$ $\quad v_1 = -\frac{\sqrt{3}}{3} C_2$

$$\vec{V} = \begin{bmatrix} -\sqrt{3}/3 \\ 1 \end{bmatrix} C_2 = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} C_2$$

$$y(x) = C_1 \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} e^{2x} + C_2 \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} e^{-2x}$$

$y' = Ay + f(x)$

$y_p = \begin{bmatrix} a \\ b \end{bmatrix}$ $y_p' = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $f(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} a & \sqrt{3}b \\ -a\sqrt{3} & b \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$a + \sqrt{3}b + 1 = 0$

$-a\sqrt{3} + b = 0$ $\quad b = a\sqrt{3}$

$a + \sqrt{3}a\sqrt{3} + 1 = 0$

$a + 3a = -1 \quad 4a = 1 \quad a = -1/4 \quad b = -\sqrt{3}/4$

Solution unique:

$$\boxed{y(x) = C_1 \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} e^{2x} + C_2 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{-2x} - \frac{1}{4} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}}$$

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4.13. $A = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$ $y_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $y' = Ay$ avec $y(0) = y_0$.

$(A - \lambda I) = \begin{vmatrix} 1-\lambda & \sqrt{3} \\ \sqrt{3} & -1-\lambda \end{vmatrix}$ $\quad (1-\lambda)(-1-\lambda) - (\sqrt{3}\cdot\sqrt{3})$

$-1 + \lambda - \lambda + \lambda^2 - 3 = 0 \quad \lambda^2 = 4$

$\lambda_1 = 2 \quad \lambda_2 = -2$

voire numéro 4.10.

$$\vec{U} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} C_1 + \vec{V} = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} C_2$$

Solution générale $y(x) = C_1 \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} e^{2x} + C_2 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{-2x}$

Conditions initiales

$$x = 0 \to C_1 \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} (1) + C_2 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} (1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\sqrt{3}\,C_1 + C_2 = 1 \qquad C_1 - C_2\sqrt{3} = 0. \qquad C_1 = C_2\sqrt{3}$

$3C_2 + C_2 = 1 \qquad C_2 = 1/4 \qquad C_1 = \frac{\sqrt{3}}{4}$

Solution unique:

$$y(x) = \frac{\sqrt{3}}{4}\begin{bmatrix}\sqrt{3}\\1\end{bmatrix}e^{2x} + \frac{1}{4}\begin{bmatrix}1\\-\sqrt{3}\end{bmatrix}e^{-2x}$$

6.9. $y'' - 3y' + 2y = 0$

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \dots$$
$$y'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \dots$$
$$y''(x) = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \dots$$

on Remplace dans l'équation initiale →

$$2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \dots$$
$$-3a_1 - 6a_2x - 9a_3x^2 - 12a_4x^3 - 15a_5x^4 + \dots$$
$$+2a_0 + 2a_1x + 2a_2x^2 + 2a_3x^3 + 2a_4x^4 + 2a_5x^5 + \dots$$

$$\left.\begin{aligned} x^0 &\to 2a_2 - 3a_1 + 2a_0 = 0\\ x^1 &\to 6a_3 - 6a_2 + 2a_1 = 0\\ x^2 &\to 12a_4 - 9a_3 + 2a_2 = 0\\ x^3 &\to 20a_5 - 12a_4 + 2a_3 = 0\end{aligned}\right\}$$

4 équations / 5 inconnues. Solution en fonction de a_1 et a_0.

$$\boxed{a_2 = \frac{3a_1 - 2a_0}{2}}$$

$$a_3 = \frac{6a_2 - 2a_1}{6} = a_2 - \frac{a_1}{3}, \text{ Remplace } a_2 \to$$

$$a_3 = \frac{3a_1}{2} - a_0 - \frac{a_1}{3} = \boxed{\frac{7a_1 - 6a_0}{6} = a_3}$$

$$a_4 = \frac{9a_3 - 2a_2}{12}, \text{ Remplace } a_2 \text{ et } a_3.$$

$$a_4 = \frac{9\left(\frac{7a_1 - 6a_0}{6}\right) - 2\left(\frac{3a_1 - 2a_0}{2}\right)}{12} = \frac{63a_1 - 54a_0 - 18a_1 + 12a_0}{72}$$

$$a_4 = \frac{45a_1 - 42a_0}{72} \to \boxed{a_4 = \frac{5}{8}a_1 - \frac{7}{12}a_0}$$

$$a_5 = \frac{12a_4 - 2a_3}{20} = \frac{12\left(\frac{45a_1 - 42a_0}{72}\right) - 2\left(\frac{7a_1 - 6a_0}{6}\right)}{20}$$

$$= \frac{\frac{45a_1 - 42a_0}{6} + \frac{-14a_1 + 12a_0}{6}}{20} = \frac{31a_1 - 30a_0}{120} = \boxed{a_5 = \frac{31}{120}a_1 - \frac{1}{4}a_0}$$

$$\boxed{y(x) = a_0 + a_1x + \left(\frac{3a_1 - 2a_0}{2}\right)x^2 + \left(\frac{7a_1 - 6a_0}{6}\right)x^3 + \left(\frac{5}{8}a_1 - \frac{7}{12}a_0\right)x^4 + \left(\frac{31}{120}a_1 - \frac{1}{4}a_0\right)x^5 + \dots}$$

6.28. $f(x) = e^{2x}$ $-1 < x < 1$ 3 premiers coefficients + approximation.

posons $f(x) = \sum_{m=0}^{\infty} a_m P_m(x)$ $-1 < x < 1$

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$

$$a_m = \frac{2m+1}{2}\int_{-1}^{1} f(x)\, P_m(x)\, dx$$

$$m = 0: \; a_0 = \frac{1}{2}\int_{-1}^{1} e^{2x}(1)\, dx = \frac{1}{4}\left[e^{2x}\right]_{-1}^{1}$$

$$\boxed{a_0 = \frac{1}{4}(e^2 - e^{-2})}$$

$m = 1$: $a_1 = \frac{3}{2}\int_{-1}^{1} e^{2x} x\, dx$ → intégration par parties

$u = x$ $du = dx$

$dv = e^{2x}dx$ $v = \frac{e^{2x}}{2}$

$$\frac{3}{2}\left[\frac{xe^{2x}}{2}\Big|_{-1}^{1} - \int_{-1}^{1}\frac{e^{2x}}{2}dx\right]$$

$$= \frac{3}{2}\left[\frac{xe^{2x}}{2}\Big|_{-1}^{1} - \frac{e^{2x}}{4}\Big|_{-1}^{1}\right] = \frac{3}{2}\left[\frac{e^2}{2} + \frac{e^{-2}}{2} - \frac{e^2}{4} + \frac{e^{-2}}{4}\right]$$

$$= \frac{3}{2}\left[\frac{e^2}{4} + \frac{3e^{-2}}{4}\right] \quad \boxed{a_1 = \frac{3}{8}(e^2 + 3e^{-2})}$$

$$m = 2: \; a_2 = \frac{5}{2}\int_{-1}^{1} e^{2x}\frac{1}{2}(3x^2 - 1)\, dx$$

$$\frac{5}{4}\left[\int_{-1}^{1} 3x^2e^{2x}dx - \int_{-1}^{1} e^{2x}dx\right] \to \frac{5}{4}\left[\frac{-e^{2x}}{2}\Big|_{-1}^{1} + 3\int_{-1}^{1} x^2e^{2x}dx\right]$$

intégration par parties

$u = x^2$ $du = 2x\,dx$

$dv = e^{2x}dx$ $v = \frac{e^{2x}}{2}$

$$\frac{5}{4}\left[\frac{-e^{2x}}{2}\Big|_{-1}^{1} + \frac{3x^2e^{2x}}{2}\Big|_{-1}^{1} - 3\int\frac{e^{2x}}{2}2x\,dx\right]$$

$$\frac{5}{4}\left[\frac{-e^{2x}}{2}\Big|_{-1}^{1} + \frac{3x^2e^{2x}}{2}\Big|_{-1}^{1} - \int 3e^{2x}x\,dx\right]$$

$$\frac{5}{4}\left[\frac{-e^{2x}}{2}\Big|_{-1}^{1} + \frac{3x^2e^{2x}}{2}\Big|_{-1}^{1} - \frac{3xe^{2x}}{2}\Big|_{-1}^{1} + \frac{3e^{2x}}{4}\Big|_{-1}^{1}\right]$$

$$\frac{5}{8}\left[e^{-2x}\Big|_{-1}^{1} + 3x^2e^{2x}\Big|_{-1}^{1} - 3xe^{2x}\Big|_{-1}^{1} + \frac{3e^{2x}}{2}\Big|_{-1}^{1}\right]$$

$$\frac{5}{8}\left[-e^2 + e^{-2} + 3e^2 - 3e^{-2} + (-3e^2 - 3e^{-2}) + \frac{3}{2}e^2 - \frac{3}{2}e^{-2}\right]$$

$$= \frac{5}{8}\left[\frac{e^2}{2} - \frac{13e^{-2}}{2}\right] \rightarrow \boxed{a_2 = \frac{5}{16}\left[e^2 - 13e^{-2}\right]}$$

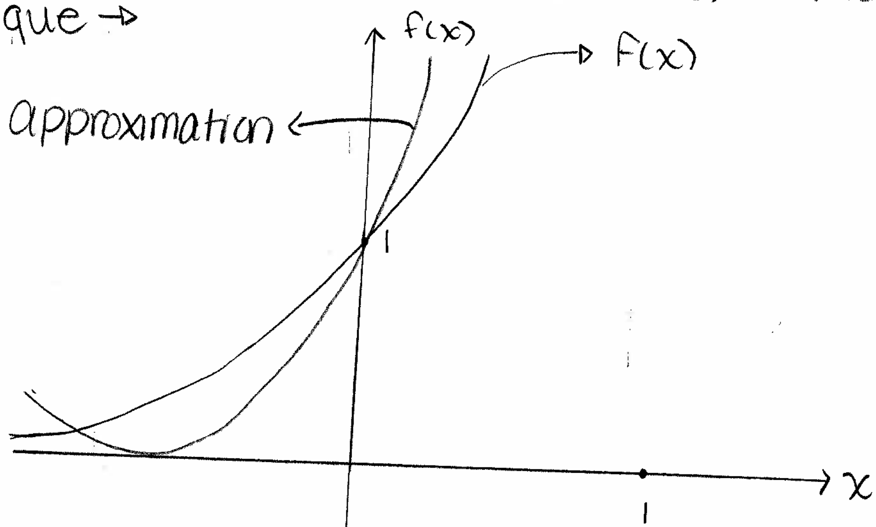
Approximation $\rightarrow$ $f(x) = a_0 P_0(x) + a_1 P_1(x) + a_2 P_2(x)$

$$f(x) = \frac{1}{4}(e^2 - e^{-2}) + \frac{3}{8}(e^2 + 3e^{-2})x + \frac{5}{16}(e^2 - 13e^{-2})\left(\frac{1}{2}(3x^2-1)\right)$$

$$f(x) = 1{,}813430204 + 2{,}923148231x + 2{,}638920664x^2 - 0{,}879640221$$

$$f(x) = 2{,}638920664x^2 + 2{,}923148231x + 0{,}933789983$$

Graphique $\rightarrow$



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6,32 $I = \int_{0,3}^{1,7} e^{-x^2}\,dx.$ $\quad b = 1{,}7 \quad a = 0{,}3$

$$x = \frac{(b-a)t + b + a}{2} \qquad dx = \frac{(b-a)}{2}\,dt. \qquad f(x) = e^{-x^2}$$

$$x = \frac{1{,}4t + 2}{2} = 0{,}7t + 1 \qquad dx = 0{,}7dt$$

$$I = \int_{-1}^{1} e^{-(0{,}7t+1)^2}(0{,}7)\,dt = 0{,}7\int_{-1}^{1} e^{-(0{,}7t+1)^2}\,dt.$$

formule Gauss à 3 points $\rightarrow$

$$I = \left(\frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)\right) \times 0{,}7$$

$$0{,}7\left(\frac{5}{9}(0{,}8109372)+\frac{8}{9}(0{,}367879441)+\frac{5}{9}(0{,}09269550 1)\right)$$

$I = 0{,}58031548$. ✓

10,16. 3 premier pas ode23 ordre 3.
3 pas suivants ABM3 - $h = 0{,}1$ / 6 décimales près

$y' = x^2 + 2y^2 \quad y(0) = 1. \quad f(x) = y' = x^2 + 2y^2$

méthode ode 23.

n	x_n	y_n	K_1	K_2	K_3	K_4
0	0	1	0,2	0,24225	0,279840	0,313284
1	0,1	1,249568	0,313284	0,397735	0,482242	0,559174
2	0,2	1,666095	0,559175	0,763386	1,009859	1,252652
3	0,3	2,493644	1,252652.	1,959093	3,155079	4,676564

$x_n = x_0 + hn \rightarrow 0 + 0{,}1n \rightarrow x_n = 0{,}1n$

$n=0$: $K_1 = 0{,}1 f(0, 1) = 0{,}2$

$K_2 = 0{,}1 f(0 + (1/2)(0{,}1), 1 + (1/2)K_1) = 0{,}24225$

$K_3 = 0{,}1 f(0 + (3/4)(0{,}1), 1 + (3/4)K_2) = 0{,}279840$

$K_4 = 0{,}1 f(0 + 0{,}1, 1 + (2/9)K_1 + (1/3)K_2 + (4/9)K_3) = 0{,}313284$

$y_1 = 1 + \frac{2}{9}K_1 + \frac{1}{3}K_2 + \frac{4}{9}K_3 = 1{,}249568$

$n=1$: $K_1 : 0{,}1 f(0{,}1, 1{,}249568) = 0{,}313284$

$K_2 : 0{,}1 f(0{,}1 + 0{,}5(0{,}1), 1{,}249568 + 0{,}5 \cdot K_1) = 0{,}397735$

$K_3 : 0{,}1 f(0{,}1 + 0{,}75(0{,}1), 1{,}249568 + 0{,}75(K_2)) = 0{,}482242$

$K_4 : 0{,}1 f(0{,}1 + 0{,}1, 1{,}249568 + (2/9)K_1 + (1/3)K_2 + (4/9)K_3) = 0{,}559174$

$y_2 = 1{,}249568 + \frac{2}{9}K_1 + \frac{1}{3}K_2 + \frac{4}{9}K_3 = 1{,}666095$

$n=2$: $K_1 : 0{,}1 f(0{,}2, 1{,}666095) = 0{,}559175$

$K_2 : 0{,}1 f(0{,}2 + 0{,}5(0{,}1), 1{,}666095 + 0{,}5 K_1) = 0{,}763386$

$K_3 : 0{,}1 f(0{,}2 + 0{,}75(0{,}1), 1{,}666095 + 0{,}75 K_2) = 1{,}009859$

$K_4 : 0{,}1 f(0{,}2 + 0{,}1, 1{,}666095 + (2/9)K_1 + (1/3)K_2 + (4/9)K_3 = 1{,}252652$

$y_3 = 1{,}666095 + \frac{2}{9}K_1 + \frac{1}{3}K_2 + \frac{4}{9}K_3 = 2{,}493644$

$n=3$. $K_1 : 0{,}1 f(0{,}3, 2{,}493644) = 1{,}252652$

$K_2 : 0{,}1 f(0{,}3 + 0{,}5(0{,}1), 2{,}493644 + 0{,}5K_1) = 1{,}959093$

$K_3 : 0{,}1 f(0{,}3 + 0{,}75(0{,}1), 2{,}493644 + 0{,}75 K_2) = 3{,}155079$

$K_4 : 0{,}1 f(0{,}3 + 0{,}1, 2{,}493644 + (2/9) K_1 + (1/3) K_2 + (4/9) K_3) =$
$4{,}676564$

Erreur :

n	E
0	-0,001768555
1	-0,004925555
2	-0,019591041
3	-0,157738138

$$E = -\frac{5}{72} K_1 + \frac{1}{12} K_2 + \frac{1}{9} K_3 - \frac{1}{8} K_4$$

méthode de ABM3

3 pas suivants :

$y_0 = 1$

$y_1 = 1{,}249568$

$y_2 = 1{,}666095$

$y_3 = 2{,}493644$

$$y^p_{n+1} = y_n^c + \frac{h}{12}(23 f_n^c - 16 f_{n-1}^c + 5 f_{n-2}^c)$$

$$y^c_{n+1} = y_n^c + \frac{h}{12}(5 f_{n+1}^p + 8 f_n^c - f_{n-1}^c)$$

$f_k^c = f(x_k, y_k^c)$

$f_k^p = f(x_k, y_k^p)$

$$y_6^p = y_5^c + \frac{0{,}1}{12}(23 f_5^c - 16 f_4^c + 5 f_3^c) = 172{,}987678$$

$$y_6^c = y_5^c + \frac{0{,}1}{12}(5 f_6^p + 8 f_4^5 - f_4^c) = 2568{,}747122$$

$$E = \frac{-1}{10}[y_6^c - y_6^p] = -239{,}575944$$

$$y_4 p = y_3^c + \frac{0{,}1}{12}(23 f_3^c - 16 f_2^c + 5 f_1^c) = 4{,}272835$$

$$y_4^c = y_3^c + \frac{0{,}1}{12}(5 f_4^p + 8 f_3^c - f_2^c) = 4{,}805864$$

$$E = -\frac{1}{10}[y_4^c - y_4^p] = -0{,}0533029.$$

$$y_5 p = y_4^c + \frac{0{,}1}{12}(23 f_4^c - 16 f_3^c + 5 f_2^c) = 12{,}256122$$

$$y_5 c = y_4^c + \frac{0{,}1}{12}(5 f_5^p + 8 f_4^c - f_3^c) = 20{,}320002$$

$E = -0{,}806388$ ✓