MAT2784 A
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Devoir 3

D 3.1

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$\boxed{2.5}$ $y'' + 9y = 0 \qquad y(0) = 0 \qquad y'(0) = 1$

$y = e^{\lambda x}$

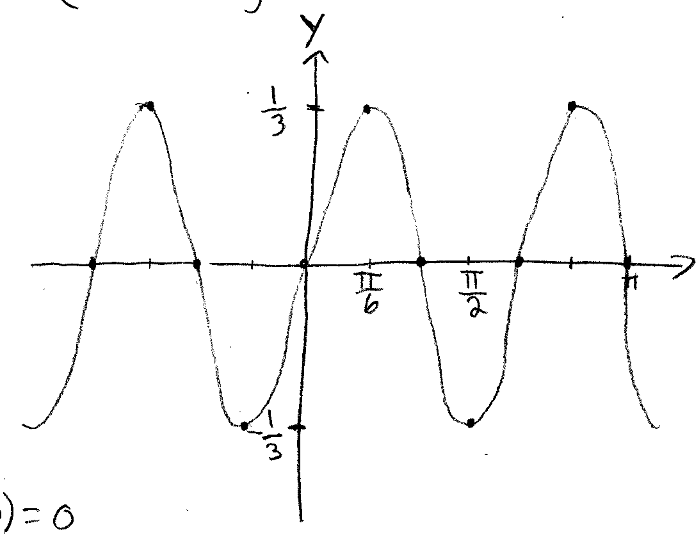
$\lambda^2 e^{\lambda x} + 9e^{\lambda x} = 0 \qquad e^{\lambda x}(\lambda^2 + 9) = 0$

$\lambda^2 + 9 = 0$

$\lambda = \sqrt{-9} = \pm 3i$

$y_1 = e^{3xi} \qquad y_2 = e^{-3xi}$

$y = c_1 e^{3xi} + c_2 e^{-3xi}$



- $0 = c_1 + c_2$ // car $y(0) = 0$
- $y' = 3ic_1 e^{3xi} - 3ic_2 e^{-3xi}$

$1 = 3i(c_1 - c_2)$ // car $y'(0) = 1$

$\frac{1}{3i} = 2c_1 \qquad c_1 = \frac{1}{6i} \qquad c_2 = \frac{-1}{6i}$

$\therefore \quad y = \frac{e^{3xi} - e^{-3xi}}{6i}$

$e^{xi} = \cos x + i \sin x$

$y = \frac{\cos 3x + i\sin 3x - \cos(-3x) - i\sin(-3x)}{6i}$

$y = \frac{\sin 3x + \sin 3x}{6}$

$$\boxed{y = \frac{\sin 3x}{3}}$$

D3.2

$\boxed{2.9}$ $y''+4y=0$ $\qquad y(0)=1 \qquad y'(0)=2$

trouver amplitude et periode

$$\lambda^2+4=0$$
$$\lambda=\pm 2i$$

$y_1=e^{2xi} \qquad y_2=e^{-2xi}$

$\therefore\; y=c_1e^{2xi}+c_2e^{-2xi}$ $\qquad y=c_1(\cos 2x+i\sin 2x)+c_2(\cos -2x+i\sin -2x)$

- $1=c_1+c_2 \qquad c_1=1-c_2$ // car $y(0)=1$
- $y'=2ic_1e^{2xi}-2ic_2e^{-2xi}$
  $2=2i(c_1-c_2)$ // car $y'(0)=1$
  $\frac{1}{i}=c_1-c_2 \qquad c_2=c_1-\frac{1}{i}$

$c_1=1-c_1+\frac{1}{i} \qquad 2c_1=1+\frac{1}{i} \qquad c_1=\frac{1}{2}+\frac{1}{2i}$

$c_2=1-c_1 \qquad c_2=1-\frac{1}{2}-\frac{1}{2i}=\frac{1}{2}-\frac{1}{2i}=c_2$

$$\therefore\; y=\frac{e^{2xi}}{2}+\frac{e^{2xi}}{2i}+\frac{e^{-2xi}}{2}-\frac{e^{-2xi}}{2i}$$

$$y=\frac{\cos 2x}{2}+\cancel{\frac{i\sin 2x}{2}}+\cancel{\frac{\cos 2x}{2i}}+\frac{i\sin 2x}{2i}+\frac{\cos -2x}{2}+\cancel{\frac{i\sin -2x}{2}}-\cancel{\frac{\cos -2x}{2i}}-\frac{i\sin -2x}{2i}$$

$$y=\cos 2x+\sin 2x$$

$$\text{Amplitude}=\frac{\max(y(x))-\min(y(x))}{2}=\frac{2\sqrt{2}}{2}=\sqrt{2}$$

$$\text{Période}=\frac{2\pi}{2}=\pi$$

$\boxed{2.16}$ $x^2 y'' + x y' + 4y = 0$

posons $y = x^m$ alors selon Euler-Cauchy

$m^2 + (a-1)m + b = 0$ $a = 1$ $b = 4$

$m^2 + 4 = 0$ $m = \pm 2i$

//On a deux racines complexes, donc

$U_1 = x^{2i} = e^{2i\ln x} = \cos(2\ln x) + i\sin(2\ln x)$
$U_2 = x^{-2i} = e^{-2i\ln x} = \cos(2\ln x) - i\sin(2\ln x)$

$y_1 = \cos(2\ln x)$ $y_2 = \sin(2\ln x)$

$\boxed{y = c_1 \cos 2\ln x + c_2 \sin 2\ln x}$

$= c_1 \cos(\ln x^2) + c_2 \sin(\ln x^2)$

$\boxed{2.17}$ $x^2y'' + 4xy' + 2y = 0$ $\quad y(1) = 1 \quad y'(1) = 2$

• posons $y = x^m$ alors (Euler-Cauchy)

! $m^2 + (a-1)m + b = 0$ $\quad a = 4 \quad b = 2$

$m^2 + 3m + 2 = 0$

∴ alors $m_1 = -2 \quad m_2 = -1$

On a deux racines réelles

$$y(x) = \frac{c_1}{x^2} + \frac{c_2}{x}$$

$1 = c_1 + c_2 \qquad c_1 = 1 - c_2$ // car $y(1) = 1$

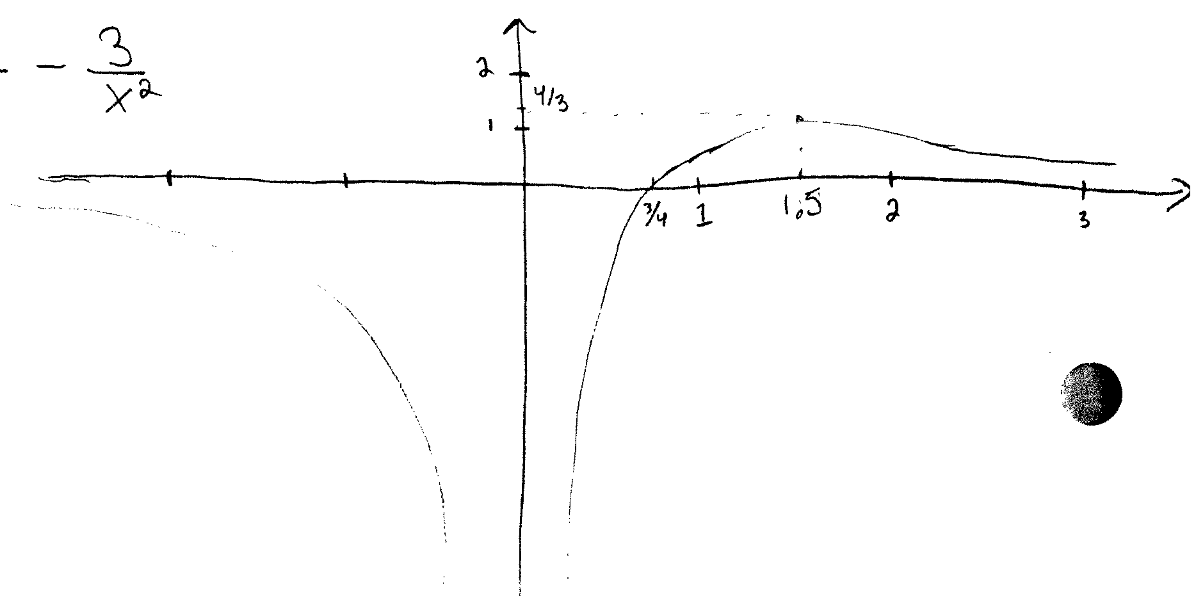
$$y' = -\frac{2c_1}{x^3} - \frac{c_2}{x^2}$$

$2 = -2c_1 - c_2 \qquad -2 - 2c_1 = c_2$ // car $y'(1) = 2$

∴ $c_1 = 1 + 2 + 2c_1 \qquad c_1 = -3$

∴ $c_2 = -2 - 2 \cdot -3 \qquad c_2 = 4$

$$y = \frac{4}{x} - \frac{3}{x^2}$$



$\boxed{3.7}$ $y''' - y'' - y' + y = 0 \qquad y(0) = 0 \quad y'(0) = 5 \quad y''(0) = 2$

equation linéaire homogene

$y = e^{\lambda x} \quad y' = \lambda e^{\lambda x} \quad y'' = \lambda^2 e^{\lambda x} \quad y''' = \lambda^3 e^{\lambda x}$

$e^{\lambda x}(\lambda^3 - \lambda^2 - \lambda + 1) = 0$

$\Rightarrow \quad \lambda^3 - \lambda^2 - \lambda + 1 = 0$

$(\lambda - 1)(\lambda^2 - 1) = 0$

$\lambda_1 = 1 \quad \lambda^2 = 1$

$\lambda_2 = -1 \quad \lambda_3 = 1$

<u>Solution générale</u>

$y(x) = C_1 e^x + C_2 e^{-x} + C_3 x e^x$

$y'(x) = C_1 e^x - C_2 e^{-x} + C_3 x e^x + C_3 e^x$

$y''(x) = C_1 e^x + C_2 e^{-x} + C_3 x e^x + C_3 e^x + C_3 e^x$

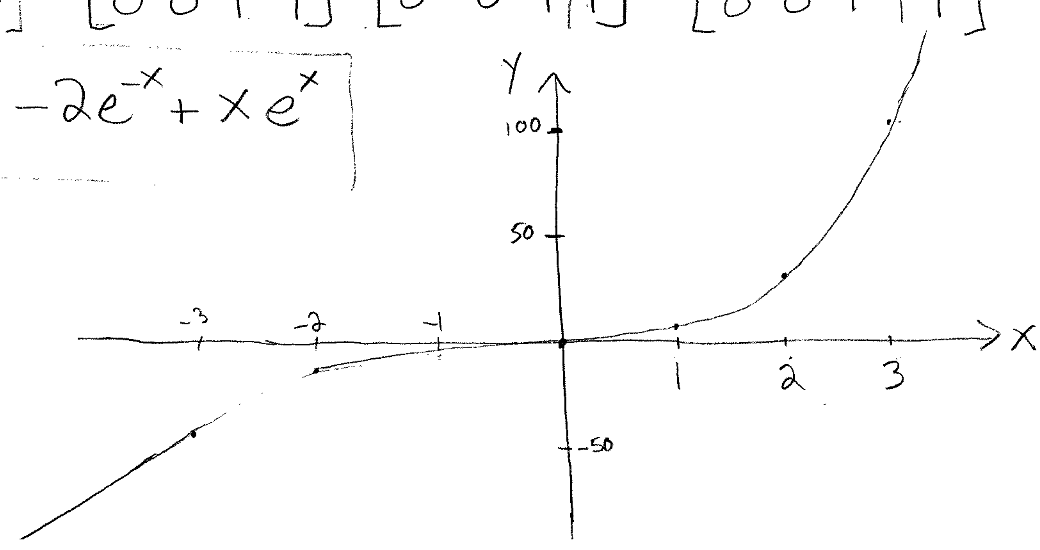
$y(0) = C_1 + C_2 = 0$

$y'(0) = C_1 - C_2 + C_3 = 5$

$y''(0) = C_1 + C_2 + 2C_3 = 2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 5 \\ 1 & 1 & 2 & 2 \end{array}\right] \sim \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 5 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\boxed{y(x) = 2e^x - 2e^{-x} + x e^x}$$



$\boxed{3.10}$ $y_1(x) = 1+x$ $\quad y_2(x) = x$ $\quad y_3(x) = x^2$

$y_1' = 1 \quad y_2' = 1 \quad y_3' = 2x$

$y_1'' = 0 \quad y_2'' = 0 \quad y_3'' = 2$

$$W = \begin{vmatrix} 1+x & x & x^2 \\ 1 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2(1+x) - 2x = 2 + 2x - 2x = 2$$

$W \neq 0$ donc y_1 y_2 et y_3 sont linéairement indépendantes

$\boxed{\text{Numero surprise}}$ Accélération de Steffensen

n	$g_4(x)$ $\sqrt{10/(4+x)}$	S_n	Z_1	Z_2
0	1,5	1,5	1,348399725	1,367376372
1	1,348399724	1,365265223	1,365225534	1,365230583
2	1,367376372	1,365230013	1,365230013	1,365230013
3	1,364957015	1,365230013414	1,365230013414	1,365230013414
4	1,365264748	NaN	NaN	NaN
5	1,365225594			
6	1,365230576			
7				

$\boxed{\text{Le zero est à } x = 1,36523001341410}$

$g'(1,3652300134141) = -0,12722\ldots \neq 0$
Donc la convergence de la récurrence est 1

Le manuel dit (p161) que le processus de Steffensen transforme une récurrence d'ordre 1 en ordre 2.

$\boxed{\text{Donc la convergence est d'ordre 2.}}$

$\boxed{8.1}$ Construire les polynômes de Lagrange de dégré 1 et 2

$f(x) = \ln(x+1)$ $\quad x_0 = 0 \;,\; x_1 = 0{,}6 \;,\; x_2 = 0{,}9$

evaluer à $x = 0{,}45$

trouver erreur actuelle

points : $\underset{(x_0\,;\,f_0)}{(0\,,\,\ln 1)}$ $\underset{(x_1\,,\,f_1)}{(0{,}6\,;\,\ln 1{,}6)}$ $\underset{(x_2\,,\,f_2)}{(0{,}9\,,\,\ln 1{,}9)}$

<u>Degré 1</u>, avec les 2 premiers points

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x - 0{,}6}{-0{,}6} = 1 - \frac{5x}{3}$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0} = \frac{x}{0{,}6} = \frac{5x}{3}$$

$$p_1(x) = f(x_0)L_0(x) + f(x_1)L_1(x) = \ln 1 - \frac{\ln 1 \cdot 5x}{3} + \frac{\ln 1{,}6 \cdot 5x}{3}$$

$$\boxed{p_1(x) = \frac{5}{3} \cdot x \cdot \ln 1{,}6}$$

<u>Evaluer à $x = 0{,}45$</u>

$$p_1(x) = \frac{5}{3} \cdot 0{,}45 \cdot \ln 1{,}6 = \boxed{0{,}352502721}$$

$$\ln(0{,}45 + 1) = 0{,}371563556$$

<u>Erreur</u>

$$\epsilon = \ln(0{,}45+1) - p_1(x) = 0{,}0190608$$

✓

<u>Degré 2</u> avec les 3 points

$$L_0 = \frac{(x-0,6)(x-0,9)}{(0-0,6)(0-0,9)} = \frac{x^2-1,5x+0,54}{0,54}$$

$$L_1 = \frac{(x-0)(x-0,9)}{(0,6-0)(0,6-0,9)} = \frac{x^2-0,9x}{-0,18}$$

$$L_2 = \frac{(x-0)(x-0,6)}{(0,9-0)(0,9-0,6)} = \frac{x^2-0,6x}{0,27}$$

$$p_2(x) = 0 \cdot L_0 + \ln 1{,}6 \cdot \left(\frac{x^2-0,9x}{-0,18}\right) + \ln 1{,}9 \cdot \left(\frac{x^2-0,6x}{0,27}\right)$$

$$p_2(x) = \ln 1.6 \cdot \left(\frac{x^2-0,9x}{-0,18}\right) + \ln 1.9 \left(\frac{x^2-0,6x}{0,27}\right)$$

<u>Evaluer à $x = 0,45$</u>

$$p_2(x) = \boxed{0,368290611}$$

$$\ln(0,45+1) = 0,371563556$$

<u>Erreur</u>

$$\epsilon = \ln(1,45) - p_2(0,45) = \boxed{0,003272945}$$ ✓