MAT 5187, Assignment 3, Exercise 1

Taken form the Lecture notes for MAT 2384, pp. 270–271, or MAT 2784, pp. 276–277 (see my home page).

REPRODUCE THE RESULTS OF THE FOLLOWING EXAMPLES 5.16 AND 5.17 ON MATLAB OR EQUIVALENT RESULTS ON OTHER SOFTWARES AND PRINT YOUR RESULTS.

Example 5.16 Use the five Matlab ode solvers to solve the non-stiff differential equations

$$y'' + (10^q + 1)y' + 10^q = 0 \qquad \text{on} \quad [0, 1],$$

with initial conditions

$$y(0) = 2, \qquad y'(0) = -10^q - 1,$$

for $q = 1$ and compare the number of steps used by the solvers.

Solution. The function M-file `exp5_16.m` is

```
function uprime = exp5_16(x,u)
global q
A=[0 1;-10^q -1-10^q];
uprime = A*u;
```

The following commands solve the initial value problem.

```
>> clear
>> global q; q = 1;
>> xspan = [0 1]; u0 = [2 -(10^q + 1)]';
>> [x23,u23] = ode23('exp5_16',xspan,u0);
>> [x45,u45] = ode45('exp5_16',xspan,u0);
>> [x113,u113] = ode113('exp5_16',xspan,u0);
>> [x23s,u23s] = ode23s('exp5_16',xspan,u0);
>> [x15s,u15s] = ode15s('exp5_16',xspan,u0);
>> whos
  Name          Size           Bytes  Class

  q              1x1               8  double array (global)
  u0             2x1              16  double array
  u113          26x2             416  double array
  u15s          32x2             512  double array
  u23           20x2             320  double array
  u23s          25x2             400  double array
  u45           49x2             784  double array
  x113          26x1             208  double array
```

```
  x15s        32x1              256  double array
  x23         20x1              160  double array
  x23s        25x1              200  double array
  x45         49x1              392  double array
  xspan        1x2               16  double array

Grand total is 461 elements using 3688 bytes
```

From the table produced by the command `whos` one sees that the nonstiff ode solvers `ode23`, `ode45`, `ode113`, and the stiff ode solvers `ode23s`, `ode15s`, use 20, 49, 26, and 25, 32 steps, respectively. □

Example 5.17 Use the five Matlab ode solvers to solve the stiff differential equations

$$y'' + (10^q + 1)y' + 10^q = 0 \qquad \text{on} \quad [0, 1],$$

with initial conditions

$$y(0) = 2, \qquad y'(0) = -10^q - 1,$$

for $q = 5$ and compare the number of steps used by the solvers.

SOLUTION. Setting the value $q = 5$ in the program of Example **??** we obtain the following results for the `whos` command.

```
clear
global q; q = 5;
xspan = [0 1]; u0 = [2 -(10^q + 1)]';
[x23,u23] = ode23('exp5_16',xspan,u0);
[x45,u45] = ode45('exp5_16',xspan,u0);
[x113,u113] = ode113('exp5_16',xspan,u0);
[x23s,u23s] = ode23s('exp5_16',xspan,u0);
[x15s,u15s] = ode15s('exp5_16',xspan,u0);
whos
  Name         Size          Bytes  Class

  q             1x1              8  double array (global)
  u0            2x1             16  double array
  u113     62258x2          996128  double array
  u15s       107x2            1712  double array
  u23      39834x2          637344  double array
  u23s        75x2            1200  double array
  u45     120593x2         1929488  double array
  x113     62258x1          498064  double array
  x15s       107x1             856  double array
```

```
  x23      39834x1          318672  double array
  x23s        75x1             600  double array
  x45     120593x1          964744  double array
  xspan        1x2              16  double array

Grand total is 668606 elements using 5348848 bytes
```

From the table produced by the command `whos` one sees that the nonstiff ode solvers `ode23`, `ode45`, `ode113`, and the stiff ode solvers `ode23s`, `ode15s`, use 39 834, 120 593, 62 258, and 75, 107 steps, respectively. It follows that nonstiff solvers are hopelessly slow and expensive to solve stiff equations. $\square$

MAT 5187, Assignment 3, Exercise 2

Exercise 5.3.1. (page 156 in Lambert 1991)
Find a solution of the third-order conditions for which $c_2 = c_3$ and $b_2 = b_3$; the resulting explicit method is known as *Nyström's third-order method.*

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Exercise 5.5.1. (page 162 in Lambert 1991)
(i) Given the differential system $u' = uv$, $v' = u + v$, calculate, by direct differentiation, $u^{(3)}$ and $v^{(3)}$ in terms of u and v.
(ii) Let $y = [u, v]^T$ and $f = [uv, u + v]^T$. Calculate $f^{(1)}(f^{(1)}(f))$ and $f^{(2)}(f, f)$ and check that $y^{(3)} = f^{(1)}(f^{(1)}(f)) + f^{(2)}(f, f)$ gives the result obtained in (i).