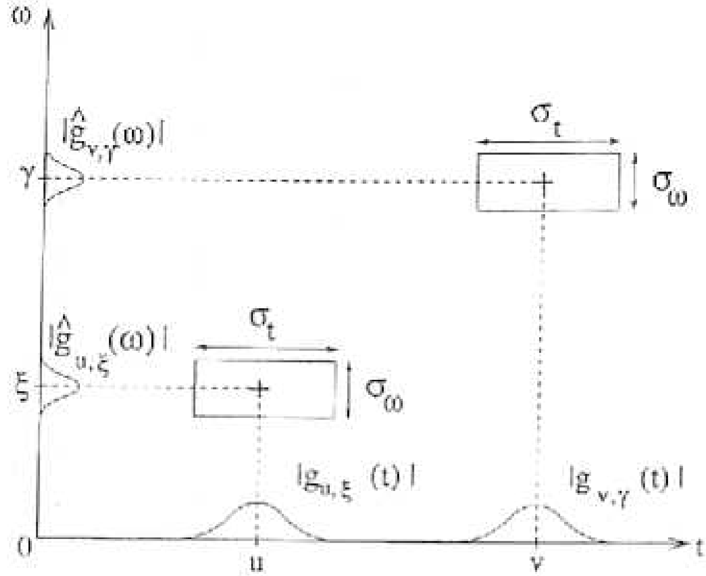

FIGURE 1.1 Time-frequency boxes ("Heisenberg rectangles") representing the energy spread of two Gabor atoms.

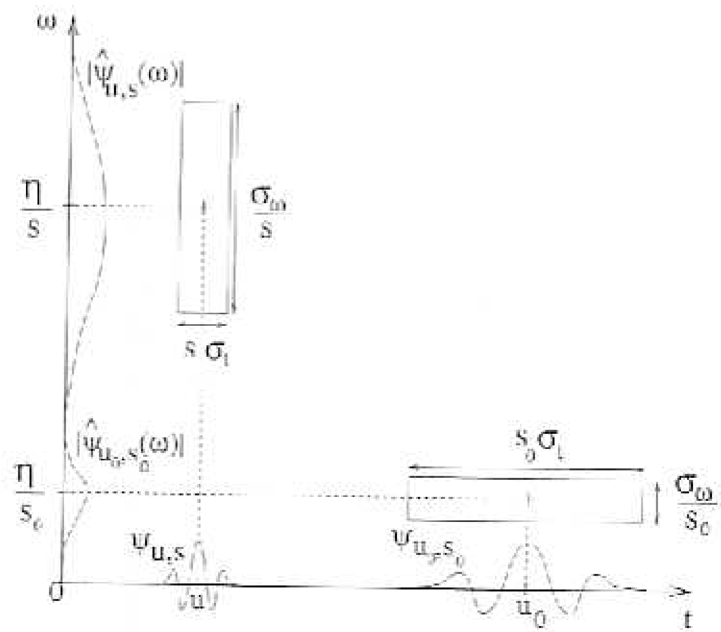

FIGURE 1.2 Time-frequency boxes of two wavelets $\psi_{u,s}$ and ψ_{u_0,s_0}. When the scale s decreases, the time support is reduced but the frequency spread increases and covers an interval that is shifted towards high frequencies.

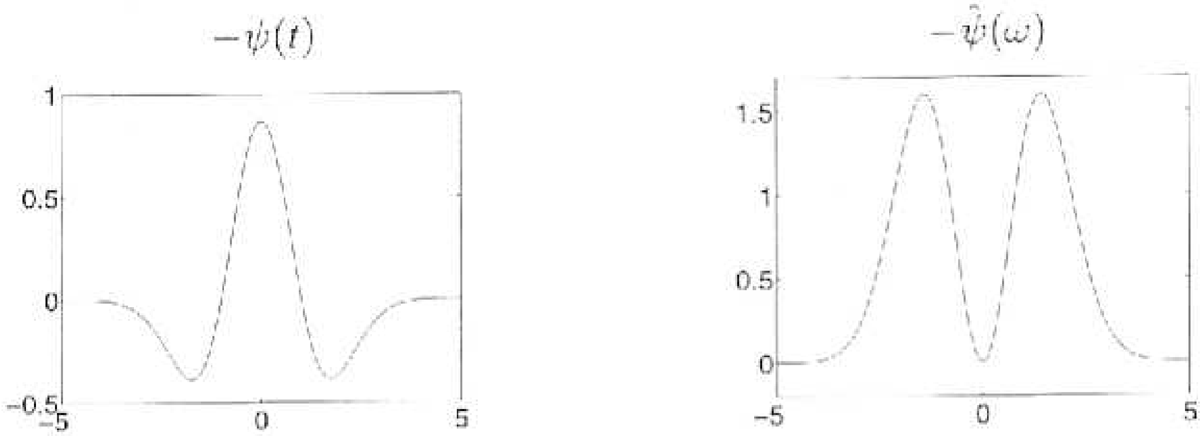

FIGURE 4.6 Mexican hat wavelet (4.34) for $\sigma = 1$ and its Fourier transform.

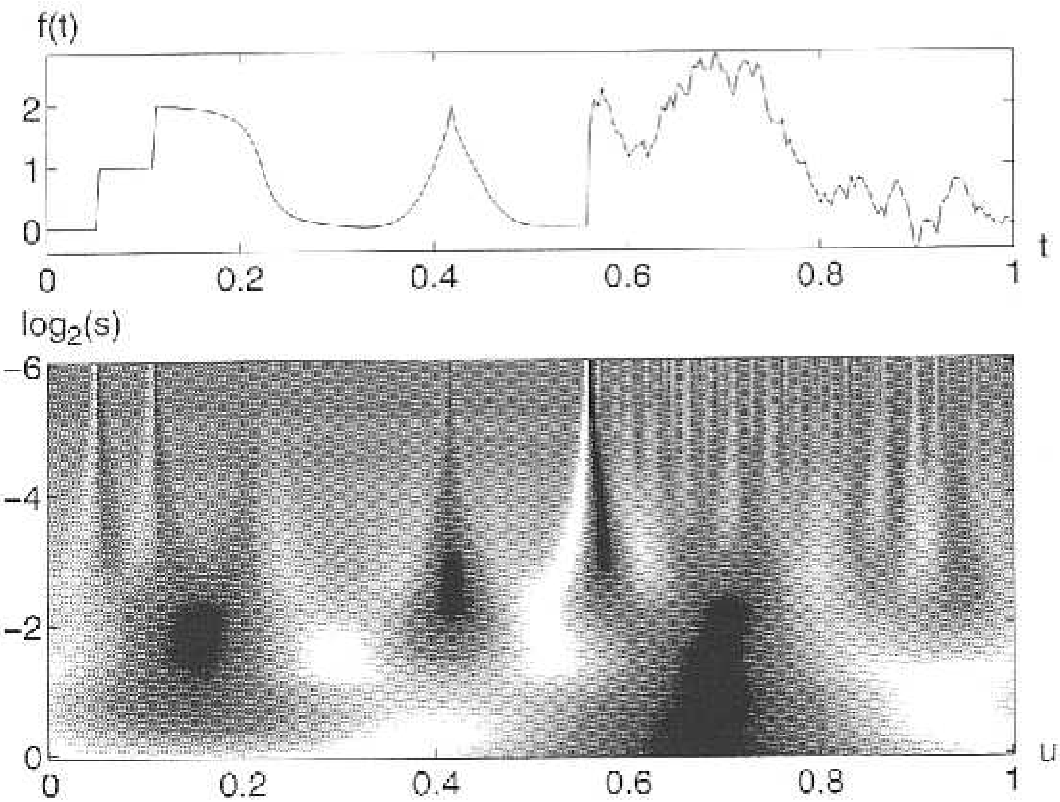

FIGURE 4.7 Real wavelet transform $Wf(u, s)$ computed with a Mexican hat wavelet (4.34). The vertical axis represents $\log_2 s$. Black, grey and white points correspond respectively to positive, zero and negative wavelet coefficients.

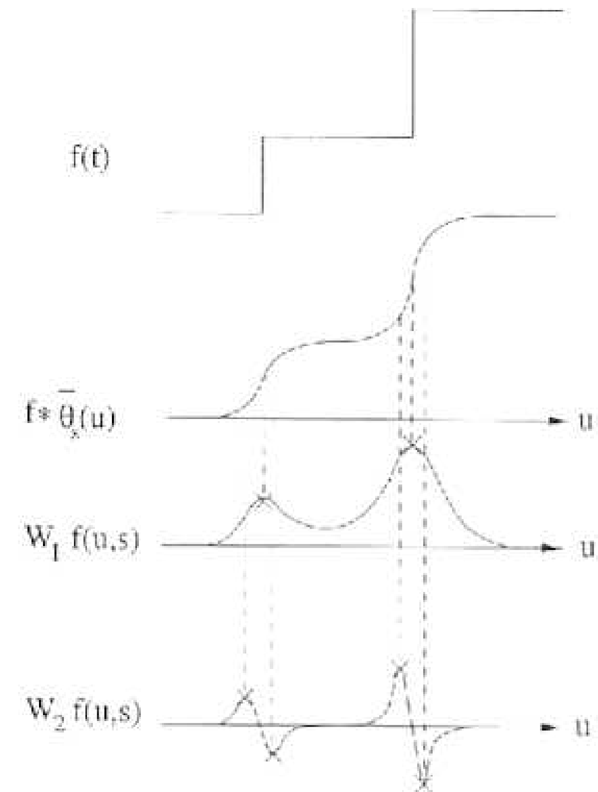

FIGURE 6.4 The convolution $f \star \bar{\theta}_s(u)$ averages f over a domain proportional to s. If $\psi = -\theta'$ then $W_1 f(u,s) = s\frac{d}{du}(f \star \bar{\theta}_s)(u)$ has modulus maxima at sharp variation points of $f \star \bar{\theta}_s(u)$. If $\psi = \theta''$ then the modulus maxima of $W_2 f(u,s) = s^2 \frac{d^2}{du^2}(f \star \bar{\theta}_s)(u)$ correspond to locally maximum curvatures.

Theorem 6.5 (HWANG, MALLAT) *Suppose that ψ is $\mathbf{C}^n$ with a compact support, and $\psi = (-1)^n \theta^{(n)}$ with $\int_{-\infty}^{+\infty} \theta(t)\,dt \neq 0$. Let $f \in \mathbf{L}^1[a,b]$. If there exists $s_0 > 0$ such that $|Wf(u,s)|$ has no local maximum for $u \in [a,b]$ and $s < s_0$, then f is uniformly Lipschitz n on $[a-\epsilon, b-\epsilon]$, for any $\epsilon > 0$.*

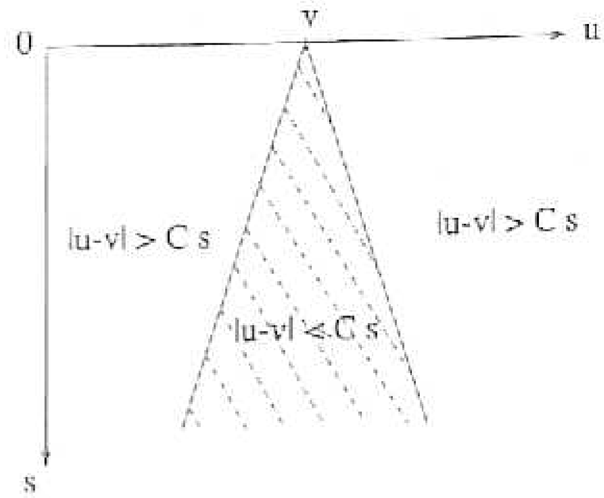

FIGURE 6.2 The cone of influence of an abscissa v consists of the scal
points (u, s) for which the support of $\psi_{u,s}$ intersects $t = v$.

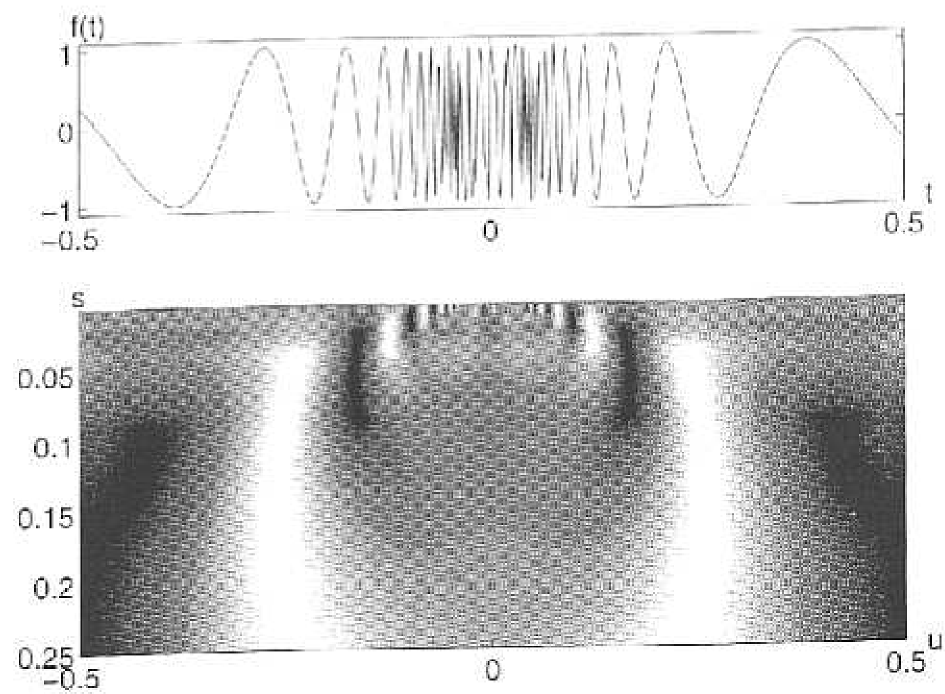

FIGURE 6.3 Wavelet transform of $f(t) = \sin(a\,t^{-1})$ calculated with $\psi =$
where θ is a Gaussian. High amplitude coefficients are along a parabola below
cone of influence of $t = 0$.

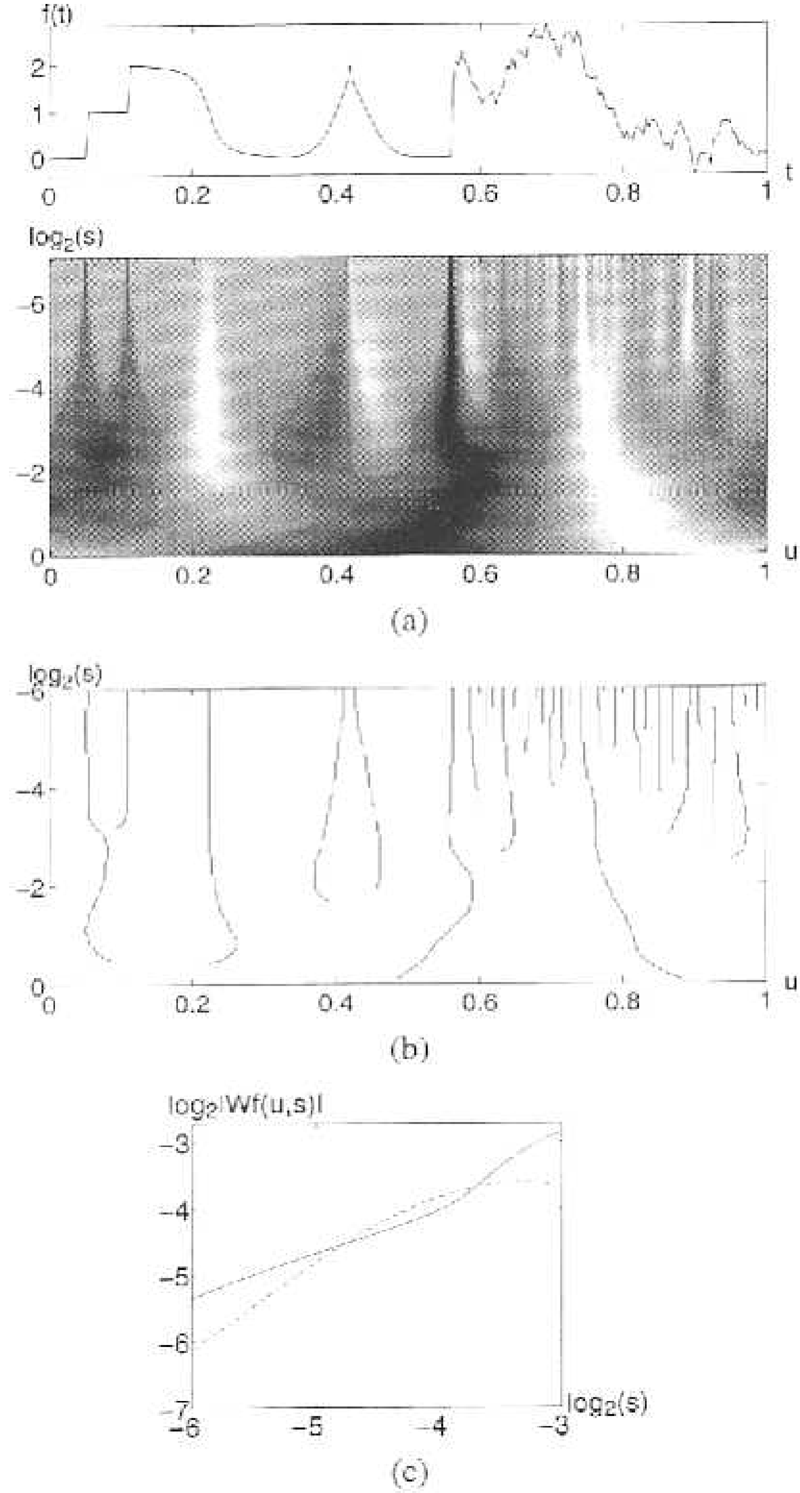

FIGURE 6.5 (a): Wavelet transform $Wf(u,s)$. The horizontal and vertical axes give respectively u and $\log_2 s$. (b): Modulus maxima of $Wf(u,s)$. (c): The full line gives the decay of $\log_2 |Wf(u,s)|$ as a function of $\log_2 s$ along the maxima line that converges to the abscissa $t = 0.05$. The dashed line gives $\log_2 |Wf(u,s)|$ along the left maxima line that converges to $t = 0.42$.

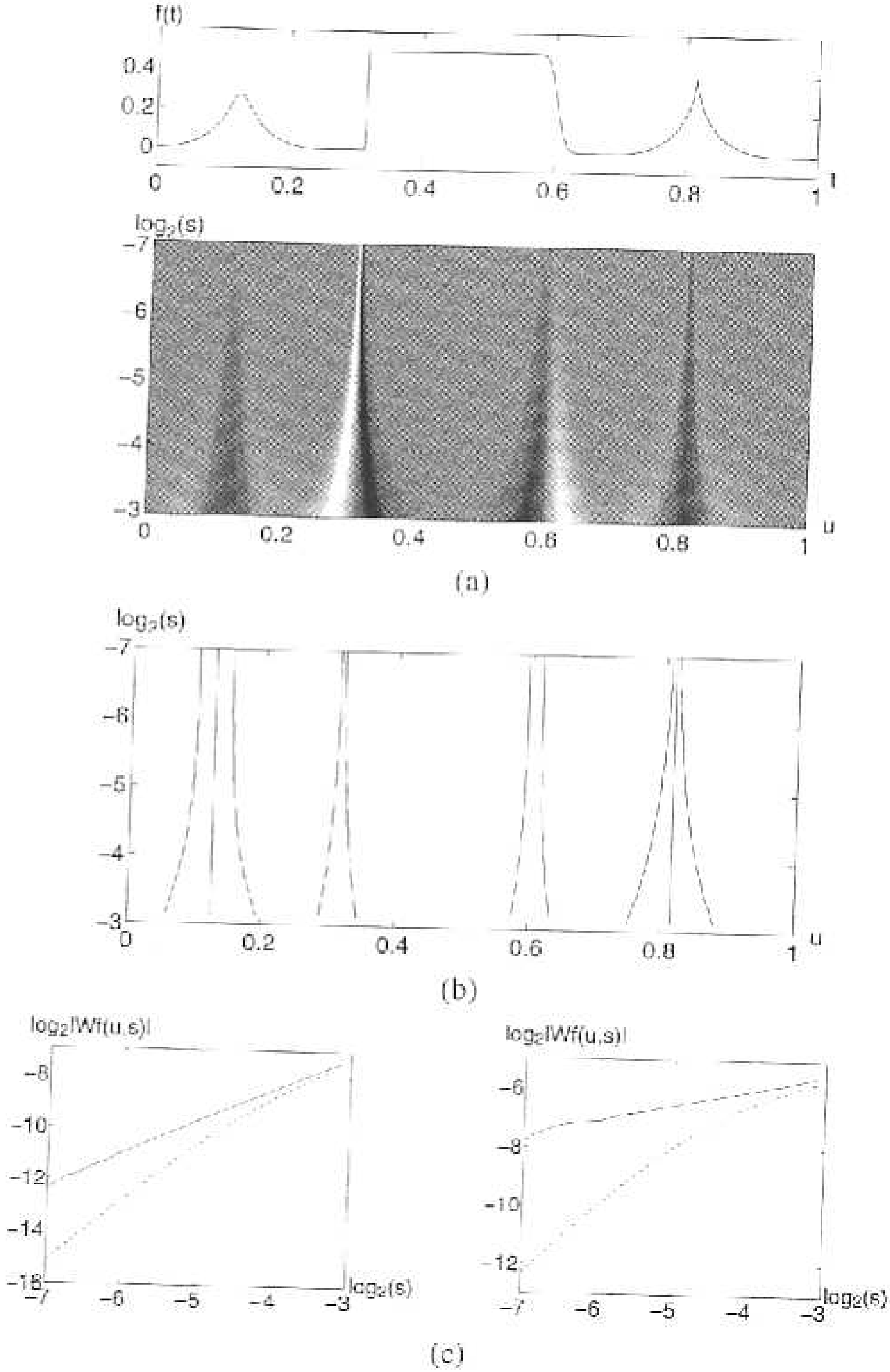

FIGURE 6.6 (a): Wavelet transform $Wf(u,s)$. (b): Modulus maxima of a wavelet transform computed $\psi = \theta''$, where θ is a Gaussian with variance $\beta = 1$. (c): Decay of $\log_2 |Wf(u,s)|$ along maxima curves. In the left figure, the solid and dotted lines correspond respectively to the maxima curves converging to $t = 0.81$ and $t = 0.12$. In the right figure, they correspond respectively to the curves converging to $t = 0.38$ and $t = 0.55$. The diffusion at $t = 0.12$ and $t = 0.55$ modifies the decay for $s \leq \sigma = 2^{-5}$.

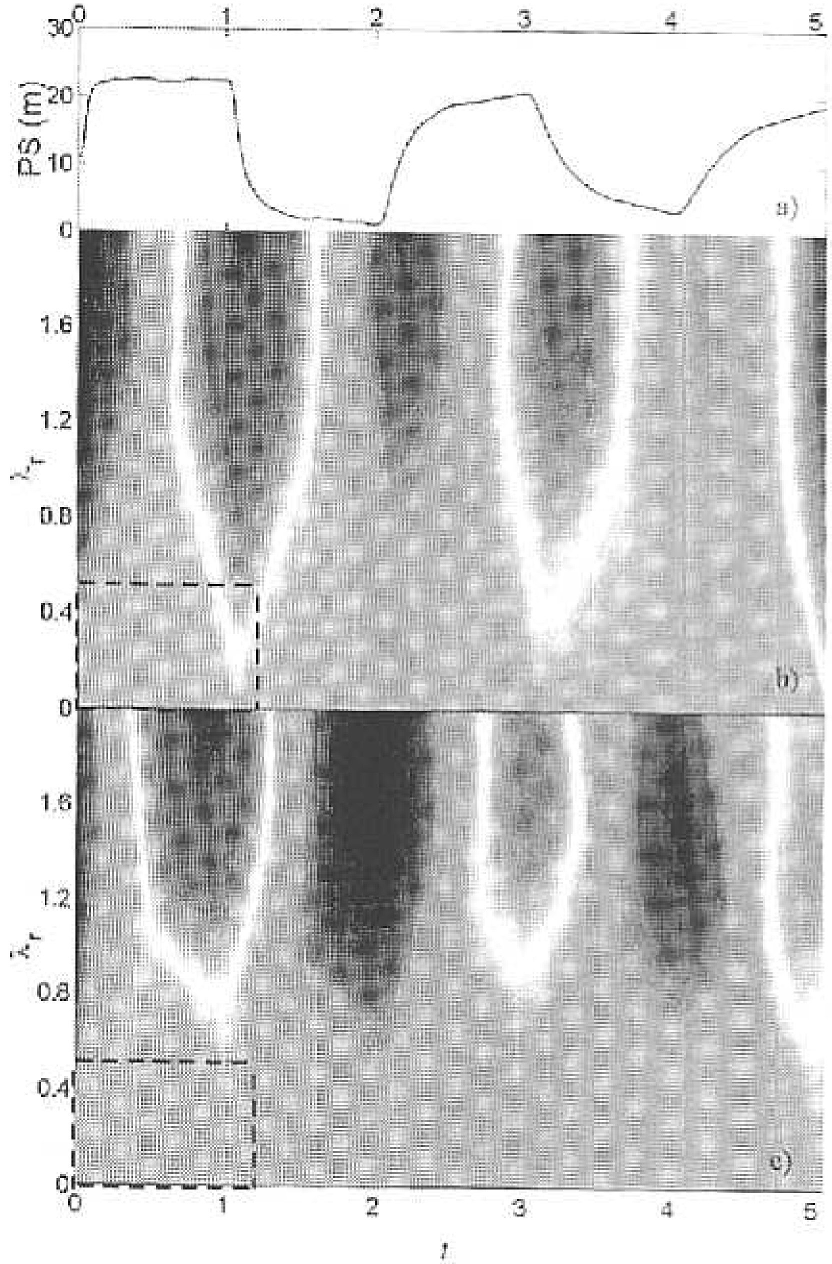

Fig. 4. Pressure signal (a), and db1 (b) and db2 (c) wavelet transform for test no. 1. Areas contoured by dashed lines are enlarged in Figs. 6 and 7, respectively.

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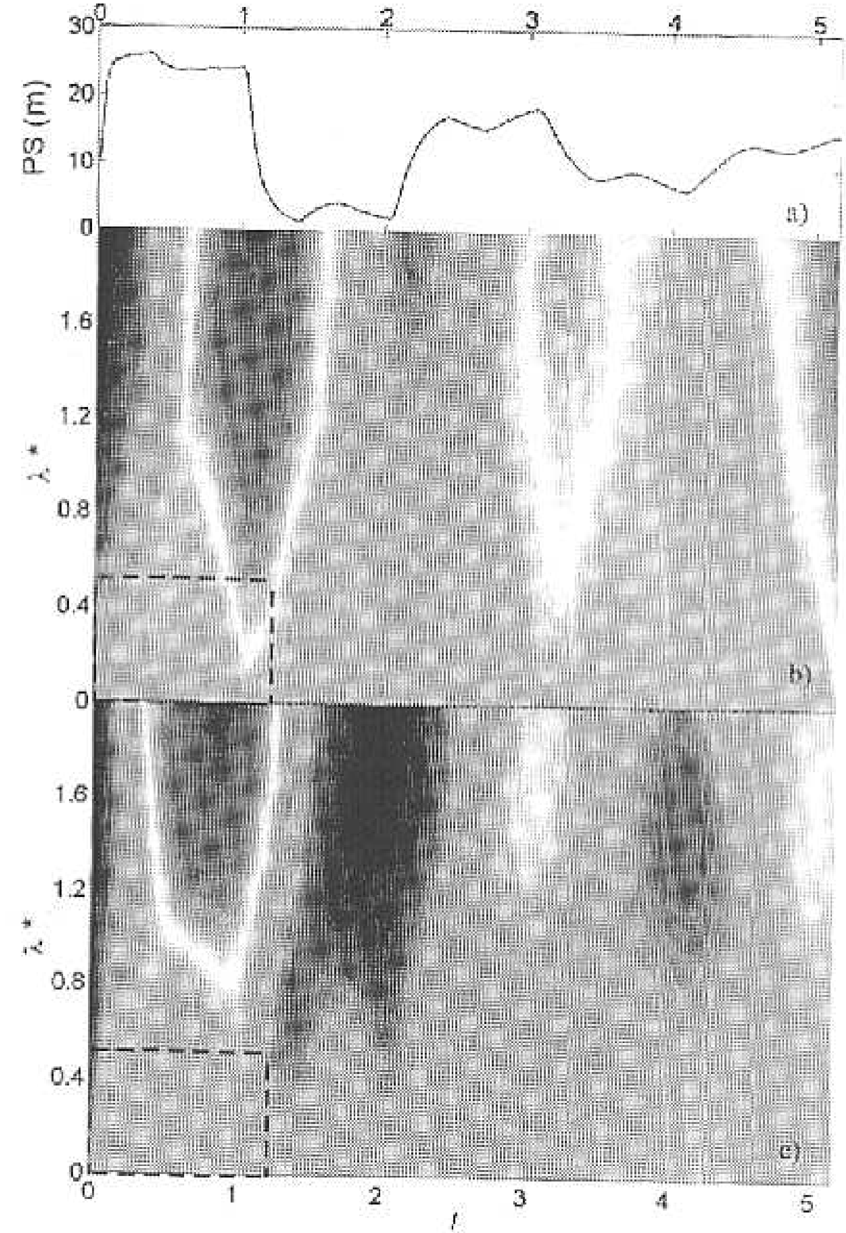

Fig. 8. Pressure signal (a), and db1 (b) and db2 (c) wavelet transform for test no. 2. Areas contoured by dashed lines are enlarged in Figs. 9 and 10, respectively.

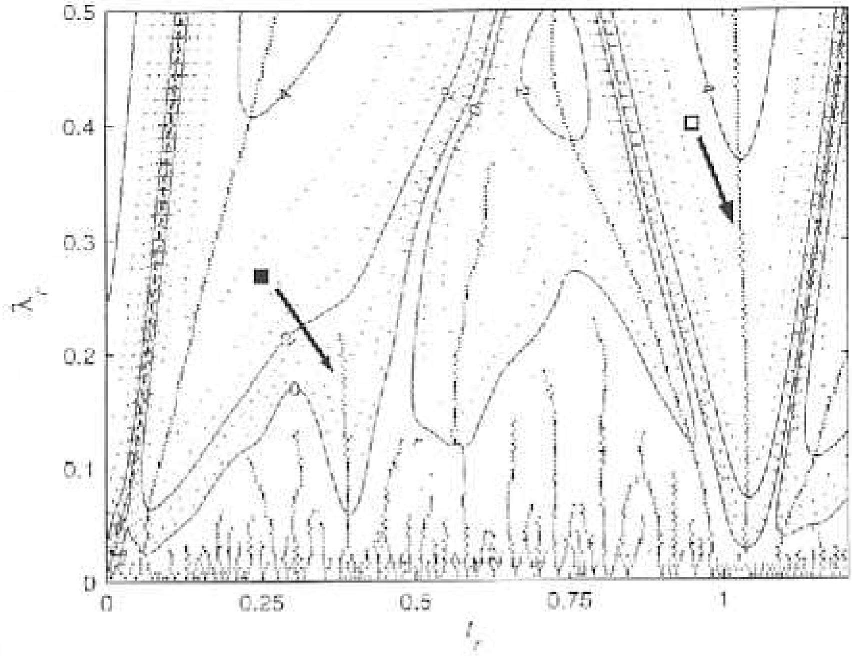

Fig. 10. Wavelet transform (logarithmic scale of the absolute value) of the db2 wavelet coefficients of Fig. 8c (test no. 2), with the local maxima (dots). Hollow (filled) square indicates the ridge related to the reservoir (leak) reflected wave.

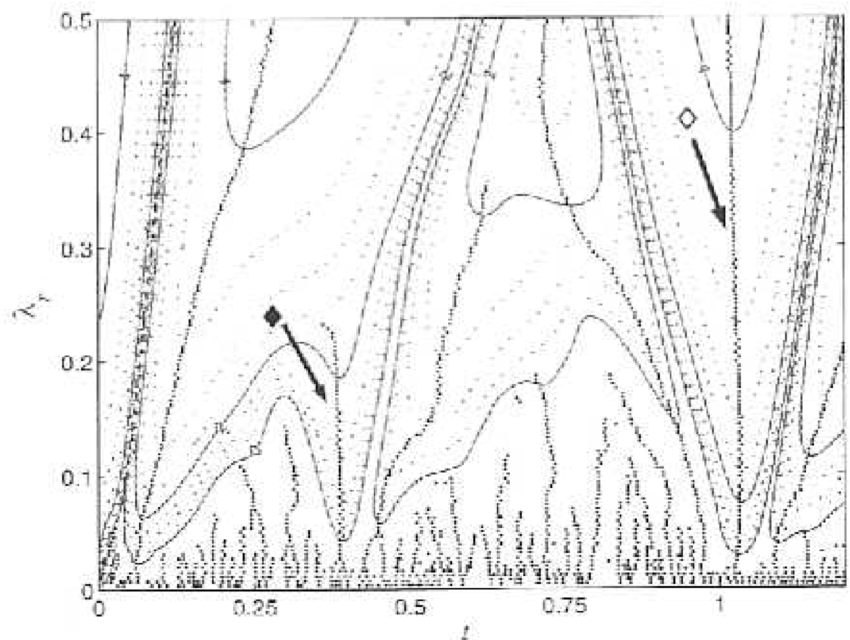

Fig. 11. Wavelet transform (logarithmic scale of the absolute value) of the db2 wavelet coefficients (test no. 3), with the local maxima (dots). Hollow (filled) diamond indicates the ridge related to the reservoir (leak) reflected wave.

Local maxima lines corresponding to leak reflected waves can be distinguished from the others because their maximum slope is much steeper than other maxima lines. Such a behavior is related to the strength of the singularity induced in the pressure signal by the leak reflected wave and can hence be measured by Hölder's

maps of Figs. 7, 10, and 11, the local maxima lines corresponding to the reservoir and leak reflected waves are shown in Fig. 12. The local maxima lines shown in this figure are marked by the same symbols in the wavelet maps. Precisely, hollow (filled) symbols denote the pressure waves reflected by the reservoir (leak) while

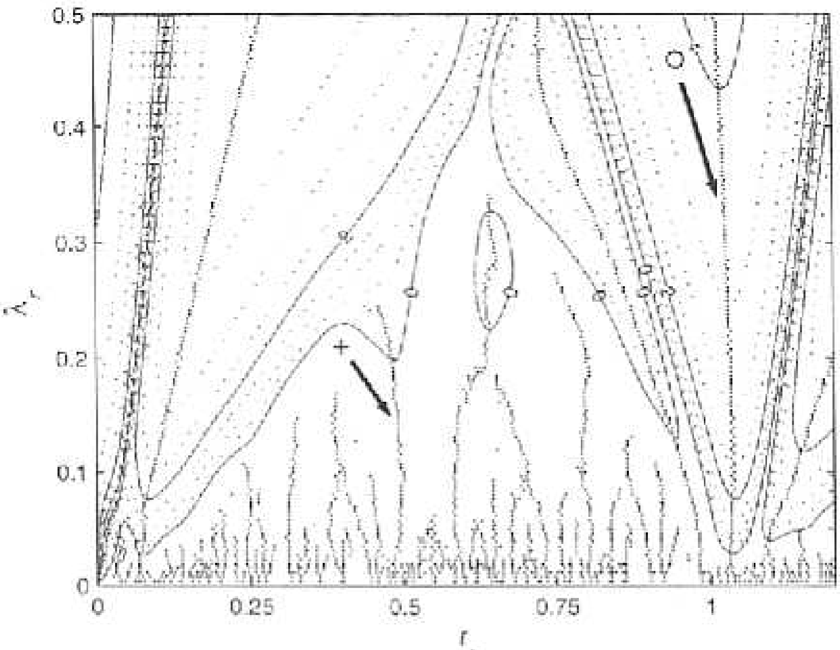

Fig. 7. Wavelet transform (logarithmic scale of the absolute value) of the db2 wavelet coefficients of Fig. 4c (test no. 1), with the local maxima (dots). Hollow circle indicates the ridge related to the reservoir-reflected wave; cross symbol indicates another ridge.

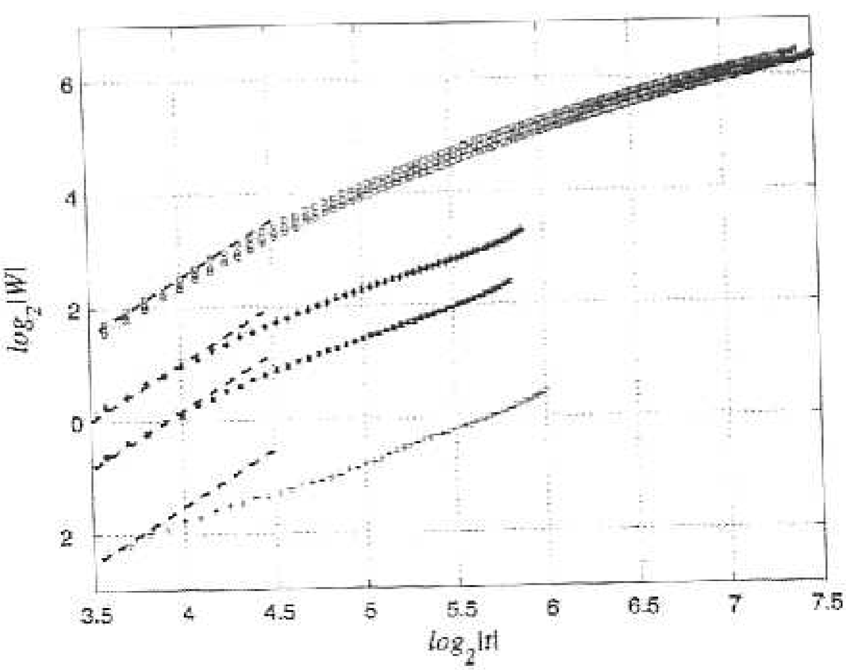

Fig. 12. Local maxima lines for db2 wavelet transforms from Figs. 7, 10, and 11. Hollow (filled) symbols indicate the ridges related to the reservoir (leak) reflected waves, with circle, square, and diamond symbols referring to test nos. 1, 2, and 3, respectively. Cross symbols refer to test no. 1.