VIMS2002 International Symposium on
Virtual and Intelligent Measurement Systems
Mt. Alyeska Resort, AK, USA, 19-20 May 2002

Dealing with Uncertain Measurements in Virtual Representations for Robot Guidance

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Abstract

When measurements are collected with real sensors, it is known that they are always submitted to some level of uncertainty due to the imperfection of practical systems. Often, it might be acceptable to neglect uncertainty on measurements in order to simplify the development. But such a simplification can also result in critical situations under some specific circumstances such as the building of virtual maps for robot guidance. Erroneous data can easily lead to collisions between the robot and its environment which might result in important damages both to the robotic system and its surroundings. This papers presents an analysis of how uncertainty originating from multiple sources including uncertain spatial relationships in a robotic workcell can be propagated up to the scene representation. Various ways are studied to propagate the uncertainty over a chain of geometrical transformations in 3-D space and an approach is proposed to merge the propagated uncertainty into an occupancy grid that models a cluttered environment.

I. Introduction

In the context of autonomous robotics, most applications require that a virtual representation of the environment is built prior to the determination of movements. This is of paramount importance for cluttered environments whose structure is not known *a priori*. The availability of such a model allows the robot to avoid collisions and eventually to optimize its trajectory [1, 10]. Unfortunately, building a complete model of a complex scene implies many important constraints. Because of the fact that vision sensors have a limited field of view, measurements must be taken successively from different viewpoints in order to collect data on the entire structure of the environment [6, 8]. This requirement to move the sensor might introduce significant distortion or contradictory information in the virtual representation if not carefully processed. Maintaining the correlation between data gathered from various viewpoints is difficult because of the uncertainty on the sensor measurements but also because of the uncertainty introduced by the positioning device that is used to move the sensor up to these different viewpoints.

This paper proposes an examination of how the numerous sources of error existing in a vision-based measurement setup can be dealt with in the construction of a virtual representation. The desired model is a probabilistic representation of space cluttering encoded as an occupancy grid. This work is an extension to a previously developed strategy for building probabilistic occupancy models of 3D cluttered environments from raw range measurements in a computationally tractable way. The initial algorithm only dealt with uncertainty on the distances provided by the range sensor. The goal of the extended approach is now to take into account some supplementary knowledge on the uncertainty distributions that characterize both the sensor and the positioning device.

Kalman filters are powerful tools to deal with such noisy data [3, 4] but they tend to process the measurements and the errors simultaneously. When the uncertainty originates from different devices with different magnitudes, it might be interesting to process errors independently from the measurements. Since the uncertainty sources are distributed among the components of the modeling system, some means might be developed to propagate the uncertainty from there respective reference frame to that attached with the model representation where the final information is to be encoded.

Jacobian matrices can be used to propagate the uncertainty. Smith *et al.* [14] have developed the equations for a propagation over a chain of two uncertain geometrical transformations. This approach reaches a significant complexity when supplementary uncertain transformations are added as it is often the case in robotics systems where many components are mobile. The case is still worse if inverse transformation expressions need to be used.

Julier and Uhlmann [7] proposed a probabilistic approach to propagate the uncertainty through general nonlinear relationships. The complexity of this approach also depends on the number of independent reference frames that are used to fully describe the robotic workcell. However, covariance matrix estimation which is usually complex is replaced by an intermediate probabilistic distribution which is less computationally intensive.

Both of these uncertainty propagation schemes are detailed and their suitability to the probabilistic model building application in a 3-dimensional workspace where a range sensor is mounted on an arm manipulator is analyzed. Next the uncertainty propagation technique is transposed in the model building process. We then demonstrate how this critical

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information can be added to the virtual environment representation that is to be used to plan collision-free movements for the robot.

II. Probabilistic modeling scheme

In the context of mobile robot navigation, Elfes proposed a framework for building 2-D probabilistic occupancy maps of the environment using a sensor error model and a Bayesian probabilistic approach to combine data from multiple viewpoints. This results in an occupancy grid covering the robot workspace in which cells are tagged with the probability that an object lies in the corresponding area.

In previous works [9, 11], techniques have been proposed to circumvent the computational explosion that results from the addition of a third dimension to the original scheme. One of these approaches proposes a new formalism of the Bayesian occupancy probability estimation that consists in using a closed-form approximation of the characteristic occupancy probability distribution function (OPDF) obtained by Elfes when the sensor measurements are characterized by a Gaussian error distribution as shown in figure 1. As one can predict, the occupancy probability reaches a maximum value at the object surface while it is set to 0% (empty space) between the sensor and the object surface. Behind the object surface, the probability decreases to 50% only (unknown state) as this area is occluded by the object from the sensor viewpoint.

Working on raw range measurements, this approximated OPDF is used to directly compute the occupancy probability of a given volume of 3-D space centered on the sensor viewpoint. This process is repeated for each viewpoint visited by the sensor and the resulting occupancy probability distributions are temporarily stored into local occupancy grids. Next, an intersection search algorithm merges all local occupancy grids into a global Cartesian multiresolution occupancy grid. The merging process is speeded up by taking advantage of the multiresolution property of the model to avoid useless volume matching. When some points in 3-D space have been measured from more than one viewpoint, a Bayesian function is used to compute the resulting occupancy probability. This leads to the reinforcement of the occupancy probability as coherent matching measurements are extracted from numerous local occupancy grids. Figure 2 summarizes the occupancy probability estimation and the merging process.

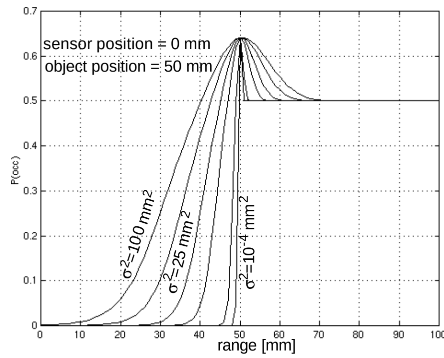

Figure 1: Approximative occupancy probability profiles for various levels of uncertainty.

III. Multi-viewpoints acquisition setup

In the previously described modeling scheme, only the range sensor uncertainty was taken into account under the form of a Gaussian error distribution on the distance measurement. The spatial relationships between each local OPDF reference frame was considered to be error free. But when a manipulator is used to move the sensors to different viewpoints like in the experimental setup depicted in figure 3, it is not safe to assume that the sensor position and orientation are known without any uncertainty. In practice, most sensors, and especially range sensors, require that a positioning device is used in order to allow them to scan a significant area as their field of view is very limited, eventually to a single line or even a single point. Very precise coordinate measuring machines (CCMs) can be used to estimate the sensor position, but these are more of obstacles in a real robotic workcell as they cannot actively contribute to the task. Robotic arms provide good repeatability but relatively poor accuracy in comparison with that of high quality range sensors. However they are widely used in practice because of their relative facility to program and their versatility. When some means are developed to estimate the registration between the viewpoints [5] there is still some remaining uncertainty. Registration techniques do not provide a complete information on geometrical transformations between the camera and the model reference frames, but only between two respective views.

Given the setup shown in figure 3 where a laser range sensor is mounted on a 6-DOFs robot arm, a number of

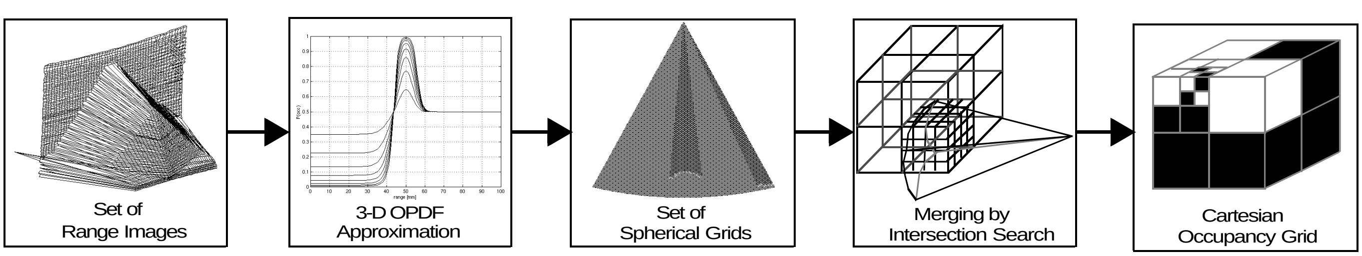

Figure 2: Probabilistic modeling scheme.

uncertain relationships can be defined. Figure 4 shows the main geometrical transformations to be considered as uncertainty sources in this setup. Ellipses denote the uncertainty on the corresponding pointers between reference frames.

Since range data gathered from any viewpoint are defined with respect to a local R_{camera} while the model is built with respect to R_{scene} to facilitate further manipulations of the model, initial measurements X_c must be transposed from R_{camera} to R_{scene} as follows:

$$X_s = T_{sw}^{-1} T_{rw} T_{cr} X_c \quad (1)$$

where T_{sw}, T_{rw} and T_{cr} are the homogeneous transformation matrices between the various reference frames. X_s and X_c are the range measurements in the scene and in the camera reference frames respectively.

Each component in equation (1) is characterized by some uncertainty which can be expressed in terms of a covariance matrix, Γ. The uncertainty on range measurements depends on the sensor characteristics. Uncertainty on the geometrical transformations depends on mechanical calibration of the assembly and on the precision of the manipulator used to hold the camera.

For each measurement from a given viewpoint, the sensor provides the coordinates, X_c, of a point located on the object and its covariance matrix, Γ_c with respect to the camera reference frame. Simultaneously, the configuration of the robot, T_{rw}, is measured along with its uncertainty, Γ_{rw}. Similar information might be estimated for the camera configuration with respect to the robot end effector, $[T_{cr}, \Gamma_{cr}]$ and for the reference frame rigidly attached to the scene model, $[T_{sw}, \Gamma_{sw}]$.

In order to keep uncertainty processing tractable, the assumption is made that the uncertainty along each axis is independent from the uncertainty along other axes. This makes the covariance matrices diagonal. The spatial relationships and covariance matrices are all defined with respect to different reference frames while the ultimate goal is to merge all measurements into a single reference frame rigidly attached to the scene while taking into account the different sources of uncertainty. This justifies the necessity to propagate the uncertainty through the transformation chain in order to obtain quantitative uncertainty values along each axis of R_{scene}.

The following sections examine strategies to propagate the uncertainty associated with each spatial transformation and propose an approach to encode this supplementary information into the probabilistic occupancy virtual representation.

IV. Propagation with Jacobian matrices

Smith *et al.* [13] have developed the expressions to propagate uncertainty when only two spatial transformations in 3-D space are considered. Here, the development is extended for a chain of three spatial relationships in 3-D as required by the acquisition setup.

The measurements coordinates expressed with respect to R_{scene}, X_s, depend on 21 parameters:

$$X_s = f(t_{sw}^{-1}, t_{rw}, t_{cr}, x_c) \quad (2)$$

where t_{sw}^{-1}, t_{rw} and t_{cr} are the parameters of the transformations matrices T_{sw}^{-1}, T_{rw} and T_{cr} respectively (3 translations + 3 rotations each). x_c corresponds to the 3 parameters of the measurement point X_c.

The covariance matrix of the measurements with respect to R_{scene}, Γ_s, taking into account all sources of uncertainty, can be expressed as:

$$\Gamma_s = J_f \cdot \Gamma_{\gamma_{sw}^{-1}, \gamma_{rw}, \gamma_{cr}, \gamma_c} \cdot J_f^T \quad (3)$$

where J_f is the Jacobian matrix of the nonlinear function f evaluated around the mean value $\hat{x}$. $\Gamma_{\gamma_{sw}^{-1}, \gamma_{rw}, \gamma_{cr}, \gamma_c}$ is a global covariance matrix composed of four diagonal submatrices containing respectively the covariance parameters γ_{sw}^{-1}, γ_{rw}, γ_{cr}, γ_c associated with Γ_{sw}^{-1}, Γ_{rw}, Γ_{cr} and Γ_c. As each transformation and each measurement is independent of each other, the global covariance matrix is also diagonal. Therefore, equation (3) can be developed as follows:

$$\Gamma_s = J_{f_{t_{sw}^{-1}}} \Gamma_{\gamma_{sw}^{-1}} J_{f_{t_{sw}^{-1}}}^T + J_{f_{t_{rw}}} \Gamma_{\gamma_{rw}} J_{f_{t_{rw}}}^T + J_{f_{t_{cr}}} \Gamma_{\gamma_{cr}} J_{f_{t_{cr}}}^T + J_{f_{x_c}} \Gamma_{\gamma_c} J_{f_{x_c}}^T \quad (4)$$

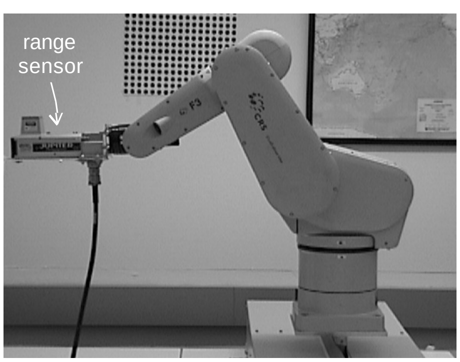

Figure 3: Range data acquisition setup.

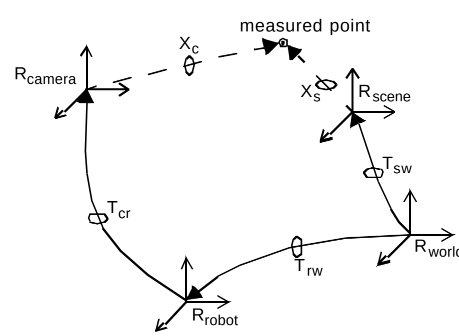

Figure 4: Uncertain transformations in the acquisition setup.

But covariance matrices associated with inverse geometrical transformations must be processed in a slightly different way [14] such that:

$$\Gamma_{\gamma_{sw}^{-1}} = J_{t_{sw\varnothing}^{-1}} \Gamma_{\gamma_{sw}} J_{t_{sw\varnothing}^{-1}}^{T} \tag{5}$$

where

$$J_{t_{sw\varnothing}^{-1}} = \frac{\delta X'}{\delta X} \tag{6}$$

when

$$X' = T_{sw}^{-1} X \tag{7}$$

Finally, the uncertainty propagation equation (4) can be expressed as follows:

$$\begin{aligned} \Gamma_s = \; & J_{f_{t_{sw}^{-1}}} J_{t_{sw\varnothing}^{-1}} \Gamma_{\gamma_{sw}} J_{t_{sw\varnothing}^{-1}}^{T} J_{f_{t_{sw}^{-1}}}^{T} + J_{f_{t_{rw}}} \Gamma_{\gamma_{rw}} J_{f_{t_{rw}}}^{T} \\ & + J_{f_{t_{cr}}} \Gamma_{\gamma_{cr}} J_{f_{t_{cr}}}^{T} + J_{f_{x_c}} \Gamma_{\gamma_c} J_{f_{x_c}}^{T} \end{aligned} \tag{8}$$

Applying equations (1) and (8) to every range measurement with respect to R_{camera}, X_c, provides the corresponding coordinates with respect to R_{scene}, X_s, as well as the uncertainty on X_s taking into account every sources of error in the acquisition setup.

Even though this approach is very rigorous, its complexity directly depends on the number of chained transformations through which the measurements must be propagated. It is also sensitive to the fact that some transformations are inverted with respect to the definition of their parameters. The resulting processing is computationally intensive and the overhead added to the model building algorithm is therefore significant.

V. Propagation through reference frame relationships

Julier and Uhlmann [7] have proposed a strategy to propagate uncertainty across nonlinear relationships such as spatial transformations between two reference frames. The approach consists in applying the nonlinear relationship to a probabilistic distribution centered on the measured point in the initial reference frame whose dimensions are augmented to match the total number of degrees of freedom implied in the nonlinear relationship and in the measurement. The initial probabilistic distribution is characterized by the variance along each axis of the augmented reference frame, R_1. Applying the nonlinear relationship to this distribution results in a new probabilistic distribution defined with respect to the axes of a second reference frame, R_2. Estimating the mean and the covariance along each axis of the new distribution provides the measurement coordinates and the uncertainty values relative to the second reference frame.

In the context of 3-D probabilistic modeling from multiple viewpoints with three uncertain transformations, the nonlinear relationship depends on 21 parameters as shown in the previous section. Therefore, the initial and the final reference frames must but be expanded to 21 dimensions for the technique to be applied. As we are only interested in the coordinates and the uncertainty of measurements along the x, y and z axes of R_{scene} to compute the model, only these three parameters are really relevant. This allows to reduce the computational complexity by avoiding the computation of the other values along the virtual axes of R_{scene}.

Nevertheless, in accordance with Julier and Uhlmann, the probabilistic distribution in the initial reference frame must be composed of a minimal set of *2n* points for a *n*-dimensional system (here $n = 21$) in order to preserve the probabilistic distribution moments in both frames. This means that for each range measurement, the nonlinear relationship of equation (1) must be computed for *2n* points before the mean and the covariance along the first three axes (x,y,z) of R_{scene} are estimated.

The first step of this approach consists in extracting the square root of $n \cdot \Gamma_{\gamma_{sw}^{-1}, \gamma_{rw}, \gamma_{cr}, \gamma_c}$ to estimate the standard deviation along each of the 21 axes in the augmented camera reference frame. The Cholesky method [12] is well suited for this task. The *2n* points, *pt[i]*, included in the probabilistic distribution sample are defined as each column of the two matrices resulting from the root extraction (plus and minus signs). The mean value of each parameter is added to these coordinates in order to center the sample on the real data values.

Next, the nonlinear relationship is applied to camera-based *2n* points in order to compute the corresponding *2n* points with respect to R_{scene}.

$$X_{R_{scene}^{augmented}}[i] = T_{sw}^{-1} T_{rw} T_{cr} X_{R_{camera}^{augmented}}[i] \tag{9}$$

The mean $(\hat{x}_{R_{scene}}, \hat{y}_{R_{scene}}, \hat{z}_{R_{scene}})$ and the variance $\left(\sigma^2_{x_{R_{scene}}}, \sigma^2_{y_{R_{scene}}}, \sigma^2_{z_{R_{scene}}}\right)$ are finally estimated on these points.

The Julier and Uhlmann approach eliminates the need for a repetitive estimation of the Jacobian matrices and equation (8). But for each range measurement, *2n* points must be processed through a nonlinear function. The complexity of the approach is then dependent on the number of degrees of freedom implied in the nonlinear relationship. However, it is possible to reduce the number of points to process from 42 to 30 by setting the scene reference frame aligned with the world reference frame ($R_{scene} = R_{world}$) at the expense of a lost of some flexibility.

VI. Uncertainty integration into probabilistic octree modeling

As range measurement are collected from many different viewpoints in a 3-D modeling application, the uncertainty propagation procedure must be consistent with the necessity to merge multiple local occupancy grids built locally around each viewpoint. In a previous approach, Elfes [2] proposed a

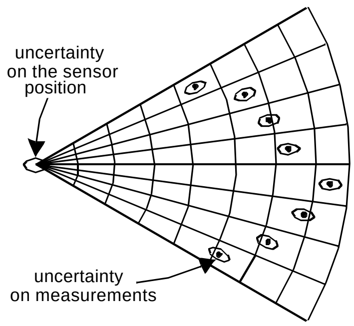

Figure 5: Uncertainty ellipsoids on measurements and sensor position/orientation.

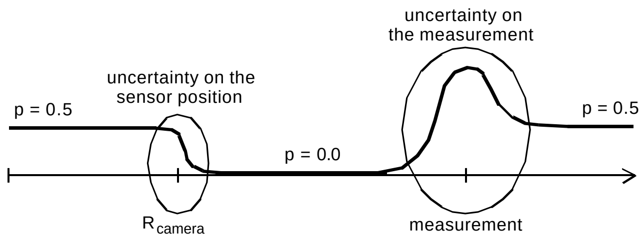

Figure 6: Extended closed-form OPDF.

blurring convolution process to take into consideration the uncertainty on a mobile robot position on which a sonar sensor was mounted to create a planar map of the environment. Such a strategy could be extended to 3-D space but would hardly deal with the uncertainty on rotation transformations as the blurring process is defined along the axes of the reference frames only. This could lead to important deviation of the uncertainty estimation with respect to R_{scene}.

An adaptation of the closed-form OPDF computation is proposed that allows taking into account both the uncertainty on position and orientation originating from all sources. The technique consists in transforming each measurement from R_{camera} to R_{scene} taking advantage of the Julier and Uhlmann approach described previously. This results in a distribution of measurement points referred to R_{scene} $(X_{R_{scene}}, \Gamma_{R_{scene}})$ which depends both on the position and on the orientation uncertainties. The modified closed-form OPDF approximation provides knowledge on the uncertainty of the sensor position, which corresponds to the origin of the corresponding local spherical occupancy grid as shown in figure 5. The ellipses represent the uncertainty level on measurements and on the origin of the area scanned by the sensor. To incorporate this supplementary information on the sensor location, the initial closed-form OPDF approximation is extended behind the sensor as shown in figure 6. As previously mentioned, the occupancy probability, p, is 0 (*empty*) between the sensor and the object surface because there is only empty space. Nearby the object, the probability grows in a Gaussian-like manner and drops to 0.5 (*unknown*) behind the object surface. A similar behavior is expected around the sensor position. Behind the sensor, the state of the space remains unknown ($p = 0.5$). As the sensor position is not perfectly known because of the uncertainty on the transformation chain, the step between unknown area (behind the sensor) and empty space (in front of the sensor) is not accurately located. For this reason, the occupancy probability near the origin of R_{camera} drops down in a Gaussian-like manner from 0.5 to 0.0. Figure 6 shows a 1-D projection of the updated closed-form OPDF approximation. In practice, this probabilistic distribution must be extrapolated to 3-D.

The OPDF can also be defined on a parametric straight line aligned with the laser beam, $\vec{d}_i = \vec{u} + \lambda\vec{v}_i$ as shown in figure 7. This allows to directly process measurements that have been transformed to equivalent coordinates with respect to R_{scene}. As a result, both the uncertainties on range measurements, on the sensor position and on the model reference frame position are taken into account when new data are added to the local spherical grid. The OPDF parametric straight line must be reevaluated for each measurement since the laser beam has a different orientation for each point. This allows considering non uniform uncertainty distributions which characterize most range sensors.

Considering that measurements from a given viewpoint are gathered in a short period of time in comparison with

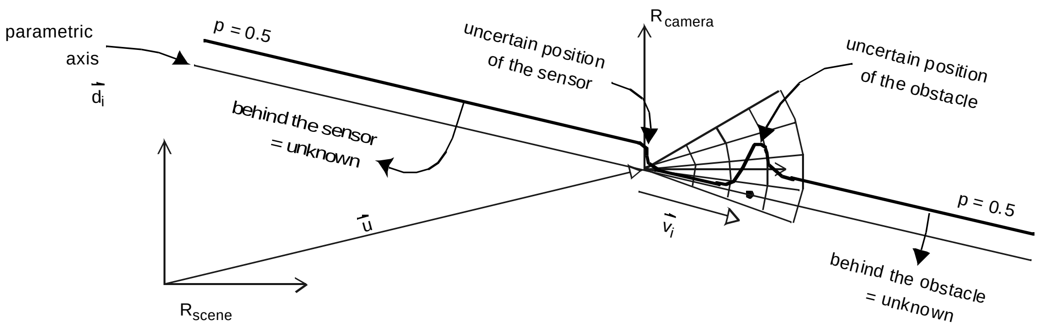

Figure 7: Parametric closed-form OPDF.

perturbations frequency, the origin of the local spherical grid is assumed to be constant but uncertain. This assumption allows the original range data merging strategy based on intersection search between local spherical grids and a global Cartesian grid to be applied even with the addition of uncertainty processing.

The main interest of this approach based on uncertainty propagation techniques is that it allows to merge both the uncertainty on the range measurements that depends on the sensor characteristics and the uncertainty on the sensor position/orientation that depends on the mechanical system used to position the camera. All this information, which is critical in many applications, can be processed without the evaluation of Jacobian and covariance matrices. Moreover, as the transformed measurements relative to R_{scene} are used to estimate the occupancy probability distribution rather than measurements relative to R_{camera} as in Elfes' approach, both the position and the orientation uncertainties are taken into account. This strategy also appears to be less computationally expensive than the evaluation of Jacobian matrices in spite of the fact that 42 points must be processed for each measurement. This is explained by the fact that this processing is simply a product of 3 constant matrices for a given viewpoint of the sensor.

VII. Conclusion

A strategy to deal with uncertainties on measurements and on range sensor position/orientation in a multi-viewpoints acquisition setup for 3-D probabilistic modeling has been presented. The Smith *et al.* Jacobian-based propagation scheme has been extended to a chain of three geometrical transformations with one inverse component between R_{camera} and R_{scene}. The Julier and Uhlmann probabilistic distribution propagation scheme through nonlinear transformations has also been investigated and appears to be more suitable for the probabilistic information merging algorithm currently used in our modeling system. Its capacity to take into account the uncertainty on both the position and the orientation without complex matrix derivation and evaluation is an important advantage. Finally, an adaptation of the closed-form approximation of the occupancy probability distribution function to suit the requirements of uncertainty processing is presented. Future work is planned to remove the assumption of fixed sensor origin during the scanning phase from a given viewpoint. But the merging strategy between local occupancy grids and the final Cartesian grid must first be revisited.

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