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Fuzzy Logic Inference for Occupancy State Modeling and Data Fusion

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Abstract ***- Autonomous robotic systems require a detailed model of space occupancy to be built from sensory information in order to navigate safely in their environment. Probabilistic occupancy models have been proposed that use conditional probabilities evaluation to merge redundant measurements. These approaches provide meaningful representation of space but require important approximations to remain computationally tractable for high dimensionality. As a result, the strict definition of probability is denatured. The present paper proposes an exploration of the fuzzy logic paradigm as a modeling tool for occupancy mapping in the context of workspace representation for robotic applications. A computationally tractable fuzzy logic inference engine is introduced that allows data fusion to construct a robot workspace representation in a more intuitive way while preserving desirable characteristics achieved by probabilistic modeling schemes.***

I. INTRODUCTION

Modeling the workspace of an autonomous or semi-autonomous robotic device for planning its actions is a critical issue, especially when the robot has to operate in a cluttered workspace whose structure is not known *a priori*. Assessing the risk to make the robot to circulate through given regions of space is mandatory in order to prevent damages to the device as well as to the environment. Unfortunately, as sensors have a limited field of view, building a complete model of a complex scene usually requires that measurements are taken successively from different viewpoints [7, 9]. Data fusion must then be operated on the datasets in order to merge information from all viewpoints in a coherent representation.

Many frameworks have been proposed for merging information on the basis of a proper registration [3, 15] between successive viewpoints [6, 8, 11, 16, 17]. However, not all of these strategies allow to take advantage of the fact that redundant information is often collected when measurements are taken in a same region of space from different viewpoints due to the overlap between the respective sensors' field of view. Basic approaches such as minimization or maximization of the estimates corresponding to a same point in space have been widely used in the computation of deterministic occupancy representations that associate a limited set of tags (*empty*, *occupied* or *unknown*) to each region of space. Unfortunately, it is widely recognized that such schemes do not ensure a proper classification of contradictory data and reamin sensitive to perturbations.

A computational framework using the Bayes theorem has been introduced to build probabilistic occupancy maps on planar terrains for mobile robot guidance [4]. This approach computes the conditional probability, normalized between 0.0 (*empty*) and 1.0 (*occupied*), that a region of space is cluttered by obstacles. Unfortunately, under its formal definition, this scheme reveals to be computationally untractable for 3-D spaces. However, it demonstrates very attractive characteristics as a controlled refinement process occurs when numerous consistent and/or contradictory measurements are presented to the data fusion engine.

Previous work has led to a tractable extension of the bayesian data merging framework to 3-D space that relies on a closed-form approximation of the original behavior of the evolving occupancy probability [13]. The scheme has been further extended to improve the refinement process and to account for uncertainty originating from various sources [14]. However, this scheme remains a probabilistic one.

From the perspective of collision avoidance in robotics, the term "probability" might be somewhat abusive in the context where strict conditional probabilities are no longer computed but rather replaced by a closed-form approximation demonstrating a similar behavior in general. What is of interest for the robot operation and the computerized path planner is to assess the relative risk associated with the choice of making the device to circulate in a given area of the workspace.

Pursing this goal and taking into account the new trends in computer vision and robotics to represent uncertain information using fuzzy descriptors [2] and to merge multisensor data with help of fuzzy inference [5, 10], this paper proposes a fuzzy inference engine that can advantageously be used as a data fusion mechanism for consistent/contradictory range measurements in the construction of occupancy models in replacement of the previous probabilistic scheme. The problem is examined in the context of volumetric scene modeling for path planning and collision avoidance in robotic applications.

Bi- and tridimensional fuzzy occupancy grids are considered for the virtual representation of environment as they allow to directly monitor the cluttering state of space. These grids consists of a recursive subdivision of a bounded region of space into square or cubic cells up to a given level of resolution. Each cell is tagged with a fuzzy descriptor of the occupancy state of space. The main aspects that are considered in the development of this fuzzy data fusion framework are: 1) the behavior of the occupancy state as supplementary data (consistent or contradictory) are merged with the current state of space, 2) the capability to deal with uncertainty in the measurement process (both from the sensor and the positioning devices), and 3) the computational efficiency of the approach as large datasets generally need to be processed.

The following sections present the characteristics of the probabilistic occupancy modeling scheme in order to define a base of reference for the proposed fuzzy occupancy representation. Then the fuzzy inference structure is introduced and the behavior of the occupancy model that results from the merge of range measurements using this fuzzy modeling/fusion scheme is demonstrated. Its performances are analyzed with respect to the three aspects mentioned previously and the suitability of a fuzzy workspace representation for path planning operations is discussed.

II. PROBABILISTIC MODELING SCHEME

The construction of an occupancy grid model consists in estimating the probability that a grid cell is occupied based on range measurements collected on the surface of objects.

A. Initial probabilistic scheme

Elfes [4] introduced a bayesian two-step procedure for building 2-D probabilistic occupancy maps of planar surfaces. The sensor measurements accuracy is assumed to follow a gaussian distribution. Measurements are mapped on a discretized grid containing a probability of occupancy for each of its cells. Under some assumptions about the status of successive cells and based on the current configuration of the grid, the probability that the state, $s(C_i)$, of a given cell, C_i, is occupied for a measured range value, r, is estimated as:

$$P(s(C_i) = OCC|r) = \frac{P(r|(s(C_i) = OCC)) \cdot P(s(C_i) = OCC)}{\sum\limits_{s(C_i) = \{OCC, EMP\}} P(r|s(C_i)) \cdot P(s(C_i))} \quad (1)$$

where

$$P(r|(s(C_i) = OCC)) = \sum_{\{G_{s(C_i)}\}} P(r|(s(C_i) = OCC), G_{s(C_i)}) \cdot P(G_{s(C_i)}|(s(C_i) = OCC)) \quad (2)$$

for a given configuration, $G_{s(C_i)}$.

In a second phase, the probability of occupancy of grid cells is updated by means of a merging process that follows Bayes theorem.

$$P(s(C_i) = OCC|P_1, P_2) = \frac{P_1 P_2}{P_1 P_2 + (1 - P_1)(1 - P_2)} \quad (3)$$

where P_1 and P_2 are respectively the probability that is already contained in a given cell of the model and the probability estimated from the latest measurement following eq. (1).

Even though this approach reveals to be mathematically consistent with the definition of probability, the first step implies the computation of the probability of occupancy for all possible configurations of the grid as shown in figure 1 for a simplistic 1-D case. Here, 16 grid configurations must be processed before updating the probability of each cell when a new range measurement is provided by the sensor. In general, 2^{n^m} configurations have to be considered for a m-dimensional grid having n cells along each of its side. Experiments demonstrated that the computation of such conditional probabilities becomes computationally untractable in the 3-D case.

0 0 0 0	0 0 0 1	0 0 1 0	0 0 1 1
0 1 0 0	0 1 0 1	0 1 1 0	0 1 1 1
1 0 0 0	1 0 0 1	1 0 1 0	1 0 1 1
1 1 0 0	1 1 0 1	1 1 1 0	1 1 1 1

Fig. 1. 16 possible configurations for a 1-D discretized occupancy grid with 4 cells (0=empty, 1=occupied).

B. Revisited probabilistic scheme

In order to prevent the computational explosion that results from the implementation of eq. (1) and (2) for 3-D space occupancy estimation, an extended approach [13] has been proposed that consists in using a closed-form approximation of the Occupancy Probability Distribution Function (OPDF) obtained with the initial scheme as shown in figure 2. Assuming that the sensor error can be modeled by a gaussian distribution, we observe that the estimated probability of occupancy is close to zero near the sensor (located at 0 mm). This probability rises until it reaches a maximum at the location of the surface of an object (located at 50 mm in this example) and then drops to 0.5 for the area which is in occlusion behind the object. A probability of 0.5 in this area means that the occupancy state remains unknown.

These observations conducted to the development of an experimental closed-form expression that approximates the shape of the OPDF. The main advantage of this closed-form expression is that is eliminates the need to explore all possible grid configurations in the computation of the occupancy probability for a given cell of the occupancy grid. Furthermore, it makes the probability evaluation process independent from the number of cells in the grid. The closed-form expression that is proposed to model the shape of the OPDF for a range sensor in the 1-D case is given by:

$$P(\rho) = \frac{7}{20} + \frac{3}{20}\left(1 + e^{-\left(\frac{2((\rho-\bar{\rho})+2\sigma_\rho)}{\sigma_\rho}\right)}\right)^{-1} + \frac{3}{20} \cdot e^{-\left(\frac{(\rho-\bar{\rho})^2}{\sigma_\rho^2}\right)} \quad (4)$$

where $\bar{\rho}$ is the range measurement and σ_ρ is its variance.

The application of this approximated OPDF for the estimation of occupancy probability revealed to be computationally tractable for 3-D workspace modeling. Moreover, when this closed-form expression is applied in combination with the Bayes theorem, eq. (3), to merge former probability estimates contained in the grid along with those resulting from new measurements, a progressive refinement of the occupancy probability is observed as several measurements are collected on the same point of the surface of objects. This behavior is illustrated in figure 3.

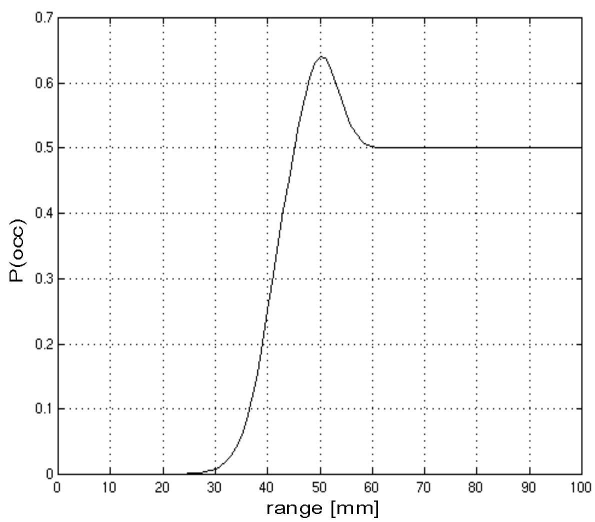

Fig. 2. Characteristic shape of the OPDF resulting from conditional probability estimation with Bayes merge.

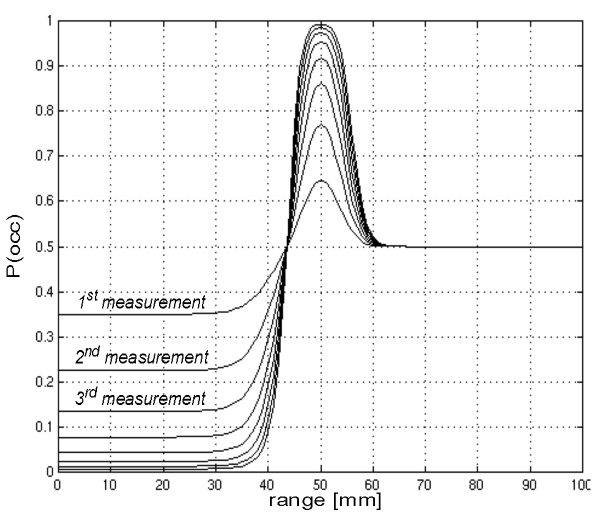

Fig. 3. Refinement of the occupancy probability with fusion of consistent/contradictory measurements.

III. FUZZY MODELING SCHEME

When the later closed-form approximation of the OPDF is used to make occupancy probability estimation tractable, the strict mathematical definition of probability is approximated. Instead, a relative assessment of the risk for the robot to circulate in a given area is monitored in the model of the environment on a comparative basis with other regions of the workspace. Therefore, a parallel between the resulting virtual representation and a fuzzy logic description of the cluttering state of the environment can be established.

The present section introduces a fuzzy inference engine that allows to generate such a fuzzy description of a robot workspace in terms of occupancy level while reproducing the desirable characteristics achieved by the probabilistic modeling scheme previously described. The main characteristics considered are: 1) a progressive refinement of the occupancy state when consistent/contradictory measurements are merged, 2) the possibility to represent uncertainty on measurements and registration, and 3) a computationally tractable approach.

A typical fuzzy inference engine has been developed to operate on range measurements collected from a laser range finder. The system follows the classical structure of a fuzzy logic inference engine in which the distribution of membership functions and the rules have been adapted to suit the requirements of the application.

A. Mapping measurements to fuzzy inputs

The inputs to the system correspond to a mapping of the occupancy state of space along the line traversed by the laser beam during data collection for each point on the object's surface as shown in figure 4. This area is discretized up to a given resolution and the fuzzyfication process operates successively on each cell defined by the discretization step.

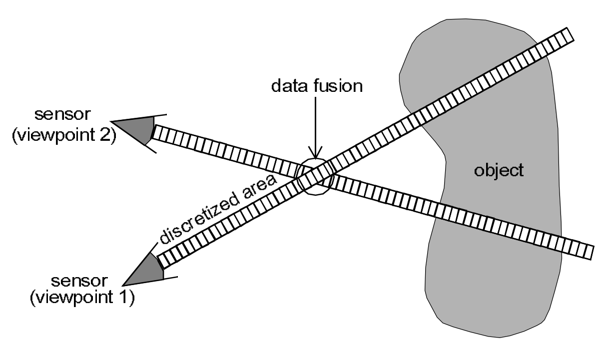

Fig. 4. Mapping of the discretized area along the laser beam to the input of the fuzzy inference engine.

The crisp input to the fuzzyfication process for each cell is defined as the *distance* between the actual measurement provided by the sensor and the discrete position of the given cell with respect to the sensor.

$$\text{distance} = \text{cell position} - \text{sensor measurement} \quad (5)$$

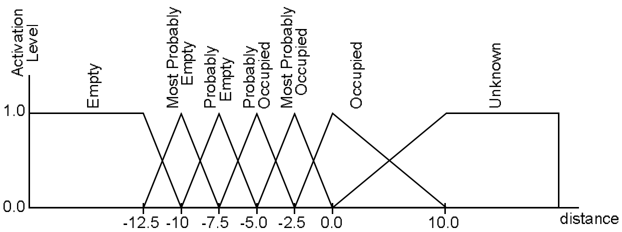

Fig. 5. Input membership functions distribution.

Membership functions of the input are classified in an order that matches with the typical occupancy distribution when a sensor collects measurements on a surface, taking into account occlusions. That is, starting from empty space just in front of the sensor, it evolves up to unknown space behind the surface of the object, with a region of occupied space around the surface of the object. Figure 5 shows the distribution of membership functions for the inputs. We observe the correspondence between the fuzzy labels and the occupancy probability distribution shown in figure 2. Especially, the fact that the *unknown* tag is located on the far right end indicates that the sensor is located on the left side of the mapping (along with negative distances as the cells are closer to the sensor than the measurement value). Numerical values shown here correspond to the examples presented in section 4.

When a new measurement is collected, a corresponding fuzzy representation is created along the line traversed by the laser beam. Each cell contains a list of activated members along with their respective level of activation, thus describing if this area of space is emptied, occupied or in an intermediate or unknown state. These fuzzy descriptors are then ready to be used for a data fusion process to occur when other measurements along the same laser line or in the same region of space will be collected as illustrated in figure 4.

B. Evaluation rules for data fusion

When two measurements are available for a same cell of the workspace, they need to be merged to achieve a consistent representation. Fuzzy evaluation rules are then applied to the set of fuzzy inputs in order to combine them. Evaluation rules are defined in such a way that the refinement process previously observed on the occupancy state of space occurs depending on the nature of the information provided. Table 1 defines the set of rules that have been used in the present experimentation to validate the approach. The number of rules is determined by the number of fuzzy membership functions.

It is also noted that the table should not be symmetrical (in terms of conclusion labeling) as new inputs must influence the occupancy state in an appropriate way to led to the refinement of the occupancy state. One input (here input 2) represents the current state of the model while the other (input 1) is associated with the new fuzzyfied measurement. This table of rules can also be extended to *n* dimensions in order to allow a simultaneous merge of *n* fuzzy inputs.

Table 1. Data fusion rules.

Input 2 (model current state) \ Input 1 (new measurement)	EMP	M.P. EMP	P. EMP	P. OCC	M.P. OCC	OCC	UNK
EMP	EMP	M.P. EMP	M.P. EMP	M.P. EMP	P. EMP	UNK	EMP
M.P. EMP	EMP	M.P. EMP	P. EMP	P. EMP	UNK	P. EMP	M.P. EMP
P. EMP	M.P. EMP	M.P. EMP	P. EMP	UNK	P. OCC	P. OCC	P. EMP
P. OCC	P. EMP	P. EMP	UNK	P. OCC	M.P. OCC	M.P. OCC	P. OCC
M.P. OCC	P. OCC	UNK	P. OCC	P. OCC	M.P. OCC	OCC	M.P. OCC
OCC	UNK	P. OCC	P. OCC	M.P. OCC	M.P. OCC	OCC	OCC
UNK	M.P. EMP	M.P. EMP	P. EMP	P. OCC	M.P. OCC	M.P. OCC	UNK

C. Fuzzy outputs

The membership functions distribution for outputs uses the same tags as those found on the inputs as shown in figure 6. This is mandatory as an output is expected to be used as an input in a forthcoming fusion operation with a new measurement. The crisp output variable represents the occupancy state of space or, in the context of robotics, an assessment of the risk of collision between the robot and its environment. The distribution of membership functions is slightly modified with respect to the inputs in order to map the risk with a linear distribution similar to that obtained from the probabilistic scheme (between 0.0 and 1.0). As a result, the *unknown* tag is now located in the center of the distribution as it is associated with a risk of 50% (unknown state of space).

The defuzzyfication process does not need to be applied as long as the model is not to be interpreted by an external device, e.g. a path planner or a rendering tool. The model can easily be encoded under his fuzzy representation where each cell contains a list of activated fuzzy members and their respective level of activation in accordance with the occupancy state of the region that it represents. This way the model is ready for further refinement by merging new fuzzyfied range measurements.

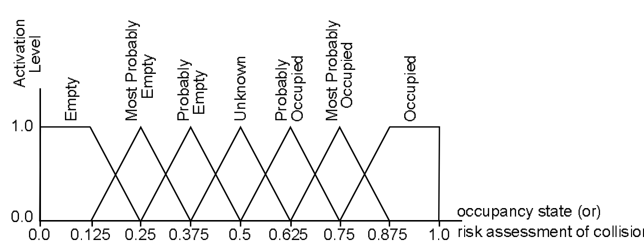

Fig. 6. Output membership functions distribution.

IV. EXPERIMENTAL RESULTS

Experimentation has been conducted with the proposed fuzzy logic inference engine in order to assess its capability to encode occupancy state. The refinement process has also been examined and the definition of membership functions and evaluation rules has been tuned such that a suitable behavior is achieved. The following curves illustrate the evolution of the occupancy state along the laser beam line as consistent and/or contradictory measurements are merged in the model.

Figure 7a shows the normalized distribution of occupancy for a single raw range measurement after defuzzyfication. The surface of the object is located where the distance equals 0 mm. Here the fuzzy representation of the input has been directly defuzzyfied without any rule evaluation (any fusion with other data) to observe the initial occupancy state distribution resulting from the discretized fuzzyfication process.

Next, the same input is merged with a initial occupancy distribution of 0.5 (unknown) everywhere in order to validate the refinement process in the occupancy state. The fusion result is shown in figure 7a. We see that the occupancy state progressively evolves from 0.5 to lower or higher levels depending on where the cells are located with respect to the surface of the object. This refinement process is pursued by merging the same measurement (at 0 mm) for a second time with the distribution that resulted from the previous operation. As a result, figure 7b shows a stronger confidence in space state both in empty and occupied areas after the fusion with a second consistent measurement.

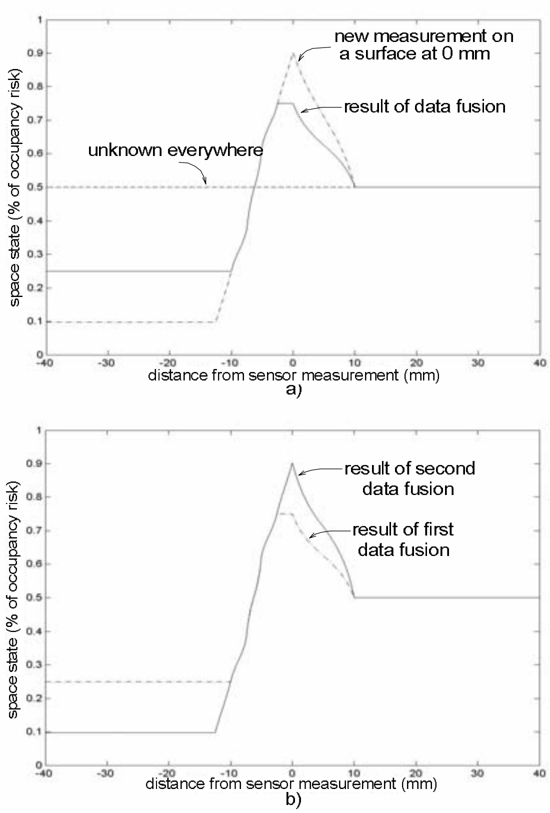

Fig. 7. Double fusion of a range measurement with an initial distribution corresponding to an unmapped area.

The proposed inference engine has also been evaluated in terms of its response to contradictory data. In figure 8, an occupancy state distribution corresponding to a range measurement shifted by 10 mm to the right with respect to the original one is merged twice with the result of the later fusion operation shown in figure 7. We observe the modifications to the distribution that result from the first merge as well as the evolution of the distribution after a second merge with the shifted data.

Again, the progressive refinement process is observed as new conclusions are not drawn drastically after a single contradictory measurement. The rate at which the distribution is upgraded actually depends on the refinement of the membership functions definition. If a larger number of members are used, the transformation between occupancy states requires a larger number of consistent data confirming the information. This makes the system very flexible to various requirements of the task, especially for safety concerns.

Finally, it is interesting to observe how the fuzzy inference engine behaves following perturbations from contradictory measurements. Figure 9 shows how the distribution progressively retrieves the information about the original position of the object surface (at 0 mm) after having been perturbed. For this purpose, the perturbed map obtained in figure 8 is merge three times with the initial distribution corresponding to the surface of the object at 0 mm. We observe that the peak of occupancy risk is brought back to its initial position after the second fusion with the consistent measurement. However, another peak is preserved around +10 mm as a result of the perturbation. This behavior is perfectly suitable as this area of space (behind the actual surface of the object where distance > 0) cannot be measured from the sensor viewpoint (located on the far left side) as it is in occlusion.

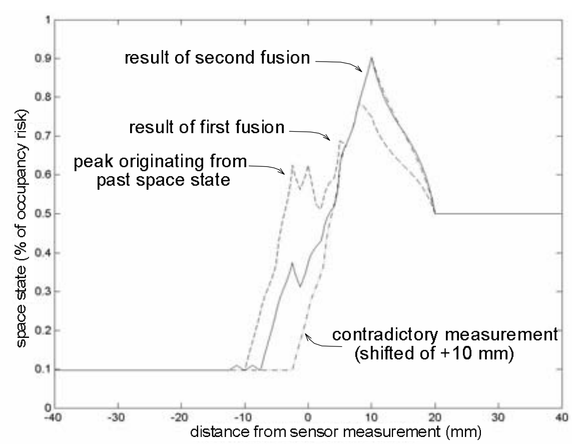

Fig. 8. Fusion with contradictory measurements.

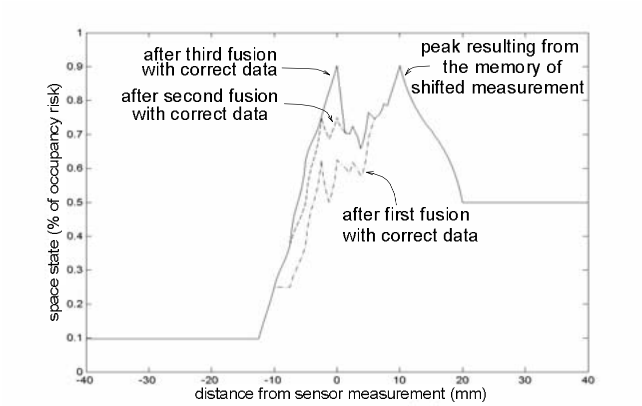

Fig. 9. Fusion of the perturbed model with the correct measurement.

As a result, we observe that such a fuzzy logic encoding can be suitable for mapping the occupancy state of space. Membership functions and evaluation rules can be defined to successfully implement the progressive refinement behavior obtained with a probabilistic scheme while avoiding the complexity of conditional probability estimation. The resulting encoding is more intuitive than approximate probabilistic estimates that are required to make the other approach tractable with large environments. As with a probabilistic scheme, the uncertainty on measurements and registration is encoded in the model as it influences the level of activation of membership functions. Finally, the computational workload of fuzzy logic inference engine can easily be kept tractable as only a very limited number of evaluation rules are activated during a given fusion process.

This approach also reveals to be suitable for robotics applications where path planning and collision avoidance are critical. As the fuzzy representation can directly be converted to a linear distribution representing the risk assessment for collisions in a given region of space, the same path planning techniques can be applied both for a probabilistic or a fuzzy occupancy representation of the environment [1,12].

V. CONCLUSION

This work demonstrates that fuzzy logic can advantageously be used in the construction of continuous occupancy models for robot path planning and collision avoidance. Fuzzy descriptors provide a consistent and more intuitive representation of the cluttering state of space and allow to handle large quantities of information in a tractable manner. A fuzzy logic inference engine is proposed that implements data fusion with similar behaviors as those observed under the application of the Bayes theorem for the risk assessment of collisions in robotic applications. This strategy is also under experimentation for modeling of complete 3-D spaces following a search for spatial intersections between fuzzy models of the occupancy state in complex environments.

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