

# The quest For duplicability Benoit Valiron

Setting: Quantum computation with classical control

\* two units of data: bit (classical)  
qubit (quantum)

\* one qubit: has a state, represented by a normalized vector in  $\mathcal{H} = \mathbb{C}^2$ .

n qubits: have a state in  $\mathcal{H}^{\otimes n}$ .

Corollary: non-locality of data:

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$  is the state of 2 qubits,

living in  $\mathcal{H} \otimes \mathcal{H}$

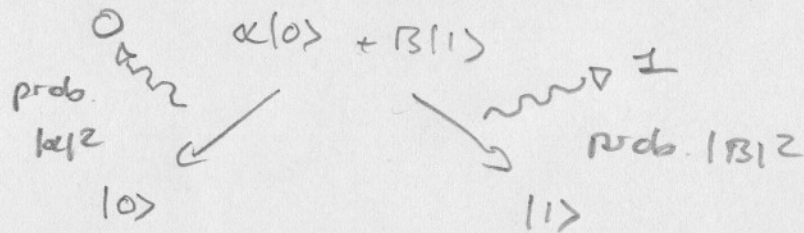
qubit 1  
is here

qubit 2  
is here.

but one cannot separate them  $\rightarrow$  entangled states.

\* Operations on qubits:

- unitary maps on the states  $\rightarrow$  reversible
- measurements:



- $\rightarrow$  destructive operation
- $\rightarrow$  probabilistic

\* Thesis: non-duplicability no map

$$\alpha|0\rangle + \beta|1\rangle \not\rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle)$$

\* Want classical control: tests, abstractions, etc.

Basic Framework:

\* want to be in a sym. mon. category:  
 $(\mathcal{E}, \otimes, I)$

\* want to be able to assert "this obj is duplicable"  
 $\rightarrow$  want a Functor  $! : \mathcal{E} \rightarrow \mathcal{E}$ .

a duplicable object does not need to be duplicated:  
 $\rightarrow$  want a map  $!A \xrightarrow{E_A} A$

a duplicable object is duplicable:

$\rightarrow$  want an iso  $!A \xrightarrow{S_A} !!A$

and we get a idempotent comonad  $(!, \mathcal{E}, \delta)$ :

Data:

$\left[ \begin{array}{l} (\mathcal{E}, \otimes, I) \text{ sym. mon. cat} \\ (!, \delta, \mathcal{E}) \text{ idemp. comonad} \end{array} \right]$

i.e.

$\left[ \begin{array}{l} \text{ReFlexive adjunction:} \\ \mathcal{E}^c \xrightarrow{I} (\mathcal{E}, \otimes, I) \end{array} \right]$

Need some more:

A pair of duplicable obj is duplicable:

$$!A \otimes !B \xrightarrow{d_{A,B}^!} !(A \otimes B)$$

$$I \xrightarrow{d_I^!} !T \quad (\text{For symmetry}).$$

These are not isos: if you can have as many pairs of socks as you want, you can't just throw the right ones away: we don't have weakening -

Updated Data:

$$\left[ \begin{array}{l} (e, \otimes, I) \text{ sym. mon. cat} \\ (!, s, \varepsilon, d^!, d') \text{ monoidal} \\ \text{idempotent comonad} \end{array} \right]$$

i.e.

$$\left[ \begin{array}{l} (e^L, \otimes, I) \text{ sym. mon.} \\ (e^L, \otimes, I) \xrightleftharpoons{!} (e, \otimes, I) \\ \text{sym. mon. refl. adjunction} \end{array} \right]$$

It's not enough:

We said "duplication", we need maps of duplication:

$$\Delta_A: !A \longrightarrow !A \otimes !A$$

$$\diamond_A: !A \longrightarrow T \quad (\text{for symmetry})$$

Together with nice enough structure:

- \*  $(!A, \Delta_A, \diamond_A)$  commutative comonoid
- \*  $\Delta_A$  and  $\diamond_A$  need to be monoidal NT's
- \* They need to be  $!$ -coalgebra morphisms
- \*  $\delta_A$  need to be comonoid morphism.

Updated data:

$$\left[ \begin{array}{l} (\mathcal{E}, \otimes, I) \text{ sym. mon. cat} \\ (!, \delta, \varepsilon, d, d', \diamond, \Delta) \text{ idemp. linear} \\ \text{exponential comonad} \end{array} \right]$$

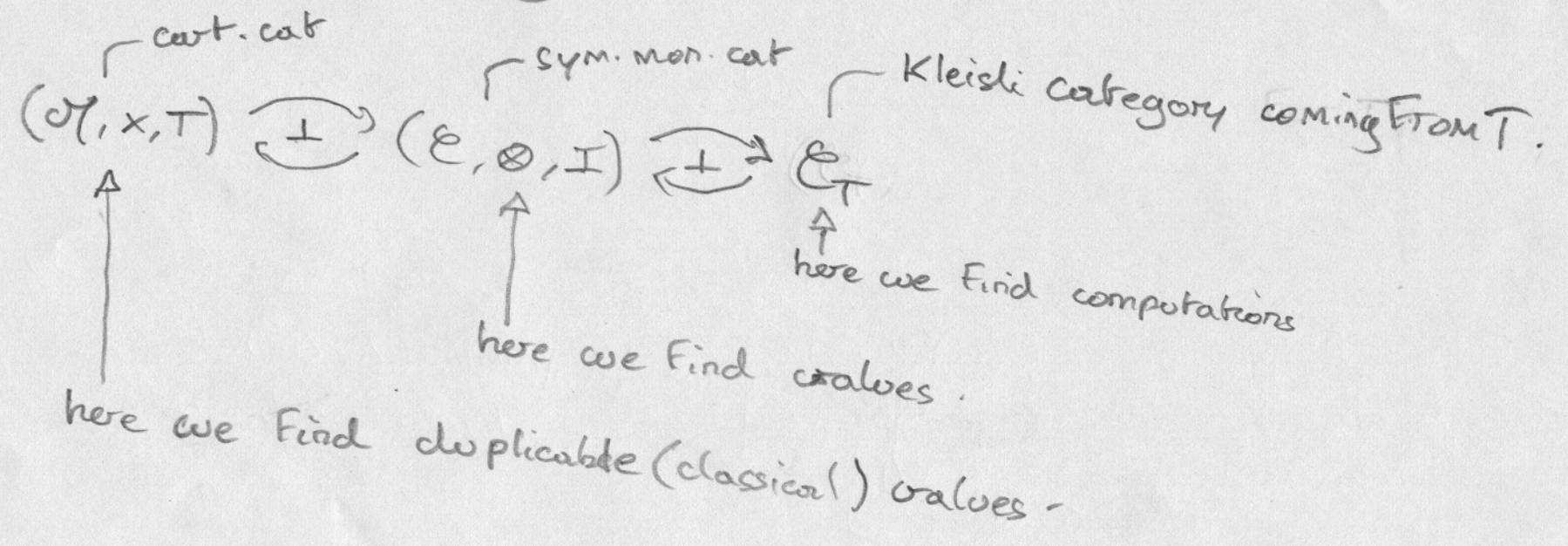
i.e.

$$\left[ \begin{array}{l} (\mathcal{A}, X, T) \text{ cart. cat.} \\ (\mathcal{E}, \otimes, I) \text{ sym. mon. cat} \\ (\mathcal{A}, X, T) \overset{\perp}{\circlearrowleft} (\mathcal{E}, \otimes, I) \text{ refl.} \\ \text{monoidal adjunction} \end{array} \right]$$

Modelo the idempotency, we have a model for intuitionistic linear logic (see Benton, Bierman, Schalk, ...).

But it is not enough: the measurement introducing side effects, we need more and we add a computational monad  $(T, \mu, \eta)$ , following Moggi's framework for computations.

Final categorical setting:



Conclusion:

- \* There's more to this: we want higher-order, recursion and additive types -
- \* The commutative diagrams need to be matched with axiomatic rules of terms manipulation -
- \* This dry set of rules need a decent model -

