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Iterative Learning-Based Fuzzy Control System

Radu-Emil Precup[1], Stefan Preitl[1], Emil M. Petriu[2], József K. Tar[3], János Fodor[3]
[1]Department of Automation and Applied Informatics, "Politehnica" University of Timisoara, Romania
[2]School of Information Technology and Engineering, University of Ottawa, Canada,
[3]Institute of Intelligent Engineering Systems, Budapest Tech Polytechnical Institution
Budapest, Hungary

Abstract: The paper discusses a delta domain approach to the design of Mamdani PI-Fuzzy Controllers using Iterative Learning Control algorithms. A speed control system for a DC drive servo-drive is implemented and experimental results are used to validate the proposed design method.

Keywords *– iterative learning Control, delta domain, fuzzy control, PI-fuzzy controller*

I. INTRODUCTION

Iterative Learning Control (ILC) allows improving the performance of the control systems (CSs) by iteratively using past experience in CS operation. An ILC system iteratively solves the parametric optimization learning problem minimizing the objective function specifying CS performance [1]..

One way to view fuzzy control is as convenient initial nonlinear approach dealing with complex or ill-defined plants designed by heuristic means incorporating human skills but without general-purpose design methods [2]. This results in difficulties concerning the analysis of the structural properties of fuzzy control systems (FCSs), stability, controllability, parametric sensitivity and robustness [3]. The aim of combining fuzzy control and ILC is to improve the performance indices of FCSs because of the benefits of both feedback due to fuzzy control and feedforward compensation due to ILC merged in the same CS structure.

The paper discusses three FCS structures using ILC algorithms. They belong to the class of low-cost automation solutions due to the transparency of the design and the simplicity of the implementation. The low-cost feature is an advantage compared to previous combinations between ILC and fuzzy control reported in the literature [4-7] that ensure also very good CS performance indices but do not include general practical implementations. Current combinations of fuzzy control and ILC include the popular Takagi-Sugeno fuzzy controllers (FCs) [4, 5] and adaptive FCs [6, 7].

Since discrete-time systems are generally represented in shift operator form with at least one shortcoming at high sampling rates reflected by the loss of information because of finite word length and arithmetic truncation [8]. One possible way to overcome this drawback is to use the delta (δ) transform in the δ domain approach to controller designs [9] with two specific features:

- the stability domain expands as the sampling period gets smaller,
- the pole transposition from the q domain (for discrete-time control systems) to the delta one.

They lead to relatively small sensitivity with respect to parametric modifications of both the controlled plant and the controller.

The δ domain characterization allows the design of a new generation of low-cost FCs in FCSs that exhibit improved CS performance. The second contribution of this paper concerns a new and attractive design method of FCSs with Mamdani PI-fuzzy controllers (PI-FCs). The method is simple and transparent for the user in contrast with other combinations of fuzzy control and delta transform under the form of Takagi-Sugeno FCs treated in continuous-time [10, 11] or discrete-time [12].

The paper discusses a new design method for Mamdani PI-FCs using ILC algorithms and δ transform. A speed control system for DC drive servo-drive is then implemented and experimental results are used to validate the proposed design method.

II. PROPOSED APPROACH

The controlled plant is accepted to be characterized by the discrete-time linear time-invariant SISO system

$$y_j(k) = P(q)u_j(k) + d(k), \qquad (1)$$

where: $u_j(k),\ k \in M_u,\ M_u = \{0,1,...,N-1\}$ – control signal, $y_j(k),\ k \in M_d,\ M_d = \{m, m+1,..., N+m-1\}$ – controlled output, N – number of samples of plant inputs and output d – exogenous input signal (e.g. load disturbance input) that repeats each iteration, j – index of current iteration, q – forward time-shift operator, $P(q)$ – proper rational function of the plant, with the relative degree of $m \in N^*$ and the delay of mT_s, T_s – sampling period, k – index of current sampling interval. The transfer function $P(q)$ is supposed to be asymptotically stable. If not, it must be stabilized first in a linear CS, next the ILC is applied in cascade-like CSs. The popular Q-ILC algorithm is

$$u_{j+1}(k) = Q(q)[u_j(k) + L(q)e_j(k+1)], \qquad (2)$$

where $Q(q)$ is the Q-filter, $L(q)$ is the learning function, and $e_j(k+1)$ stands for the control error:

$$e_j(k) = r(k) - y_j(k),\ k \in M_d, \tag{3}$$

and r is the set-point (reference input).

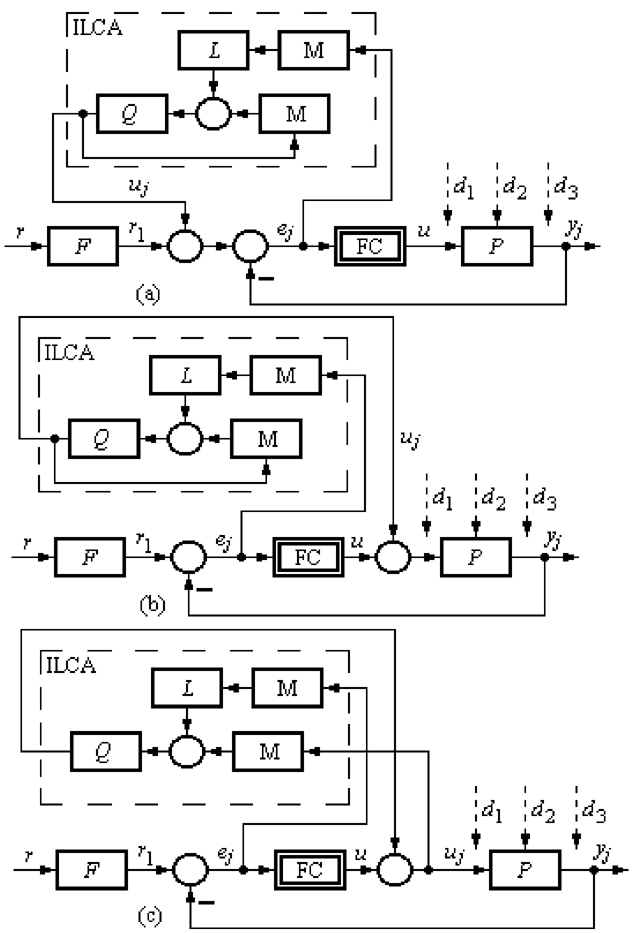

Fig. 1. ILC-based fuzzy control system structures.

The merge between the ILC algorithm (2) and the linear CSs containing feedback controllers results in several CS structures:

- CS with serial ILC, where the control signal calculated by the ILC algorithm, $u_j(k)$, is added to the reference input before the feedback loop,
- CS with parallel ILC, where $u_j(k)$ is added to the control signal produced by the feedback controller,
- CS with current-iteration ILC algorithm:

$$\begin{aligned} u_{j+1}(k) = Q(q)[u_j(k) + L(q)e_j(k+1)] + \\ + C(q)e_j(k+1), \end{aligned} \tag{4}$$

where $C(q)$ is the proper rational function of the feedback controller. Use is made of PD-type learning functions exemplified defined in (5) to simplify the ILC algorithms:

$$\begin{aligned} u_{j+1}(k) = u_j(k) + k_p e_j(k+1) + \\ + k_d[e_j(k+1) - e_j(k)], \end{aligned} \tag{5}$$

with: k_p – proportional gain and k_i – derivative gain.

The suggested FCSs are fuzzified versions of the three CSs mentioned above. They are referred to as FCS with serial ILC (Fig. 1 (a)), FCS with parallel ILC (Fig. 1 (b)) and FCS with current-iteration ILC (Fig. 1 (c)). The other variables and blocks in Fig. 1 are: d_1, d_2, d_3 – load disturbance inputs, F – set-point filter, ILCA – Iterative Learning Control Algorithm, M – memory, FC – fuzzy controller outlined as nonlinear block.

The three structures presented in Fig. 1 make use of FCs to replace the linear controllers C in order to ensure the improvement of CS performance indices in the framework of low-cost automation. For reasons of simplicity the presentation of the design method will be focused on Mamdani PI-fuzzy controllers. The paper suggests a unified design method for all three FCS structures.

Our design method makes use of the δ transform [13]:

$$\delta = (q-1)/T,\ \gamma = (z-1)/T, \tag{6}$$

where z^{-1} is its complex variable associated to discrete-time control, γ is the complex variable associated to the delta operator δ, and T is the sampling period. The stability region in case of δ domain is obtained by mapping the inner unit disc corresponding to q domain making use of (6). This results in the inner disc with center in $(-1/T, 0)$ and radius $1/T$ (Fig. 2). The δ domain stability region is variable with the sampling period as opposed to the q domain stability region. As $T \to 0$ the δ domain stability region converges to the open left half-plane, representing the stability region for the continuous-time operator.

Fig. 2 shows that the δ transform can be viewed as a tool for systems modeling between continuous- and discrete-time operators.

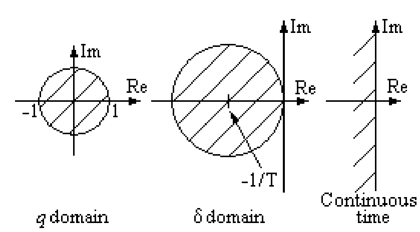

Fig. 2. Stability regions.

The δ domain approaches to controller designs can be divided in two categories [9]:

- the design is done in continuous time according to the continuous model of the plant, followed by the replace of the variable s with the corresponding delta domain variable γ,

- the direct controller design is done in the δ domain employing the δ domain model of the plant.

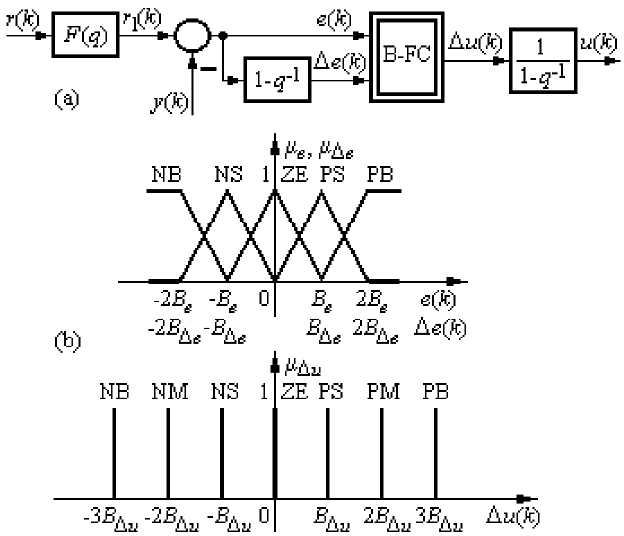

Fig. 3. PI-FC structure (a) and membership functions (b).

One remarkable advantage of the direct design applied in this paper is that the delta domain model of the plant contains already the zero-order-hold (ZOH) element. For a given continuous-time transfer function $P(s)$ the delta domain transfer function $P(\gamma)$ can be calculated:

$$P(\gamma) = \gamma/(1+T\gamma)\mathrm{T}\{L^{-1}[P(s)/s]\}, \quad (7)$$

making use of the generalized operational transform T of an original function f in the independent variable t (time) [9, 13]:

$$\mathrm{T}\{f(t)\} = T\sum_{k=0}^{\infty} f(kT)(1+\gamma T)^{-k}. \quad (8)$$

Accepting a linear PI controller with the transfer function

$$C(s) = k_C[1+1/(sT_i)], \quad (9)$$

where k_C – controller gain, T_i – integral time constant, and a linear set-point filter with the transfer function

$$F(s) = 1/(1+T_i s), \quad (10)$$

the generic structure of the Mamdani PI-FCs that discretize $C(s)$ is presented in Fig. 3 (a). B-FC is the nonlinear fuzzy block without dynamics; it includes the scaling of inputs and output in the fuzzyfication module. The fuzzyfication is done by the input and output membership functions illustrated in Fig. 3 (b).

The inference engine makes use of Mamdani's MAX-MIN compositional rule of inference assisted by the rule base presented in Table I. The defuzzyfication is done by the centre of gravity method.

Table I. Decision tale of B-FC.

Δe_k	e_k				
	NB	NS	ZE	PS	PB
PB	ZE	PS	PM	PB	PM
PS	NS	ZE	PS	PM	PB
ZE	NM	NS	ZE	PS	PM
NS	NB	NM	NS	ZE	PS
NB	NM	NB	NM	NS	ZE

The proposed design method dedicated to the accepted class of Mamdani PI-FCs in all three FCSs presented (Fig. 1) consists of the design steps 1 to 4:

- Step 1. Express the continuous-time plant model and transform it to $P(\gamma)$ by means of (7). Next, apply one controller design method in the δ domain resulting in the δ domain form of the PI controller.
- Step 2. Set the sampling period T accounting for the ZOH element and express the discrete-time equation of the-point filter $F(q)$. Then, apply (6) to obtain the discrete-time PI controller:

$$\Delta u_k = K_P(\Delta e_k + \alpha\, e_k). \quad (11)$$

- Step 3. Design one of the ILC algorithms i.e. set the parameters k_p and k_i to fulfill conditions regarding the stability and desired dynamics.
- Step 4. Apply the modal equivalence principle:

$$B_{\Delta e} = \alpha B_e,\, B_{\Delta u} = \alpha\, K_P\, B_e, \quad (12)$$

where the parameter B_e represents designer's option. Stability or sensitivity analyses of the FCS can be applied with this respect [14, 15].

III. RESULTS

A speed control system for the AMIRA DR300 nonlinear DC drive was implemented to validate the described FLC design method.

The ESO method [14] has been applied in the δ domain in the step 1 for the continuous-time plant

$$P(s) = k_P/[s(1+T_\Sigma s)], \quad (13)$$

with k_P – controlled plant gain and T_Σ – small time constant or sum of parasitic time constants. Part of the real-time experimental results is presented in Figs. 4 and 5 for the triangular modification of r and the 5 s period step modification of the d_3-type disturbance input.

The experimental results clearly illustrate the CS performance enhancement (the alleviation of the overshoot). Future research will be focused on new applications and controller structures.

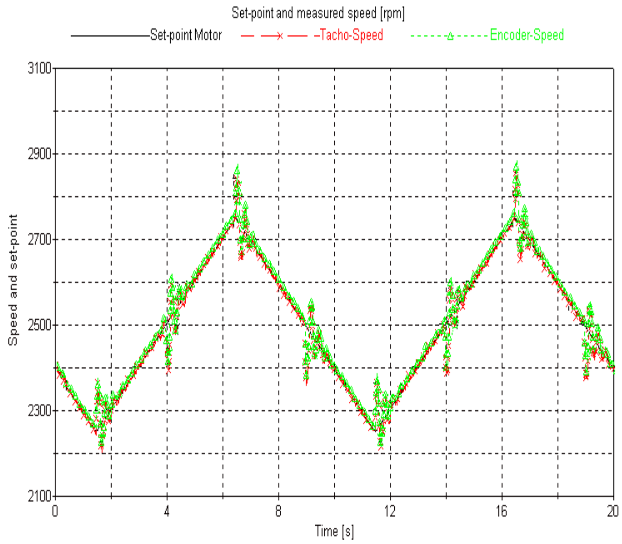

Fig. 4. Speed response of FCS without ILC.

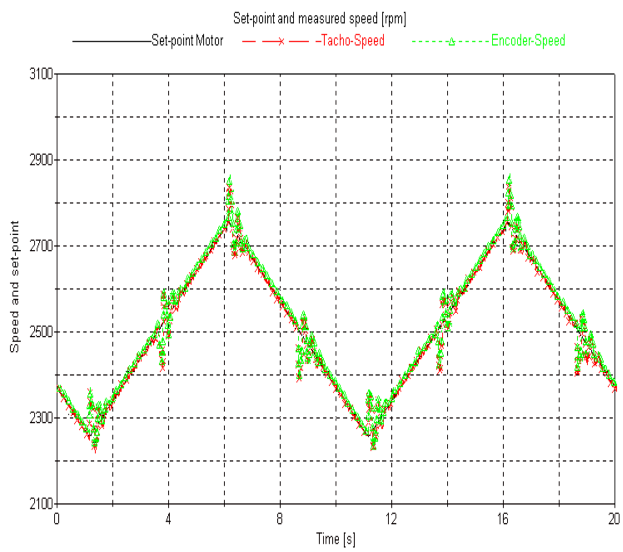

Fig. 5. Speed response of FCS with ILC.

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