CONTROL

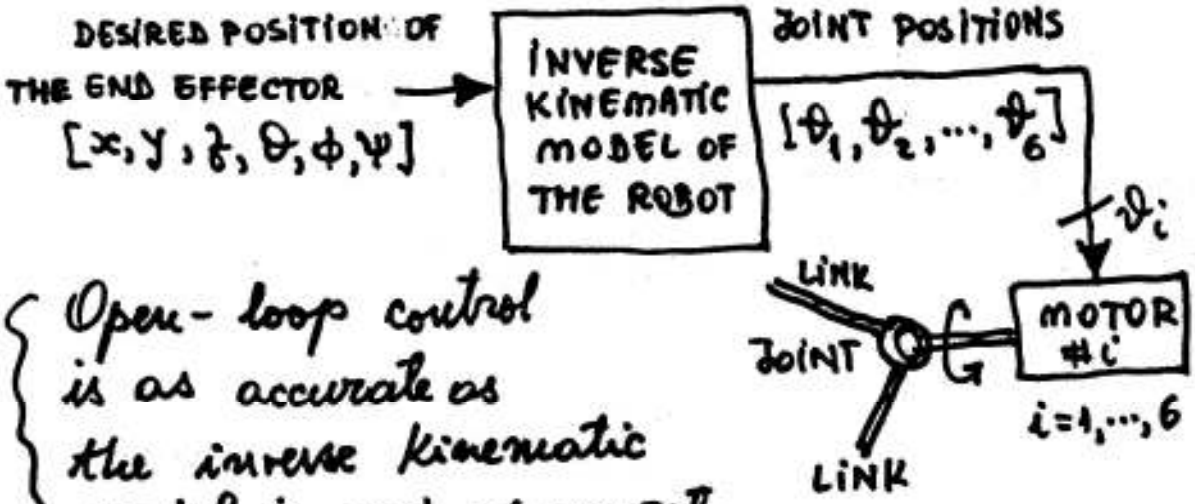
CONTROL SYSTEM = ALLOWS CORRECT EXECUTION OF THE PLANNED SEQUENCE OF MOTIONS AND FORCES EVEN IN THE PRESENCE OF UNFORSEEN ERRORS

SOURCE OF ERRORS : INACCURACIES IN THE MODEL OF THE ROBOT, MEASUREMENT ERRORS OF SENSORS, LIMITED PRECISSION IN COMPUTATION, INACCURACIES OF ACTUATORS, TOLERANCES IN WORKPIECES, FRICTION IN JOINTS, COMPLIANT LINKAGES, ...

CONTROLLING : POSITION, SPEED, FORCE, TORQUE

BASIC CONCEPTS

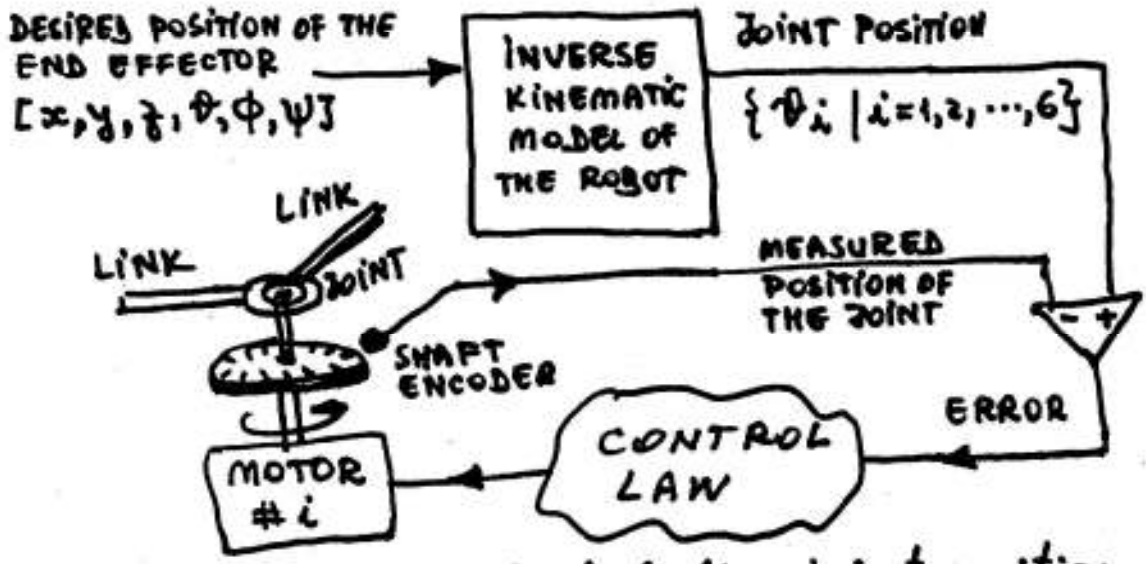
OPEN-LOOP CONTROL



Open-loop control is as accurate as the inverse kinematic model is and as accurate are the actuators and joints

CONTROL-01

- **FEEDBACK CONTROL** : THE PARAMETER THAT IS CONTROLLES IS CONTINUOUSLY MEASURED, COMPARED TO THE DESIRED VALUE OF THAT PARAMETER (REFERENCE), AND PROPER ACTION IS TAKEN ACCORDING TO A CONTROL LAW TO MINIMIZE THE ERROR (i.e. THE DIFFERENCE BETWEEN THE MEASURED AND RESPECTIVELY THE DESIRED VALUES OF THE CONTROLLED PARAMETER).



Feedback control of the joint position.

- **ADAPTIVE CONTROL** : MEASUREMENTS OF THE RESULTS OF PREVIOUS ACTIONS ARE USED TO ADDAPT THE PROCESS MODEL TO CORRECT FOR CHANGES IN THE PROCESS AND ERRORS IN THE MODEL.

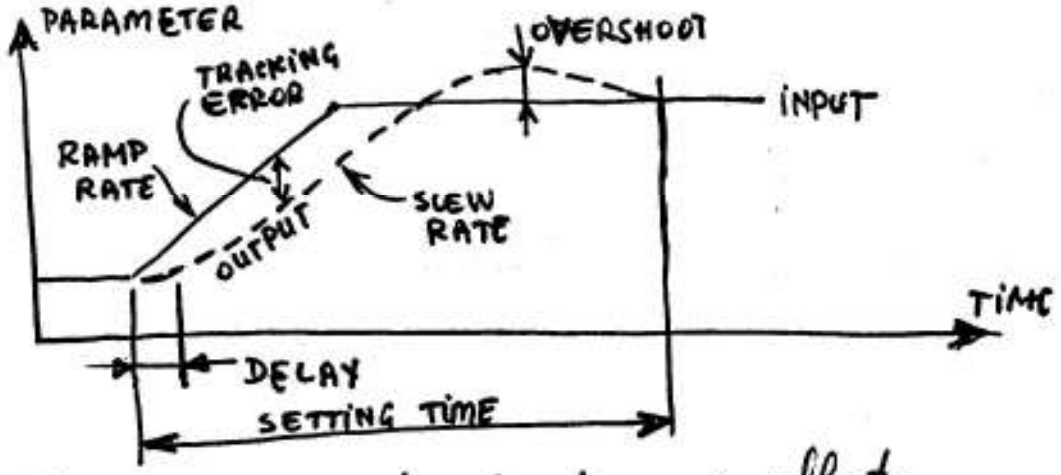
This type of feedback control can't be used for real time control of parameters that can dynamically change due to some local parameters. It is useful for the correction of model errors due to long-term variations in the environment

♦ FEEDFORWARD CONTROL : USED TO PREDICT ACTUATOR SETTINGS FOR PROCESSES WHERE FEEDBACK SIGNALS ARE DELAYED AND IN PROCESSES WHERE THE DYNAMIC EFFECTS OF DISTURBANCES MUST BE REDUCED.

TRANSFER FUNCTIONS

THE FIRST STEP WHEN DESIGNING A CONTROLLER IS TO FIND THE TRANSFER FUNCTION (PLANT MODEL) OF THE PROCESS TO BE CONTROLLED.

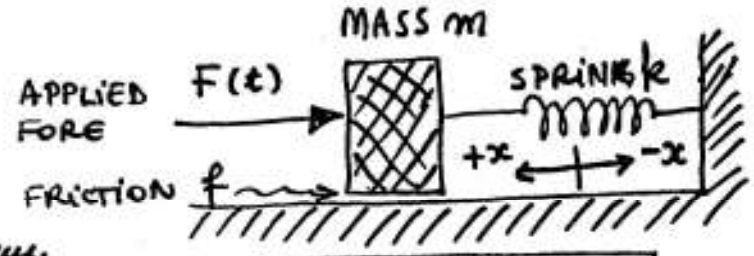
TRANSFER FUNCTIONS ARE PLANT MODELS WHICH INCLUDE DYNAMIC EFFECTS AS WELL AS THE STATIC RELATIONSHIP BETWEEN OUTPUT AND INPUT. THE MAIN DYNAMIC EFFECT OF CONCERN IS THE TIME DELAY BETWEEN WHEN THE INPUT IS CHANGED AND WHEN THE OUTPUT RESPONDS.



Tracking error due to dynamic effects.

Example

Mass-spring damper system.



$$\boxed{F(t) = m\ddot{x} + f\dot{x} + kx}$$

THE TRANSFER FUNCTION IS THE RATIO OF THE LAPLACE TRANSFORMS OF THE OUTPUT AND RESPECTIVELY THE INPUT.

$$\boxed{\frac{\text{OUTPUT}}{\text{INPUT}} = G(s) = \frac{1}{ms^2 + fs + k}}$$

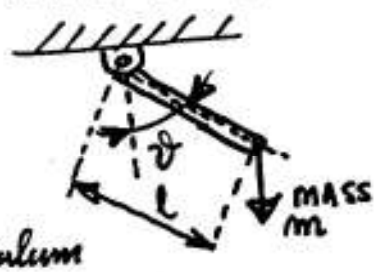
THE POLYNOMIAL IN THE DENOMINATOR, WHEN EQUATED TO ZERO, IS CALLED THE CHARACTERISTIC EQUATION.

- IN ROBOTICS, TRANSFER FUNCTIONS ARE REQUIRED FOR BOTH THE LINKS AND THE ACTUATORS.

- LINK MODEL: A LINK & JOINT CAN BE MODELLED AS A PENDULUM.

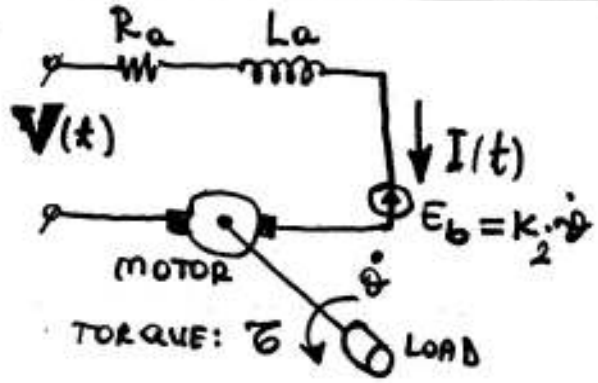
THE TORQUE APPLIED TO THE SHAFT OF THE JOINT IS A FUNCTION OF THE INERTIA, THE JOINT FRICTION, AND GRAVITY:

$$\boxed{\tau = m\ddot{\theta} + f\dot{\theta} + mgl\theta}$$



Linke & joint modelled as a pendulum

- DIRECT-CURENT MOTOR MODEL

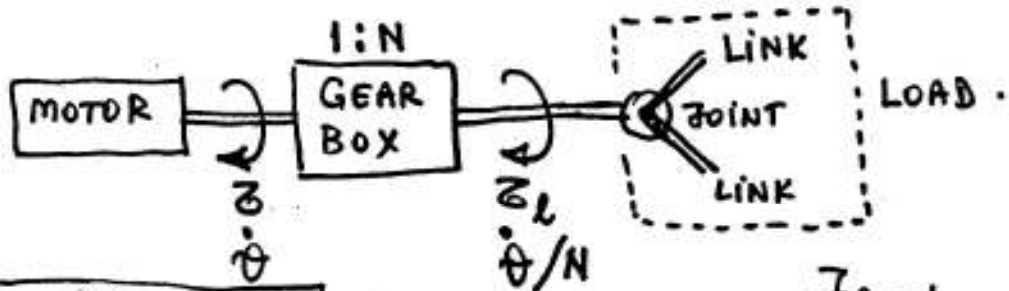


PERMANENT MAGNET D.C. MOTOR

- THE ARMATURE CURRENT $I(t)$ IS A FUNCTION OF THE APPLIED VOLTAGE $V(t)$, THE ARMATURE REESISTANCE R_a, THE ARMATURE INDUCTANCE L_a, AND THE BACK-ELECTROMOTIVE-FORCE VOLTAGE $E_b(t)$
- $E_b(t)$ IS PROPORTIONAL TO THE ANGULAR VELOCITY $\dot{\vartheta}$: $E_b(t) = K_2 \cdot \dot{\vartheta}$
- THE MOTOR TORQUE IS PROPORTIONAL TO THE ARMATURE CURRENT AND THE MAGNETIC FLUX (THAT'S CONSTANT FOR A PERMANENT MAGNET D.C. MOTOR):

$$\tau(t) = k_1 \cdot I(t)$$

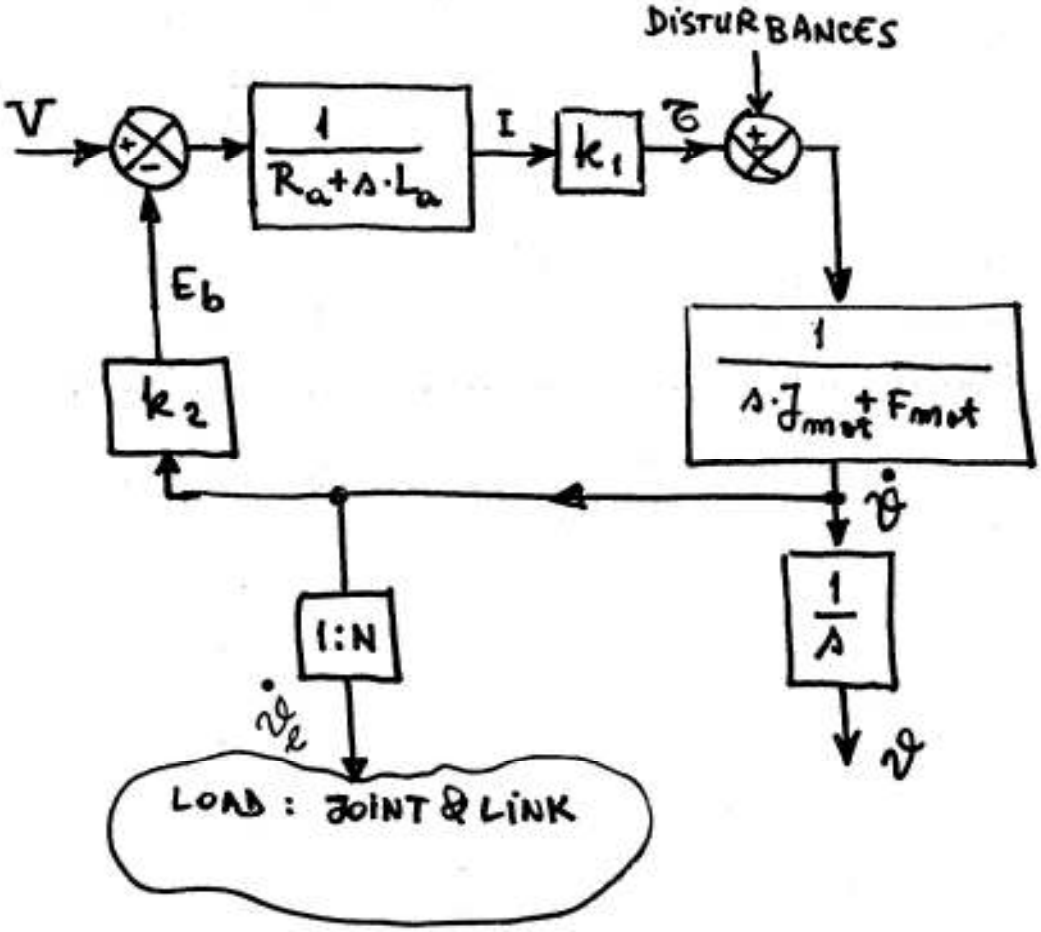
- LINK & JOINT LOAD MODEL



$$\boxed{\tau = J_{eq} \cdot \ddot{\vartheta} + F_{eq} \cdot \dot{\vartheta}}$$

EQUIVALENT INERTIA: $J_{eq} = J_{mot} + \dfrac{J_{load}}{N^2}$

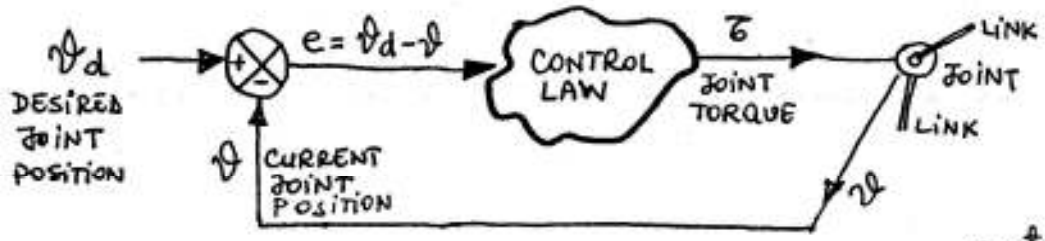
EQUIVALENT FRICTION: $F_{eq} = F_{mot} + \dfrac{F_{load}}{N^2}$



BLOCK-DIAGRAM OF A
D.C. MOTOR-DRIVEN ROBOT JOINT.

☆ FEEDBACK CONTROL OF A ROBOT JOINT

♦ **ERROR PROPORTIONAL CONTROL** : MOVE THE JOINT IN THE DIRECTION THAT MINIMIZES THE ERROR.



ERROR PROPORTIONAL CONTROL LAW: $\boxed{\tau = K_e \cdot e = K_e \cdot (\vartheta_d - \vartheta)}$ ← Constant

JOINT TORQUE = LOAD TORQUE : $\tau = J_{eq} \cdot \ddot{\vartheta} + F_{eq} \cdot \dot{\vartheta}$

$$\therefore \quad K_e \cdot (\vartheta_d - \vartheta) = J_{eq} \cdot \ddot{\vartheta} + F_{eq} \cdot \dot{\vartheta}$$

⇩

- IN THE CASE OF A FRICTIONLESS LOAD $F_{eq} = 0$, AND FOR THE SAKE OF SIMPLIFICATION $\vartheta_d = 0$:

$$\boxed{-K_e \cdot \vartheta = J_{eq} \cdot \ddot{\vartheta}} \qquad \therefore \quad \ddot{\vartheta} = -\frac{K_e}{J_{eq}} \cdot \vartheta$$

$$\therefore \vartheta = \sin \omega t \quad \Rightarrow \quad -\omega^2 \cdot \sin \omega t = -\frac{K_e}{J_{eq}} \cdot \sin \omega t$$

$$\therefore \quad \omega = \sqrt{\frac{K_e}{J_{eq}}} \quad \Rightarrow \quad \vartheta = \sin\left(\sqrt{\frac{K_e}{J_{eq}}} \cdot t\right)$$

⇨ THE PE CONTROLLER WILL HAVE AN OSCILATORY BEHAVIOR FOR FRICTIONLESS LOADS.

OSCILATING FREQUENCY ⇨ $f = \frac{1}{2\pi} \sqrt{\frac{K_e}{J_{eq}}}$

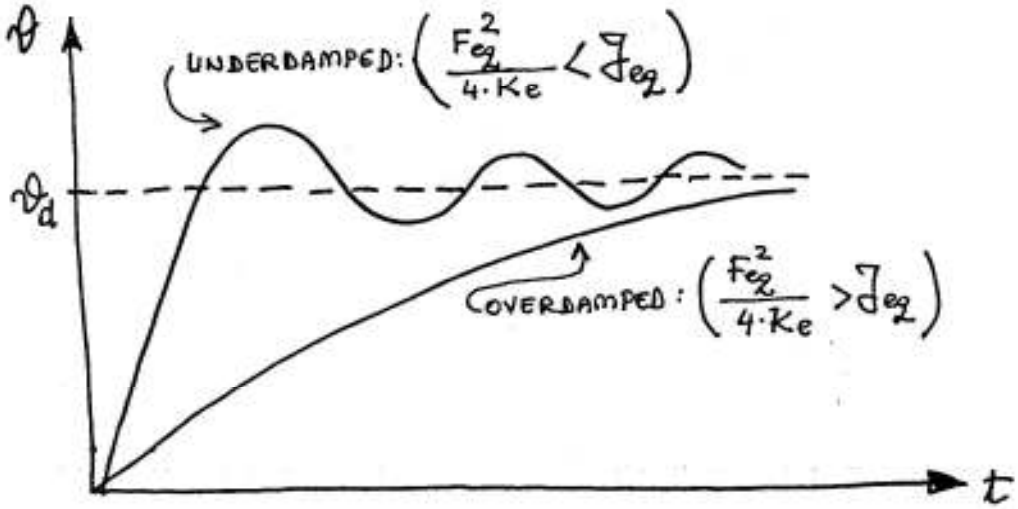
- IN THE CASE WHEN THE LOAD HAS FRICTION ($\vartheta_d = 0$ FOR THE SAKE OF SIMPLIFICATION):

$$\boxed{-K_e \cdot \vartheta = J_{eq} \cdot \ddot{\vartheta} + F_{eq} \cdot \dot{\vartheta}}$$ WHICH HAS THE SOLUTION:

$$\boxed{\vartheta = \exp\left(-\frac{F_{eq} \cdot t}{2 \cdot J_{eq}}\right) \cdot \left[C_1 \cdot \exp\left(\frac{\omega \cdot t}{2}\right) + C_2 \cdot \exp\left(\frac{-\omega t}{2}\right)\right]}$$

WHERE: $\omega = \sqrt{\left(\frac{F_{eq}}{J_{eq}}\right)^2 - \frac{4 \cdot K_e}{J_{eq}}}$

AND: $\exp\left(-\frac{F_{eq} \cdot t}{2 \cdot J_{eq}}\right)$ ⇦ DAMPING TERM



$\frac{F_{eq}^2}{4 \cdot K_e} = J_{eq}$ ⇦ CRITICALLY DAMPED

CONTROL-08

♦ THE STEADY-STATE ERROR PROBLEM

- TO HOLD A LOAD AGAINST GRAVITY THE MOTOR HAS TO CONTINUOUSLY EXERT A TORQUE. BUT WHEN THE JOINT HAS REACHED THE DESIRED POSITION THERE IS NO ERROR $\Rightarrow$ THE TORQUE $\tau = K_e \cdot (\vartheta_d - \vartheta) = 0$ $\Rightarrow$ AS THERE IS NO TORQUE THE LOAD DROPS $\therefore$ $\vartheta \neq \vartheta_d \Rightarrow$ $e = \vartheta_d - \vartheta \neq 0$ i.e. THERE IS A STEADY-STATE ERROR JUST TO KEEP THE LOAD AGAINST THE GRAVITY.

- TO ELIMINATE THE STEADY-STATE ERROR WE HAVE TO PRODUCE A TORQUE $\tau = L + K_e \cdot (\vartheta_d - \vartheta)$ WHERE L IS A CONSTANT GREAT ENOUGH TO HOLD THE LOAD AGAINST THE GRAVITY WHEN $\vartheta = \vartheta_d$.

- BUT L CAN NOT ACTUALLY BE A CONSTANT IN ROBOTIC APPLICATIONS BECAUSE THE LOAD IS NOT CONSTANT $\Rightarrow$ $L = K_i \cdot \int_{t_0}^{t} [\vartheta_d - \vartheta(t)] \cdot dt$ i.e. THE INTEGRAL OF THE ERROR WITH RESPECT TO TIME:

$$\boxed{\tau = K_e \cdot (\vartheta_d - \vartheta) + K_i \cdot \int_{t_0}^{t} [\vartheta_d - \vartheta(t)] \cdot dt}$$

PROPORTIONAL INTEGRAL CONTROL LAW

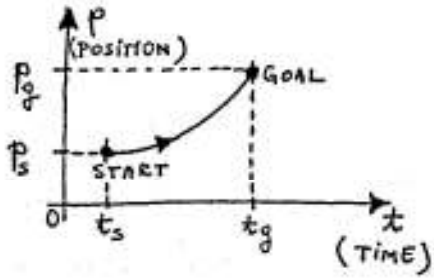
♦ THE OVERSHOOT PROBLEM

THE **PI** CONTROL LAW WORKS WELL ONLY WHEN THE JOINT MOVES SLOWLY. HOWEVER WHEN THE MOTION IS FAST THE INERTIA OF THE LOAD MAKES THE JOINT TO OVERSHOOT $\Rightarrow$ NEED FOR A BRAKING MECHANISM. IN ORDER TO PROVIDE A DEGREE OF ACTIVE BRAKING A "PROPORTIONAL DERIVATIVE" **PD** CONTROL LAW WILL HAVE THE FOLLOWING FORM:

$$\boxed{\tau = K_e \cdot (\vartheta_d - \vartheta) - K_d \cdot \dot{\vartheta}}$$

PROPORTIONAL DERIVATIVE CONTROL LAW

TRAJECTORY CONTROL POLYNOMIALS



- POLYNOMIAL TRAJECTORY MODELLING: ALLOWS TO OVERCOME THE PROBLEMS OF LIMITED ACCELERATION RATE AND INERTIA.
- SMOOTH MOTION: TRAJECTORIES SHOULD BE CONTINUOUS IN POSITION, VELOCITY, AND ACCELERATION.

$\Downarrow$

6 CONSTRAINTS: $p_s, \dot{p}_s, \ddot{p}_s, p_g, \dot{p}_g, \ddot{p}_g$

$\Downarrow$

FIND THE 6 COEFICIENTS OF A 5TH ORDER POLYNOMIAL FUNCTION OF TIME $p(t)$ THAT WILL MAKE THE CONTINUITY CONDITIONS TRUE

$$\begin{aligned} p(t) &= a_5 \cdot t^5 + a_4 \cdot t^4 + a_3 \cdot t^3 + a_2 \cdot t^2 + a_1 \cdot t + a_0 ; \\ \dot{p}(t) &= 5a_5 \cdot t^4 + 4a_4 \cdot t^3 + 3a_3 \cdot t^2 + 2a_2 \cdot t + a_1 ; \\ \ddot{p}(t) &= 20 a_5 \cdot t^3 + 12 a_4 \cdot t^2 + 6a_3 \cdot t + 2a_2 ; \end{aligned}$$

$$p(t_s) = p_s \; ; \quad p(t_g) = p_g$$
$$\dot{p}(t_s) = \dot{p}_s \; ; \quad \dot{p}(t_g) = \dot{p}_g$$
$$\ddot{p}(t_s) = \ddot{p}_s \; ; \quad \ddot{p}(t_g) = \ddot{p}_g$$

$t_s = 0 \Rightarrow$

$$\begin{cases} p(0) = a_0 = p_s \\ \dot{p}(0) = a_1 = \dot{p}_s \\ \ddot{p}(0) = 2a_2 = \ddot{p}_s \end{cases} \Rightarrow \boxed{\begin{aligned} a_0 &= p_s \\ a_1 &= \dot{p}_s \\ a_2 &= \ddot{p}_s/2 \end{aligned}}$$

$t_g = T \Rightarrow$

$$\begin{cases} T^5 \cdot a_5 + T^4 \cdot a_4 + T^3 \cdot a_3 = p_g - \left(p_s + \dot{p}_s \cdot T + \ddot{p}_s \cdot \frac{T^2}{2}\right) = C1 \\ 5T^4 \cdot a_5 + 4T^3 \cdot a_4 + 3T^2 \cdot a_3 = \dot{p}_g - (\dot{p}_s + \ddot{p}_s \cdot T) = C2 \\ 20T^3 \cdot a_5 + 12T^2 \cdot a_4 + 6T a_3 = \ddot{p}_g - \ddot{p}_s = C3 \end{cases}$$

$$\boxed{a_3 = \Delta 3/\Delta \; ; \; a_4 = \Delta 4/\Delta \; ; \; a_5 = \Delta 5/\Delta}$$ (← USING CRAMER'S RULE)

$$\Delta = \begin{vmatrix} T^5 & T^4 & T^3 \\ 5T^4 & 4T^3 & 3T^2 \\ 20T^3 & 12T^2 & 6T \end{vmatrix} = -12T^9 + 30T^9 - 20T^9 = -2T^9$$

$$\Delta 3 = \begin{vmatrix} T^5 & T^4 & C1 \\ 5T^4 & 4T^3 & C2 \\ 20T^3 & 12T^2 & C3 \end{vmatrix} = 4T^8 C3 - 12T^7 C2 - 5T^8 C3 + 20 \cdot T^7 C2 - 20T^6 C1 =$$

$$= -T^8 \cdot C3 + 8T^7 \cdot C2 - 20T^6 \cdot C1 = \ddot{p}_s \cdot T^8 - \ddot{p}_g \cdot T^8 + 8\dot{p}_g T^7 - 8\dot{p}_s T^7 - 8\ddot{p}_s \cdot T^8 -$$

$$- 20T^6 \cdot p_g + 20T^6 \cdot p_s + 20T^7 \dot{p}_s + 10T^8 \ddot{p}_s$$

$$\Delta 3 = 3T^8 \ddot{p}_s - T^8 \ddot{p}_g + 12T^7 \dot{p}_s + 8T^7 \dot{p}_g + 20T^6 p_s - 20T^6 p_g$$

$$\Delta 4 = \begin{vmatrix} T^5 & C1 & T^3 \\ 5T^4 & C2 & 3T^2 \\ 20T^3 & C3 & 6T \end{vmatrix} = 6T^6 \cdot C2 - 3T^7 \cdot C3 + 30T^5 \cdot C1 - 20T^6 \cdot C2 + 5T^7 C3 =$$

$$= 30T^5 \cdot C1 - 14T^6 \cdot C2 + 2T^7 \cdot C3 = 30T^5 p_g - 30T^5 p_s - 30T^6 \dot{p}_s - 15T^7 \ddot{p}_s -$$

$$- 14T^6 \dot{p}_g + 14T^6 \dot{p}_s + 14T^7 \cdot \ddot{p}_s + 2T^7 \cdot \ddot{p}_g - 2T^7 \ddot{p}_s$$

$$\Delta 4 = -3T^7 \cdot \ddot{p}_s + 2T^7 \cdot \ddot{p}_g - 16T^6 \cdot \dot{p}_s - 14 \cdot T^6 \cdot \dot{p}_g - 30T^5 p_s + 30T^5 \cdot p_g$$

$$\Delta 5 = \begin{vmatrix} C1 & T^4 & T^3 \\ C2 & 4T^3 & 3T^2 \\ C3 & 12T^2 & 6T \end{vmatrix} = -12T^4 \cdot C1 - 6T^5 \cdot C2 + 3 \cdot T^6 \cdot C3 +$$

$$+ C2 \cdot 12T^5 - C3 \cdot 4T^6 =$$

$$= -12T^4 \cdot C1 + 6T^5 \cdot C2 - T^6 \cdot C3 = -12T^4 \cdot p_g + 12T^4 p_s + 12T^5 \cdot \dot{p}_s +$$

$$+ 6T^6 \cdot \ddot{p}_s + 6T^5 \cdot \dot{p}_g - 6T^5 \cdot \dot{p}_s - 6T^6 \cdot \ddot{p}_s - T^6 \ddot{p}_g + T^6 \cdot \ddot{p}_s$$

$$\Delta 5 = T^6 \cdot \ddot{p}_s - T^6 \cdot \ddot{p}_g + 6T^5 \cdot \dot{p}_s + 6 \cdot T^5 \cdot \dot{p}_g + 12T^4 p_s - 12T^4 \cdot p_g .$$

$$\boxed{\begin{aligned} a_3 &= \frac{-3\ddot{p}_s + \ddot{p}_g}{2T} - \frac{6\dot{p}_s + 4\dot{p}_g}{T^2} + \frac{-10p_s + 10p_g}{T^3} \\ a_4 &= \frac{3\ddot{p}_s - 2\ddot{p}_g}{2T^2} + \frac{8\dot{p}_s + 7\dot{p}_g}{T^3} + \frac{15p_s - 15p_g}{T^4} \\ a_5 &= \frac{-\ddot{p}_s + \ddot{p}_g}{2T^3} - \frac{3\dot{p}_s + 3\dot{p}_g}{T^4} + \frac{-6p_s + 6p_g}{T^5} \end{aligned}}$$