CONTROL

CONTROL SYSTEM = ALLOWS CORRECT EXECUTION OF THE PLANNED SEQUENCE OF MOTIONS AND FORCES EVEN IN THE PRESENCE OF UNFORSEEN ERRORS

SOURCE OF ERRORS: INACCURACIES IN THE MODEL OF THE ROBOT, MEASUREMENT ERRORS OF SENSORS, LIMITED PRECISION IN COMPUTATION, INACCURACIES OF ACTUATORS, TOLERANCES IN WORKPIECES, FRICTION IN JOINTS, COMPLIANT LINKAGES, ....

CONTROLLING: POSITION, SPEED, FORCE, TORQUE

* BASIC CONCEPTS

* OPEN-LOOP CONTROL

OPEN-LOOP CONTROL is as accurate as the inverse kinematic model is and as accurate are the actuators and joints.

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**Feedback Control**: The parameter that is controlled is continuously measured, compared to the desired value of that parameter (reference), and proper action is taken according to a control law to minimize the error (i.e., the difference between the measured and respectively the desired values of the controlled parameter).

![Diagram of feedback control system]

Feedback control of the joint position.

**Adaptive Control**: Measurements of the results of previous actions are used to adapt the process model to correct for changes in the process and errors in the model.

This type of feedback control can't be used for real-time control of parameters that can dynamically change due to some local parameters. It is useful for the correction of model errors due to long-term variations in the environment.
Feedforward control: Used to predict actuator settings for processes where feedback signals are delayed and in processes where the dynamic effects of disturbances must be reduced.

Transfer functions

The first step when designing a controller is to find the transfer function (plant model) of the process to be controlled.

Transfer functions are plant models which include dynamic effects as well as the static relationship between output and input. The main dynamic effect of concern is the time delay between when the input is changed and when the output responds.

Tracking error due to dynamic effects.
**Example**

Mass-spring-damper system.

The transfer function is the ratio of the Laplace transforms of the output and respectively the input.

\[
G(s) = \frac{1}{m s^2 + f s + k} \]

The polynomial in the denominator, when equated to zero, is called the characteristic equation.

In robotics, transfer functions are required for both the links and the actuators.

- **Link model**: A link & joint can be modelled as a pendulum.

The torque applied to the shaft of the joint is a function of the inertia, the joint friction, and gravity.

\[
\tau = m \ddot{\theta} + f \dot{\theta} + mgl \dot{\theta} 
\]
**DIRECT-CURRENT MOTOR MODEL**

![Diagram of a DC motor with equations]

- The armature current $I(t)$ is a function of the applied voltage $V(t)$, the armature resistance $R_a$, the armature inductance $L_a$, and the back-electromotive-force voltage $E_b(t)$.

- $E_b(t)$ is proportional to the angular velocity $\dot{\theta}$: $E_b(t) = k_2 \cdot \dot{\theta}$.

- The motor torque is proportional to the armature current and the magnetic flux (that's constant for a permanent magnet DC motor):
  $$T(t) = k_1 \cdot I(t)$$

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**LINK & JOINT LOAD MODEL**

![Diagram of a link and joint load model with equations]

- Equivalent inertia: $J_{eq} = J_{mnt} + \frac{J_{load}}{N^2}$
- Equivalent friction: $F_{eq} = F_{mnt} + \frac{F_{load}}{N^2}$
Block-diagram of a D.C. Motor-driven Robot Joint.
FEEDBACK CONTROL OF A ROBOT JOINT

ERROR PROPORTIONAL CONTROL: MOVE THE JOINT IN THE DIRECTION THAT MINIMIZES THE ERROR.

\[ e = \Phi_d - \Phi \]

\[ \Phi_d \quad \text{DESIRED JOINT POSITION} \]
\[ \Phi \quad \text{CURRENT JOINT POSITION} \]

CONTROL LAW:

\[ \tau = K_e \cdot e = K_e \cdot (\Phi_d - \Phi) \]

\[ \text{JOINT TORQUE} = \text{LOAD TORQUE} : \tau = \tau_{eq} \cdot \dot{\Theta} + F_{eq} \cdot \ddot{\Theta} \]

\[ K_e \cdot (\Phi_d - \Phi) = \tau_{eq} \cdot \dot{\Theta} + F_{eq} \cdot \ddot{\Theta} \]

\[ \Rightarrow \]

IN THE CASE OF A FRICTIONLESS LOAD, \( F_{eq} = 0 \), AND FOR THE SAKE OF SIMPLIFICATION, \( \Phi_d = 0 \):

\[ -K_e \cdot \Theta = \tau_{eq} \cdot \ddot{\Theta} \]

\[ \Rightarrow \dot{\Theta} = -\frac{K_e}{\tau_{eq}} \cdot \Theta \]

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\[ \Rightarrow \Theta = \sin \left(\sqrt{\frac{K_e}{\tau_{eq}}} \cdot t\right) \]

\[ \Rightarrow \text{THE PE CONTROLLER WILL HAVE AN OSCILLATORY BEHAVIOR FOR FRICTIONLESS LOADS.} \]

Oscillating frequency:

\[ f = \frac{1}{2\pi} \sqrt{\frac{K_e}{\tau_{eq}}} \]

CONTROL - 07
In the case when the load has friction ($\alpha_d = 0$ for the sake of simplification):

$$-K_e \cdot \dot{\theta} = J_{eq} \cdot \ddot{\theta} + F_{eq} \cdot \dot{\theta}$$

which has the solution:

$$\theta = \exp \left( -\frac{F_{eq} \cdot t}{2 \cdot J_{eq}} \right) \cdot \left[ C_1 \cdot \exp \left( \frac{c_0 \cdot t}{2} \right) + C_2 \cdot \exp \left( -\frac{c_0 \cdot t}{2} \right) \right]$$

where:

$$c_0 = \sqrt{\left( \frac{F_{eq}}{J_{eq}} \right)^2 - \frac{4 \cdot K_e}{J_{eq}}}$$

and:

$$\exp \left( -\frac{F_{eq} \cdot t}{2 \cdot J_{eq}} \right) \leftrightarrow \text{Damping Term}$$

\[ \Theta \]

**Underdamped:** ($\frac{F_{eq}^2}{4 \cdot K_e} < J_{eq}$)

**Critically Damped:**

$$\frac{F_{eq}^2}{4 \cdot K_e} = J_{eq}$$

**Overshadowed:** ($\frac{F_{eq}^2}{4 \cdot K_e} > J_{eq}$)

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**CONTROL-08**
THE STEADY-STATE ERROR PROBLEM

• To hold a load against gravity the motor has to continuously exert a torque. But when the joint has reached the desired position there is no error \( \Rightarrow \) the torque \( \tau = K_e \cdot (\theta_d - \theta) = 0 \)
\( \Rightarrow \) as there is no torque the load drops \( \Rightarrow \) \( \theta = \theta_d - \theta_l \neq 0 \) i.e. there is a steady-state error just to keep the load against the gravity.

• To eliminate the steady-state error we have to produce a torque \( \tau = L + K_e \cdot (\dot{\theta}_d - \dot{\theta}) \) where \( L \) is a constant great enough to hold the load against the gravity when \( \theta = \theta_d \).

• But \( L \) cannot actually be a constant in robotic applications because the load is not constant \( \Rightarrow \) \( L = K_i \cdot \int_{t_0}^{t} (\theta_d - \theta(t)) \cdot dt \)
i.e. the integral of the error with respect to time:

\[
\tau = K_e \cdot (\dot{\theta}_d - \dot{\theta}) + K_i \cdot \int_{t_0}^{t} (\theta_d - \theta(t)) \cdot dt
\]

THE OVERTRESH PROBLEM

The PI control law works well only when the joint moves slowly. However, when the motion is fast the inertia of the load makes the joint to overshoot \( \Rightarrow \) need for a braking mechanism. In order to provide a degree of active braking a "proportional derivative" PD control law will have the following form:

\[
\tau = K_e \cdot (\dot{\theta}_d - \dot{\theta}) - K_d \cdot \ddot{\theta}
\]
TRAJECTORY CONTROL POLYNOMIALS

**Polyomial Trajectory Modelling**: Allows to overcome the problems of limited acceleration rate and inertia.

**Smooth Motion**: Trajectories should be continuous in position, velocity, and acceleration.

\[ \begin{align*}
\mathbf{p}(t) &= a_0 t^5 + a_1 t^4 + a_2 t^3 + a_3 t^2 + a_4 t + a_5 \\
\dot{\mathbf{p}}(t) &= 5a_0 t^4 + 4a_1 t^3 + 3a_2 t^2 + 2a_3 t + a_4 \\
\ddot{\mathbf{p}}(t) &= 20a_0 t^3 + 12a_1 t^2 + 6a_2 t + 2a_3
\end{align*} \]

\[ \begin{array}{c|c}
\mathbf{p}(t_3) &= \mathbf{p}_3 \\
\dot{\mathbf{p}}(t_3) &= \dot{\mathbf{p}}_3 \\
\ddot{\mathbf{p}}(t_3) &= \ddot{\mathbf{p}}_3
\end{array} \]

\[ \begin{array}{c|c}
\mathbf{p}(t_g) &= \mathbf{p}_g \\
\dot{\mathbf{p}}(t_g) &= \dot{\mathbf{p}}_g \\
\ddot{\mathbf{p}}(t_g) &= \ddot{\mathbf{p}}_g
\end{array} \]

\[ \begin{align*}
t_3 &= 0 \\
\Rightarrow & \begin{cases} 
\mathbf{p}(0) = a_0 = \mathbf{p}_3 \\
\dot{\mathbf{p}}(0) = a_1 = \dot{\mathbf{p}}_3 \\
\ddot{\mathbf{p}}(0) = 2a_2 = \ddot{\mathbf{p}}_3 
\end{cases}
\end{align*} \]

\[ \begin{align*}
t_g &= T \\
\Rightarrow & \begin{cases} 
T^5 a_5 + T^4 a_4 + T^3 a_3 = \mathbf{p}_g - (\mathbf{p}_3 + \dot{\mathbf{p}}_3 T + \ddot{\mathbf{p}}_3 \frac{T^2}{2}) = C_1 \\
5T^4 a_5 + 4T^3 a_4 + 3T^2 a_3 = \ddot{\mathbf{p}}_g - (\dddot{\mathbf{p}}_3 + \dddot{\mathbf{p}}_3 T) = C_2 \\
20T^3 a_5 + 12T^2 a_4 + 6Ta_3 = \dddot{\mathbf{p}}_g - \dddot{\mathbf{p}}_3 = C_3
\end{cases}
\end{align*} \]

\[ \begin{align*}
\alpha_3 &= \Delta_3 / \Delta \\
\alpha_4 &= \Delta_4 / \Delta \\
\alpha_5 &= \Delta_5 / \Delta
\end{align*} \] (Using Cramer's Rule)

\[ \Delta = \begin{vmatrix}
T^5 & T^4 & T^3 \\
5T^4 & 4T^3 & 3T^2 \\
20T^3 & 12T^2 & 6T
\end{vmatrix} = -12T^9 + 30T^3 - 20T^3 = -2T^9 \]
\[ \Delta_3 = \begin{vmatrix} T^5 & T^4 & C_1 \\ 5T^4 & 4T^3 & C_2 \\ 20T^2 & 12T^3 & C_3 \end{vmatrix} = 4T^8C_3 + 12T^7C_2 - 5T^6C_3 + 20T^5C_2 - 20T^4C_1 = \\

-10^8C_3 + 8T^7C_2 - 20T^6C_1 = P_dT^8 - P_gT^7 + 8P_dT^7 - 8P_gT^6 + 10P_dT^6 - \\

-20T^6P_d + 20T^6P_g + 20T^7P_d + 10T^7P_g, \\
\Delta_3 = 3T^8P_g - T^8P_d + 12T^7P_g + 8T^7P_d + 20T^6P_g - 20T^6P_d. \\
\Delta_4 = \begin{vmatrix} T^5 & C_1 & T^3 \\ 5T^4 & C_2 & 3T^2 \\ 20T^3 & C_3 & 6T \end{vmatrix} = 6T^8C_2 - 3T^7C_3 + 30T^6C_1 - 20T^5C_2 + 5T^4C_3 = \\

30T^5C_1 - 14T^6C_2 + 2T^5C_3 = 30T^5P_g - 30T^5P_d - 30T^6P_s - 15T^7P_s - \\

-14T^6P_g + 14T^6P_d + 14T^5P_s + 2T^5P_g - 2T^5P_d. \\
\Delta_4 = 3T^7P_g + 2T^7P_d - 16T^6P_s - 14T^6P_g - 30T^5P_s + 30T^5P_d. \\
\Delta_5 = \begin{vmatrix} C_1 & T^4 & T^3 \\ C_2 & 4T^3 & 3T^2 \\ C_3 & 12T^2 & 6T \end{vmatrix} = -12T^4C_1 - 6T^5C_2 + 3T^6C_3 + \\

-c_2 + 12T^5 - c_3 + 4T^6 = \\

-12T^4C_1 + 6T^5C_2 - T^6C_3 = -12T^4P_g + 12T^4P_d + 12T^5P_s + \\

+ 6T^5P_g + 6T^5P_d - 5T^5P_s - 6T^5P_g - 6T^5P_d + 6T^5P_s + 6T^5P_g + 12T^4P_s - 12T^4P_g. \\
\Delta_5 = T^6P_g - T^6P_d + 6T^5P_g + 6T^5P_d + 6T^5P_g + 12T^4P_s - 12T^4P_g. \\
\alpha_3 = 3P_3 + 3P_3 - \frac{6P_3 + 4P_2 + 10P_5 + 10P_g}{2T^2 + T^3}, \\
\alpha_4 = 3P_3 + 3P_3 - \frac{8P_3 + 7P_2 + 15P_5 + 15P_g}{2T^2 + T^3}, \\
\alpha_5 = \frac{3P_3 + 3P_3 - \frac{6P_3 + 6P_2}{2T^3 + T^4}}{T^5 + T^5}. \\
\text{CONTROL-11}