Stable Design of Fuzzy Controllers for Robotic Telemanipulation Applications

Radu-Emil Precup¹, Stefan Preitl¹, Emil M. Petriu², József K. Tar³, Mircea-Bogdan Rădac¹, Claudia-Adina Dragoş¹

¹“Politehnica” University of Timisoara, Romania
²University of Ottawa, Canada
³Budapest Tech, Hungary
MOTIVATION

- **Robotic telemanipulation**: considered from the perspective of the networked control systems (NCSs), which provides advantages.

- **Iterative Feedback Tuning (IFT)**:
  - gradient-based approach, based on input-output data recorded from the closed-loop system to minimize objective functions (o.f.s) specifying the control system (CS) performance indices;
  - closed-loop experimental input-output data to calculate the estimated gradient of the o.f., several experiments performed at each iteration, update law to calculate the controller parameters.

- A new class of Takagi-Sugeno PI-fuzzy controllers (PI-FCs) mapped from the IFT-based designed linear PI controller parameters. **Stability analysis** (nonlinearity vectors) as convenient way to guarantee the convergence of IFT algorithms.
IFT ALGORITHMS

- Control system structure with IFT:

- $\rho$ – parameters vector.
- First controller task: **initially stabilize** the CS $\Rightarrow$ employing another simple initial tuning method is strictly necessary. One solution: the ESO method.
IFT ALGORITHMS (cont’d 1)

General expression of the o.f. $J$:

$$J(\rho) = \frac{1}{2N} \cdot \sum_{k=1}^{N} \{ [L_y(q^{-1}) \delta y(k, \rho)]^2 + \lambda [L_u(q^{-1})u(k, \rho)]^2 \}$$

$N$ – length of each experiment, $L_y$, $L_u$ – weighting filters, to emphasize certain frequency regions, $\lambda$ – weighting constant, $\delta y$ – output error, $\delta y = y - y_d$.

Optimization problem:

$$\rho^* = \arg \min_{\rho \in SD} J(\rho)$$

(2)

SD – stability domain (constraint).

Solving (2) iteratively – Newton’s method:

$$\rho^{i+1} = \rho^i - \gamma^i (H_J(\rho^i))^{-1} est\left[ \frac{\partial J}{\partial \rho} (\rho^i) \right]$$

(3)

$i$ – index of current iteration, $est[x]$ – estimate (generally) of the variable $x$, $\gamma^i > 0$ – parameter to determine the step size, $H$ – Hessian:

$$H_J(\rho^i) = \frac{\partial}{\partial \rho} \left[ \frac{\partial J}{\partial \rho} (\rho^i) \right]$$
IFT ALGORITHMS (cont’d 2)

Calculation of estimates of gradient and Hessian of $J$: two real-time experiments (per iteration) with the CS, the first – normal one and the second – gradient one.

Normal experiment: reference input fed to the CS. Gradient experiment: the reference input is the control error in the first experiment. Subscript corresponds to experiment index:

$$
est \left( \frac{\partial J}{\partial \rho} (\rho^i) \right) = \left(1/N\right) \frac{1}{C(q^{-1}, \rho^i)} \cdot \left( \frac{\partial C}{\partial \rho} (q^{-1}, \rho^i) \right) \sum_{k=1}^{N} \{L_y^2(q^{-1}) \cdot \delta y(k, \rho^i) y_2(k, \rho^i) + \lambda L_u^2(q^{-1}) \cdot u_1(k, \rho^i) u_2(k, \rho^i) \}$$

$$
est[H_J(\rho^i)] = \left(1/N\right) \frac{1}{C(q^{-1}, \rho^i)} \left[ \frac{\partial C}{\partial \rho} (q^{-1}, \rho^i) \right] \cdot \left[ \frac{\partial C}{\partial \rho} (q^{-1}, \rho^i) \right]^T \cdot \sum_{k=1}^{N} \left[ L_y^2(q^{-1}) y_2^2(k, \rho^i) + \lambda L_u^2(q^{-1}) u_2^2(k, \rho^i) \right]$$
IFT ALGORITHMS (cont’d 3)

- IFT algorithm – steps A to E:
  
  **Step A.** Set the parameters in the o.f., the (initial) step size and the initial controller parameters, \( \rho^0 \).
  
  **Step B.** Do the two experiments with the CS and record the input-output data pairs \((u_1, y_1)\) and \((u_2, y_2)\).
  
  **Step C.** Generate the output of the reference model, \( y_d \), and calculate the output error, \( \delta y \).
  
  **Step D.** Calculate the estimates of the gradient and Hessian of J, with the control signal taken from the first experiment.
  
  **Step E.** Calculate the next set of parameters according to the update law (3).
FUZZY CONTROL SYSTEM DESIGN

- Takagi-Sugeno PI-fuzzy controller:
FUZZY CONTROL SYSTEM DESIGN (cont’d 1)

- Example of rule base (SUM and PROD operators in the inference engine, weighted area method for defuzzification):

<table>
<thead>
<tr>
<th>$\Delta e_k$</th>
<th>$e_k$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>NB</td>
</tr>
<tr>
<td>PB</td>
<td>$\Delta u_k = f_k$</td>
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<tr>
<td>PS</td>
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<tr>
<td>ZE</td>
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<tr>
<td>NS</td>
<td>$\Delta u_k = \eta f_k$</td>
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<tr>
<td>NB</td>
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- Parameter $\eta$ with typical values $0<\eta<1$ to improve the CS performance by alleviating the overshoot.

$$f_k = K_P (\Delta e_k + \alpha e_k)$$  \hspace{1cm} (8)
FUZZY CONTROL SYSTEM DESIGN (cont’d 2)

- Parameters in (8): discrete-time PI controller

\[ C(s) = k_c \left(1 + sT_i\right) / s = k_c \left[1 + 1/(sT_i)\right] \]

- Tustin’s method:

\[ K_P = k_c \left[1 - T_s / (2T_i)\right], \quad \alpha = 2T_s / (2T_i - T_s) \]

- Design: PI-FC tuning condition

\[ B_{\Delta e} = \alpha B_e \quad (12) \]

- \( B_e \) – by the stability analysis method.
FUZZY CONTROL SYSTEM DESIGN (cont’d 4)

- Input-output map of PI-FC (nonlinear)

\[ u = N(e) = \sum_{\lambda=0}^{\infty} a_{\lambda+1} \sin[(\lambda + 1) \frac{\pi}{\theta} e] \]

\[ z = \sigma b, \quad b = [b_1 \quad b_2 \quad ... \quad b_{\lambda+1}]^T, \]

\[ z = [z_1 \quad z_2 \quad ... \quad z_{\lambda+1}]^T, \quad \sigma = [\sin[(2i + 1) \frac{\pi}{\theta} e_j]_{i,j=1,\lambda+1}, \]

\[ e_j = jh_e, \quad j = 1, \lambda + 1, \]

- \( z \) – nonlinearity vector.
Matrices calculated in both steady-state regimes and transients:
\[
\Omega P = [c_{ij}]_{i=1,m, j=1,q}, \Omega Q = [d_{ij}]_{i=1,m, j=1,q}
\]

Matrix plane:
\[
M = \begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1q} \\
d_{11} & d_{12} & \cdots & d_{1q} \\
c_{21} & c_{22} & \cdots & c_{2q} \\
d_{21} & d_{22} & \cdots & d_{2q} \\
\vdots & \vdots & \ddots & \vdots \\
c_{m1} & c_{m2} & \cdots & c_{mq} \\
d_{m1} & d_{m2} & \cdots & d_{mq}
\end{bmatrix}
\]

Step-type curves:
\[
c_{\rho\eta} < c < c_{(\rho+1)\varepsilon}, \ d_{\lambda\beta} < d < d_{(\gamma+1)\beta}, \ \rho, \gamma = 1, m - 1, \ \varepsilon, \beta = 1, q - 1
\]
The intersection of the curves for \( c = d = 0 \) ⇔ the coincidence points (in the matrix plane) corresponding to the limit cycles. Solutions expressed as the two coordinates in the matrix plane, the magnitude \( A_i \) and pulsation (frequency) \( \omega \) of the input signal fed to the nonlinearity.

- **Design method of PI-FCs** – detailed presentation in the paper. It makes use of the IFT algorithm, the PI-FC tuning condition (12) and the following stability result:

A limit cycle exists and the fuzzy CS admits a periodic solution sufficiently close to
\[
e = A_0 \sin(\omega_0 t)
\]

The limit cycle is stable if for a sufficiently small value of \( \sigma \) the coincidence point is placed in the matrix plane at a transient magnitude that is larger than the magnitude \( A_0 \) of the limit cycle. Hence the system will be stable. Otherwise the system will be unstable.
**REAL-TIME EXPERIMENTAL RESULTS**

- **Case study**: PI-fuzzy controller design for plants in servo systems as actuators for telemanipulation applications: 
  \[ P(s) = \frac{k_p}{s(1 + T_\Sigma s)} \]

- **ESO method** – good compromise to desired CS performance indices by the choice of the parameter \( \beta \) within the domain \( 4 < \beta < 20 \) + assistance by the diagram:

- **PI tuning conditions**:
  \[ k_c = 1/(\beta \sqrt{\beta T_\Sigma^2 k_p}), \quad T_i = \beta T_\Sigma \]

- **Feedforward filter**:
  \[ F(s) = \frac{1}{(1 + \beta T_\Sigma s)} \]
REAL-TIME EXPERIMENTAL RESULTS (cont’d 1)

- Laboratory DC drive (AMIRA DR300): DC motor – loaded using a current controlled DC generator, mounted on the same shaft, and the drive has built-in analog current controllers for both DC machines having rated speed equal to 300 rpm, rated power equal to 30 W, and rated current equal to 2 A. Controllers – digitally implemented using a real-time control board.
Plant model: $k_P = 4900$ and $T_\Sigma = 0.035$ s. Design parameter: $\beta = 6$.

Controller parameters (sub-optimal values): $T_s = 0.01$ s, $K_P^* = 0.22$, $\alpha^* = 5.87$, $B_e^* = 0.12$, $B_{\Delta e} = \alpha^* B_e^* = 0.51$, $\eta^* = 0.76$.

Real-time results – comparison with 2-DOF PI controller for the same $\beta$: 
REAL-TIME EXPERIMENTAL RESULTS (cont’d 3)

- Left: PI, right: fuzzy, up: reference input variation, down: disturbance input variation.
REAL-TIME EXPERIMENTAL RESULTS (cont’d 4)

- Left: PI, right: fuzzy, up: reference input variation, down: disturbance input variation.
REAL-TIME EXPERIMENTAL RESULTS (cont’d 5)

- Left: PI, right: fuzzy, up: reference input variation, down: disturbance input variation.
CONCLUSIONS

- Real-time experimental results validate the proposed low-cost Takagi-Sugeno PI-FC system structure using the stable design in combination with an IFT algorithm.