**CURRENT AND CHARGE**

An electric current consists of electric charges moving in a wire.

Electric Current => \( I \)

Electric Charge => \( Q \)

\[
I = \frac{dQ}{dt}
\]

**Fig. 1** Wire carrying current \( I \) (left) and its water model (right).

- **Flow of water**: [molecules/s], [l/s]
- **Electric current**: [Ampere] = [A]

1 Ampere = 1 Coulomb/s
1 A = 1 C/s = 6.3 \times 10^{18} \text{ elementary electric charges/s}

The elementary electric charges that are actually moving are \textit{electrons} which jump between external orbits of adjacent atoms.

From a \textit{physical point of view}, the \textit{electric current} flowing through a wire is like a \textit{river of electrons}.

However as the the electrons are “negative electric charges” (due to some odd historical reasons!), from a \textit{technical point of view}, these \textit{flow of electrons is considered a negative current}!!
Small … and big currents

Engineering Unit Prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Abbr.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>$10^9$</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^6$</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
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</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3}$</td>
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<td>micro</td>
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<td>pico</td>
<td>p</td>
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<tr>
<td>femto</td>
<td>f</td>
<td>$10^{-15}$</td>
</tr>
<tr>
<td>atto</td>
<td>a</td>
<td>$10^{-18}$</td>
</tr>
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</table>

0.067 A = 67 mA

73,500 A = 73.5 kA
$I = \frac{dQ}{dt}$

Electrical charge $Q_1$ that flows over the time $t_1$ is the integral of the current from the time zero to time $t_1$.

$Q_1 = \int_{0}^{t_1} i(t) \, dt$
Kirchhoff’s Current Law

The **indestructibility of electrical currents** is expressed by Kirchhoff’s Current Law (KCL), which states that the sum of all the currents into a node is zero.

\[
\sum_{n=1}^{N} I_n = 0
\]

=> Some currents should be flowing into the node while other currents should come out of the node

- Currents flowing into the node are considered “**positive.**”
- Currents coming out are considered “**negative.**”
A current splits to enter two parallel circuit elements

- Currents flowing into the node are considered “positive” : $I_1$
- Currents coming out are considered “negative”: $I_2$ and $I_3$.

\[ I_1 - I_2 - I_3 = 0 \]
\[ \Rightarrow I_1 = I_2 + I_3 \]
CONDUCTORS, INSULATORS, SEMICONDUCTORS, AND SUPERCONDUCTORS

CONDUCTORS

….. materials through which electric currents flow relatively easily.

Metals are good conductors

• silver (tarnishes! => limited practical use);
• gold (expensive! => used as a coating to protect other metals in connectors);
• copper and aluminum (mostly used for wiring; aluminum surface oxidizes rapidly but it is considerably less expensive and lighter than copper);

Solder → an amalgam of tin & lead, having a relatively low melting temperature, used to connect electrical components
INSULATORS

… materials that do not conduct electricity:

* vacuum * dry air* ceramics * glass * plastic *rubber*dry paper*

Most wires are coated with a layer of insulation to prevent current from finding unintended paths

SUPERCONDUCTORS

… very special materials which, when cooled below their critical temperature (from a few deg. K up to more than 100^0 K for the “high-temperature superconductors”)

...are used to generate the very strong magnetic fields required for nuclear magnetic resonance imaging, and may even be used in electric motors and for levitating trains.
... materials such as silicon (Si) and germanium (Ge) which are relatively poor conductors until they are “doped” with trace quantities of other materials such as arsenic (As), phosphorous (P), or boron (B).

<table>
<thead>
<tr>
<th>GROUP III</th>
<th>GROUP IV</th>
<th>GROUP V</th>
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<tbody>
<tr>
<td><strong>B</strong></td>
<td><strong>C</strong></td>
<td><strong>N</strong></td>
</tr>
<tr>
<td>BORON</td>
<td>CARBON</td>
<td>NITROGEN</td>
</tr>
<tr>
<td>valence +3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>atomic no. 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Al</strong></td>
<td><strong>Si</strong></td>
<td><strong>P</strong></td>
</tr>
<tr>
<td>ALUMINIUM</td>
<td>SILICON</td>
<td>PHOSPHOROUS</td>
</tr>
<tr>
<td>13</td>
<td>valence +4, -4</td>
<td>valence +5</td>
</tr>
<tr>
<td>atomic no. 14</td>
<td>atomic no. 15</td>
<td></td>
</tr>
<tr>
<td><strong>Ga</strong></td>
<td><strong>Ge</strong></td>
<td><strong>As</strong></td>
</tr>
<tr>
<td>GALLIUM</td>
<td>GERMANIUM</td>
<td>ARSENIC</td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>valence +5</td>
</tr>
<tr>
<td></td>
<td></td>
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Portion of the periodic table of the elements near the element Silicon.
Semiconductors

- Impurities from Group V (P, As) are called **donors** as they contribute more free electrons to the silicon, resulting in an **n-type** silicon (in which the majority current carriers are electrons).

- Impurities from Group III (B) are called **acceptors** as they contribute more positively charged holes to the silicon, resulting in a **p-type** material (in which the majority current carriers are holes).

*Semiconductors are used to form electronic devices such as **diodes** and **transistors** (BJT = Bipolar Junction Transistor).*

![Diagrams of diodes and transistors]
In the water model, the **electric voltage** is equivalent to the **water pressure**.

The measurement unit for voltage is the **Volt** \([V]\).
RESISTANCE

WATER MODEL  ==> Any pipe hooked up to a source of water offers some resistance to the flow which goes through it. Different pipe construction parameters will affect the degree of resistance. A small-diameter pipe will offer a bigger resistance than a large-diameter one.

Similarly, the wires and other circuit components offer some resistance to the passage of electric current through them.

The magnitude of this resistance depends on the different construction parameters of these wires and other electric circuit components:
• a small-diameter wire will offer a bigger resistance than a large-diameter one;
• a long wire will offer a bigger resistance than a short wire;
• the nature of the material also affects the amount of this resistance.
The measurement unit for resistance is the *Ohm* $[\Omega]$. 

<table>
<thead>
<tr>
<th>Band color</th>
<th>Significant digit</th>
<th>Multiplier</th>
<th>Tolerance</th>
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</thead>
<tbody>
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<td>Black</td>
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<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>10</td>
<td>1%</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>100</td>
<td>2%</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>1,000</td>
<td>3%</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>10,000</td>
<td>4%</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
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<td>-</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>1,000,000</td>
<td>flame proof</td>
</tr>
<tr>
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<td>10,000,000</td>
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<tr>
<td>Gray</td>
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<td>-</td>
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<td>White</td>
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<tr>
<td>Gold</td>
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<td>0.1</td>
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</tr>
<tr>
<td>Silver</td>
<td>-</td>
<td>0.01</td>
<td>10%</td>
</tr>
<tr>
<td>No band</td>
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</tbody>
</table>
Ohm’s Law

Ohm’s Law states that the voltage $V$ drop over a resistance is equal to the value of that resistance $R$ multiplied by the current $I$ which goes through it.

\[ V = I \cdot R \]

\[ I = \frac{V}{R} = \frac{12 \text{ V}}{8 \text{ } \Omega} = 1.5 \text{ A} \]
Kirchhoff’s Voltage Law (KVL) states that the sum of all the voltages around any closed circuit is zero. This means that as follow around any loop in a specific direction, some of the voltages across components will be negative and some will be positive, but when we have closed the loop the sum of all voltages will be zero.
Loop #1:
1V + 5V + 3V - 9V = 0

Loop #2:
-3V + 12V - 4V - 5V = 0

Loop #3:
1V - 3V + 12V - 4V + 3V - 9V = 0

Move around the loop summing the voltage drops. The sign which is encountered first when reaching a component tells what to do (+ or -) with the voltage drop over that component.
Power

WATER MODEL ==> When water flows over a water wheel to power a mill, both the amount of water flowing (the equivalent of the electric current) and the height of the fall (the equivalent of the voltage) determine the power that is delivered. The product of these two parameters gives the amount of power.

Similarly, the electric power \( P \) delivered by a steady voltage \( V \) source providing a current \( I \) is expressed by the formula:

\[
P = I \cdot V
\]

The measurement unit for power is the Watt [W].

\[
1 \text{ W} = 1 \text{ A} \cdot 1 \text{ V}
\]
_energy

A charge moving through an electric field in a vacuum converts the potential energy to kinetic energy in its motion.

The quantity of energy can be expressed by the formula:

\[ U = q \cdot V \]

The measurement unit for energy is the Joule [J].

\[ 1 \text{ J} = 1 \text{ C} \cdot 1 \text{ V} \]
Energy $U$ can also be expressed as an integral of the power $P$ over an interval of time $T$:

$$ U = \int_{0}^{T} I(t) \cdot V(t) \cdot dt $$

If the voltage $V$ and current $I$ are constant in time the energy produced over the interval of time $T$ is:

$$ U = I \cdot V \cdot T $$

1 $J = 1 \, C \cdot 1 \, V = 1 \, A \cdot 1 \, s \cdot 1 \, V = 1 \, W \cdot 1 \, s$

Energy delivered by the Hydro electric utility is measured in **kilowatt-hours**:

1 $kWh = 1,000 \, W \cdot 3,600 \, s = 3,600,000 \, J$
Resistors Connected in Series

Connected “in series” means that the same current flows through all these resistors … no leaks happens.

\[ V_1 = I \cdot R_1 \quad V_2 = I \cdot R_2 \]

\[ V_{\text{tot}} = V_1 + V_2 = I \cdot R_1 + I \cdot R_2 \]

\[ V_{\text{tot}} = I \cdot (R_1 + R_2) \]

\[ R_{\text{tot}} = \frac{V_{\text{tot}}}{I} = R_1 + R_2 \]

The effective resistance of the two resistors connected in series:

\[ R_{\text{tot}} = R_1 + R_2 \]
Resistors Connected in Parallel

Connected “in parallel” means that the current entering the node splits as a river that branches in more channels.

\[
I = I_1 + I_2
\]

KCL

\[
\begin{align*}
I_1 &= \frac{V}{R_1} \\
I_2 &= \frac{V}{R_2}
\end{align*}
\]

Ohm’s law

\[
\begin{align*}
V &= I_1 \cdot R_1 \\
V &= I_2 \cdot R_2
\end{align*}
\]

The effective resistance:

\[
R_{tot} = \frac{R_1 \cdot R_2}{R_1 + R_2}
\]
Generalization => “n” resistors in parallel

\[ I = I_1 + \ldots + I_k \ldots + I_n \]

\[ I_k = \frac{V}{R_k}; \ k = 1, 2, \ldots, n \]

\[ V = I \cdot R_{\text{tot}} \]

\[ V/R_{\text{tot}} = V/R_1 + \ldots + V/R_k \ldots + V/R_n \]

\[ 1/R_{\text{tot}} = 1/R_1 + \ldots + 1/R_k \ldots + 1/R_n \]

\( R_{\text{tot}} = \) the effective resistance of the \( n \) resistors in parallel:
\{\( R_k \); \( k = 1, 2, \ldots, n \)\}
Power Dissipated in a Resistor

\[ P = I \cdot V \]

\[ V = I \cdot R \]

\[ P = I^2 \cdot R \]

\[ P = \frac{V^2}{R} \]
Schematics

- **Wire**
- **Connection**
- **No Connection** (archaic)

- **Resistor**
- **Battery**
- **Voltage Source**
- **Current Source**
- **Terminal**
Voltage Dividers

\[ V = I \cdot (R_1 + R_2) \]

\[ I = \frac{V}{R_1 + R_2} \]

\[ V_{out} = I \cdot R_2 \]

\[ V_{out} = V \cdot \frac{R_2}{R_1 + R_2} \]
Effect of the load on the voltage divider’s output

\[ V_{out} = V \cdot \frac{R_2}{R_1 + R_2} \]

\[ I^* = \frac{V}{R_1 + \{R_2\|R_L\}} \]

\[ V_{out}^* = I^* \cdot \{R_2\|R_L\} \]

\[ V_{out} = V \cdot \frac{R_2 \cdot R_L}{R_1 + R_2 \cdot R_L / (R_2 + R_L)} = V \cdot \frac{R_2 \cdot R_L}{R_1 \cdot R_L + R_2 \cdot R_L + R_1 \cdot R_2} \]

\[ V_{out}^* = V \cdot \frac{R_2}{R_1 + R_2 + R_1 \cdot R_2 / R_L} \]

\[ V_{out}^* \leq V_{out} \]
Ground

In the early days of the telegraph only a single wire was used to carry the current. The return path was provided by making a connection to the ground on both ends: emission and reception. This does make sense because the moisture and the ion content allows the soil to conduct electricity.

The term *ground* has come to mean any common connection point to which other points in an electric circuit are referenced.

- In a circuit schematic drawing all the nodes that are drawn using the same type of ground symbol are connected to a common conductor. This saves having to draw all of these wires, which might clutter the schematic.
- Voltages marked on a schematic are usually referred to ground.
- Some electronic devices housed in metal enclosures or chassis make use of what is known as a *chassis ground*.

![Diagram of ground symbols: Digital ground, Analog ground, Chassis or earth ground]
The Thévenin equivalent circuit can represent any collection of DC voltage sources and resistors as an equivalent circuit that consists of a single voltage $V_{Th}$ and a single resistor $R_{Th}$.

??? For any given circuit (even a very complex one) what would be the voltage and the resistance which a user can see looking from outside at these two terminals ???
Measurement approach to find the Thevenin equivalent circuit:

- connect a voltmeter (having a very high internal resistance so it consumes a practically negligible current from the tested circuit) across the two output terminals to measure the open-circuit voltage, \( V_{oc} \);
- replace the voltmeter with an ammeter (which has a practically negligible internal resistance) to measure the short-circuit current, \( I_{sc} \);
- calculate the Thevenin voltage, \( V_{Th} = V_{oc} \), and the Thevenin resistance \( R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{V_{oc}}{I_{sc}} \).

\[ N.B. \quad - \text{This method allows to see the effects of a load resistance across two points in a given circuit.} \\
- \text{It is an easy to apply method if you have a physical circuit measure, or if } V_{oc} \text{ and } I_{sc} \text{ can be easily calculated from the schematic.} \]
Example

\[ V_1 = 12 \text{ V} \]
\[ V_2 = 8 \text{ V} \]
\[ R_1 = 6.8 \text{ Ω} \]
\[ R_2 = 5.2 \text{ Ω} \]

\[ V_{\text{Th}} = -V_2 + V_1 = -8 \text{ V} + 12 \text{ V} = 4 \text{ V} \]
\[ R_{\text{Th}} = R_2 + R_1 = 5.2 \text{ Ω} + 6.8 \text{ Ω} \]
The Load Line and Your Car Battery

What would happen if you connect a light-emitting diode (LED) across the terminals of your car battery?

\[ V_{\text{bat}} = \frac{V_S}{2} \]

If \( R_{\text{load}} = R_{\text{int}} \) then \( V_{\text{bat}} = \frac{V_S}{2} \).

Thevenin equivalent circuit for a real battery with load.

\[ \text{I-V characteristic for a p-n LED} \]