

KINEMATICS : MANIPULATOR MOTION

- EXTEND THE MANIPULATOR MODEL TO INCLUDE TIME.
 - THE MODELLING OF MOTION ADDRESSES THE FOLLOWING TOPICS:
 - LINEAR AND ANGULAR VELOCITY OF MANIP. LINKS;
 - LINEAR AND ANGULAR ACCELERATION OF MANIPUL. LINKS;
 - DIFFERENTIAL MOTION;
 - THE MANIPULATOR JACOBIAN;
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★ OBJECTIVE OF THE MANIPULATOR'S KINEMATICS



STUDY THE MOTION OF A SET OF ROBOT ARM LINKAGES AS THE END EFFECTOR TRACES OUT A TRAJECTORY IN SPACE

PATH = A SEQUENCE OF POINTS IN SPACE THAT THE END EFFECTOR OF A ROBOT TRAVERSES AS IT MOVES FROM ONE END POINT TO THE NEXT.

TRAJECTORY = A PATH WITH TIME CONSTRAINTS, THAT IS, INCLUDES VELOCITY AND ACCELERATION AS WELL AS LOCATION AT EVERY POINT ALONG THE PATH

★ COMMON CONTROL STRATEGY \Rightarrow JOINT INTERPOLATED MOTION = CALCULATE THE CHANGE IN THE JOINT VARIABLES REQUIRED TO ACHIEVE THE FINAL LOCATION, AND TO CONTROL THE JOINT MOTORS SO THAT ALL THE JOINTS REACH THEIR FINAL VALUES AT THE SAME TIME.

\hookrightarrow THIS STRATEGY RESULTS IN SMOOTH JOINT MOTION, BUT IT MAY ALSO RESULT IN A CROOKED CARTESIAN PATH

★ APPLICATIONS NEEDING FULL SPECIFICATION OF BOTH THE TRAJECTORY OF THE END EFFECTOR AND THE PATH IT TRACES:

► WHEN MOVING AN OPEN CONTAINER OF LIQUID: Spillage can result if the acceleration is too high, or if the container is not kept in the correct orientation.

► ARC WELDING, SPRAY PAINTING, CONVEYOR-BELT TRACKING, GLUING, LASER CUTTING: Cartesian velocities and acceleration have to be controlled as well as location.

* PEOPLE CONTROL THE MOTION OF THEIR HANDS WHEN REACHING FOR AN OBJECT, AND ONLY CONTROL THE ABSOLUTE POSITION WHEN MOVING THE HAND INTO THE FINAL GRASPING POSITION.

AS SOON AS TIME OR PATH CONSTRAINTS ARE APPLIED TO THE MOTION OF THE END EFFECTOR, VELOCITY AND ACCELERATION HAVE TO BE CONTROLLED.

WE FIND THE PATH TRACED OUT BY THE END EFFECTOR BY SUBSTITUTING THE SEQUENCE OF JOINT ANGLES INTO THE KINEMATIC MODEL. TO CALCULATE VELOCITIES AND ACCELERATIONS, WE NEED KINEMATIC MODELS OF MANIPULATOR VELOCITY AND ACCELERATION.

* FINE MOVEMENT OF HUMAN HANDS INVOLVES DIFFERENTIAL MOTION BASED ON FEEDBACK FROM BOTH VISION AND TOUCH SENSES. THESE MOTIONS ARE ALSO CALLED "MOTIONS OF ACCOMODATION".

EXAMPLE OF DIFFERENTIAL MOTION IN ROBOTIC APPLICATIONS:

When using a vision system to monitor the location of the end effector, we can calculate the differential changes in position and orientation needed to place the hand on an object from successive images. By transforming these differential changes from the vision space to the Cartesian space of the robot, we can eliminate the transformation to absolute position.

★ DIFFERENTIAL MOTIONS AND THE JACOBIAN.

KINEMATIC TRANSFORMATIONS AS THEY RELATE TO VELOCITIES.

● DIFFERENTIAL MOTION

FROM THE SINGLE-AXIS
ROTATION MATRICES,
DEFINING $d_x, d_y,$ AND
 d_z AS SMALL ROTATIONS
ABOUT THE $x, y,$ AND
 z AXES: \Rightarrow

$$\text{Rot}(x, d_x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -d_x & 0 \\ 0 & d_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \text{Rot}(y, d_y) = \begin{bmatrix} 1 & 0 & d_y & 0 \\ 0 & 1 & 0 & 0 \\ -d_y & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(z, d_z) = \begin{bmatrix} 1 & -d_z & 0 & 0 \\ d_z & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

● THE GENERAL ROTATION: $\text{Rot}(x, d_x) \cdot \text{Rot}(y, d_y) \cdot \text{Rot}(z, d_z) =$

$$= \begin{bmatrix} (d_x d_y + d_z) & (-d_z) & d_y & 0 \\ (-d_y + d_x d_z) & (1 - d_x d_y d_z) & -d_x & 0 \\ 0 & (d_y d_z + d_x) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & -d_z & d_y & 0 \\ d_z & 1 & -d_x & 0 \\ -d_y & d_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

(NEGLECTING SECOND- AND THIRD-ORDER TERMS.)

A DIFFERENTIAL ROTATION d_θ ABOUT AN ARBITRARY VECTOR $\mathbf{K} = [k_x, k_y, k_z]^T$ IS EQUIVALENT TO THREE DIFFERENTIAL ROTATIONS d_x, d_y, d_z , ABOUT THE $x, y,$ AND z AXES, WHERE:

$$d_x = k_x \cdot d_\theta, \quad d_y = k_y \cdot d_\theta, \quad d_z = k_z \cdot d_\theta.$$

▷ DIFFERENTIAL ROTATIONS ARE INDEPENDENT OF THE ORDER IN WHICH THE ROTATIONS ARE PERFORMED. (THIS COMES ABOUT BECAUSE OF THE ASSUMPTION IN WHICH WE IGNORE THE SECOND- AND THIRD-ORDER TERMS.)

$$\text{Rot}(x, d_x) \cdot \text{Rot}(y, d_y) = \text{Rot}(y, d_y) \cdot \text{Rot}(x, d_x);$$

DIFFERENTIAL MOTION IS DEFINED AS A COMBINED TRANSLATION AND ROTATION:

$$T + dT = [\text{TRANS}(dx, dy, dz) \cdot \text{ROT}(K, d\theta)] \cdot T;$$

$$\therefore dT = [\text{TRANS}(dx, dy, dz) \cdot \text{ROT}(K, d\theta) - I] \cdot T.$$

⇒ DIFFERENTIAL OPERATOR

$$\Delta = \begin{bmatrix} 0 & -\delta_z & \delta_y & dx \\ \delta_z & 0 & -\delta_x & dy \\ -\delta_y & \delta_x & 0 & dz \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{TRANS}(dx, dy, dz) \cdot \text{ROT}(K, d\theta) - I;$$

GIVEN A HAND CONFIGURATION DESCRIBED BY A HOMOGENEOUS TRANSFORMATION T , WE CAN DETERMINE THE RESULTS OF APPLYING A DIFFERENTIAL CHANGE TO THAT CONFIGURATION SIMPLY BY PREMULTIPLYING BY Δ ; $dT = \Delta \cdot T$.

REMARKS

- i. THE PRINCIPLES OF DIFFERENTIAL MOTION ARE BASED UPON THE ASSUMPTION THAT THE DIFFERENTIAL MOTIONS ARE SMALL.
- ii. THE MOTION OF SMALL DIFFERENTIAL MOTIONS IS VERY APPROPRIATE TO DESCRIBE THE INSTANTANEOUS VELOCITY.
- iii. THE DIFFERENTIAL MOTION TECHNIQUE CANNOT BE USED ITERATIVELY SINCE ERRORS ACCUMULATE (WE HAVE TO LEARN HOW TO TRANSFORM VELOCITIES).

● THE JACOBIAN

THE DIFFERENTIAL DISPLACEMENT OF THE ROBOT ARM IN THE CARTESIAN COORDINATE SPACE:

$$D = \begin{bmatrix} dx \\ dy \\ d\theta \end{bmatrix}$$

⇒ IN THE JOINT SPACE:

$$D_\theta = \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ d\theta_4 \\ d\theta_5 \\ d\theta_6 \end{bmatrix}$$

$$D = J \cdot D_\theta$$

CARTESIAN SPACE

"JACOBIAN"
6x6 MATRIX

JOINT SPACE

$$D_\theta = J^{-1} \cdot D$$

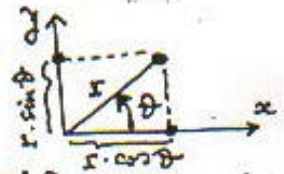
EXAMPLE

THE JACOBIAN OF THE θ - r MANIPULATOR

$$D = \begin{bmatrix} dx \\ dy \end{bmatrix} \approx \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}; \quad D_\theta = \begin{bmatrix} \dot{r} \\ \dot{\theta} \end{bmatrix}; \quad D = J \cdot D_\theta;$$

$$\begin{cases} \dot{x} \approx \frac{dx}{dt} \\ \dot{y} \approx \frac{dy}{dt} \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \cdot \begin{bmatrix} \dot{r} \\ \dot{\theta} \end{bmatrix}$$



$$\begin{cases} x = r \cdot \cos \theta \\ y = r \cdot \sin \theta \end{cases} \Rightarrow \begin{cases} \frac{dx}{dt} = \dot{r} \cdot \cos \theta - r \cdot \dot{\theta} \cdot \sin \theta; \\ \frac{dy}{dt} = \dot{r} \cdot \sin \theta + r \cdot \dot{\theta} \cdot \cos \theta; \end{cases}$$

$$J = \begin{bmatrix} \cos \theta & -r \cdot \sin \theta \\ \sin \theta & r \cdot \cos \theta \end{bmatrix};$$

$$J^{-1} = \begin{bmatrix} \frac{x}{r} & \frac{y}{r} \\ -\frac{y}{r^2} & \frac{x}{r^2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} \cos \theta & -r \cdot \sin \theta \\ \sin \theta & r \cdot \cos \theta \end{bmatrix} \begin{bmatrix} \dot{r} \\ \dot{\theta} \end{bmatrix}$$

THE MANIPULATOR "JACOBIAN"

$$D = J \cdot Dq \Rightarrow \frac{D}{dt} = J \cdot \frac{Dq}{dt}$$

VELOCITIES IN
CARTESIAN WORD.

ANGULAR VELOCITIES
IN JOINT SPACE.

$$\dot{x} = J \cdot \dot{\theta}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} \\ J_{61} & J_{62} & J_{63} & J_{64} & J_{65} & J_{66} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix}$$

→ DETERMINE THE
COEFFICIENTS OF THE
J MATRIX FROM THE
KNOWN KINEMATICS
(THE A MATRICES)
OF THE MANIPULATOR

→ THIS EQUATION IS RELATIVE TO THE ROBOT BASE FRAME : ${}^R \dot{x} = {}^R J \cdot \dot{\theta}$

• WE CAN EQUALLY WELL WRITE : ${}^H \dot{x} = {}^H J \cdot \dot{\theta}$ WHERE ${}^H J$ REPRESENTS A JACOBIAN
MATRIX RELATING JOINT VELOCITIES TO INSTANTANEOUS CARTESIAN VELOCITIES RELATIVE
TO THE HAND FRAME.

${}^H J$ IS EASIER TO COMPUTE THAN ${}^R J$!

▶ WE ESTABLISH A HAND TRANSFORM T_H (WHICH IS THE PRODUCT OF SIX A MATRICES

$$T_H = \begin{bmatrix} m_x & \sigma_x & a_x & p_x \\ m_y & \sigma_y & a_y & p_y \\ m_z & \sigma_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

▶ THE ELEMENTS OF THE HAND FRAME "JACOBIAN"
 ${}^H J$ MAY BE FOUND FOR ROTARY JOINTS AS FOLLOWS:

$$\frac{{}^H \partial x}{\partial \theta_{i+1}} = m_y i p_z - m_z i p_y ; \quad \frac{{}^H \partial y}{\partial \theta_{i+1}} = i \sigma_y p_z - i \sigma_z p_y ; \quad \frac{{}^H \partial z}{\partial \theta_{i+1}} = i a_y p_x - i a_x p_y$$

$$\frac{{}^H \partial \Phi_x}{\partial \theta_{i+1}} = i m_z ; \quad \frac{{}^H \partial \Phi_y}{\partial \theta_{i+1}} = i \sigma_z ; \quad \frac{{}^H \partial \Phi_z}{\partial \theta_{i+1}} = i a_z$$

$$\begin{cases} {}^1 T_H = A_1^{-1} \cdot T_H ; \\ {}^2 T_H = A_2^{-1} \cdot A_1^{-1} \cdot T_H ; \\ \vdots \\ {}^5 T_H = A_5^{-1} \cdot A_4^{-1} \cdot \dots \cdot A_1^{-1} \cdot T_H = A \end{cases}$$

WHEN REFERENCING AN ELEMENT OF ONE OF THESE MATRICES,
WE USE THE LEFT SUPERSCRIPT TO DESIGNATE THE MATRIX.

E.g. : m_x REFERS TO THE (1,1) TERM OF T_H , BUT ${}^2 m_x$ REFERS TO THE
(1,1) TERM OF ${}^2 T_H$.

$$A^{-1} = \frac{[A_{ij}]^T}{|A|}$$

A_{ij} = the cofactor of a_{ij}

$$A = \begin{bmatrix} r \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{bmatrix}$$

$$A_{11} = r \cos \theta ;$$

$$A_{12} = -\sin \theta ;$$

$$A_{21} = r \sin \theta ;$$

$$A_{22} = \cos \theta ;$$

$$|A| = \begin{vmatrix} r \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} = r \cdot \cos^2 \theta + r \cdot \sin^2 \theta = r.$$

$$A^{-1} = \frac{\begin{vmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{vmatrix}}{r} = \frac{\begin{vmatrix} r \cos \theta & r \sin \theta \\ -\sin \theta & \cos \theta \end{vmatrix}}{r} = \frac{\begin{vmatrix} x & y \\ -\frac{y}{r} & \frac{x}{r} \end{vmatrix}}{r}$$

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned}$$

$$A^{-1} = \begin{vmatrix} \frac{x}{r} & \frac{y}{r} \\ -\frac{y}{r^2} & \frac{x}{r^2} \end{vmatrix}.$$