**OBJECT LOCATION**

* **Cartesian Coordinates**

Cartesian coordinates used in robotics

Cartesian coordinates used in computer graphics

Corkscrew rule (right-hand rule) to define positive directions of the axes
* RECOVERING THE LOCATION OF MOVING OBJECTS: TRANSLATION*

Using a reference frame $N$ affixed to the object allows the calculation of the variable location of the point $Q$ on the moving object in a given reference frame $R$ as the sum of two vectors, a constant one representing the location of $Q$ relative to $N$ and another variable representing the location of $N$ relative to $R$:

$$^Rq = ^Rp + ^Nq$$

Recovering the 3D location of a point on a moving object in a given reference frame $R$, when the object suffers only a translation movement relative to $R$.

$$^Rp^* = ^Rp + ^Rr$$
$$^Rq^* = ^Rp^* + ^Nq$$
$$^Rq^* = ^Rp + ^Nq + ^Rr$$

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* RECOVERING THE LOCATION OF MOVING OBJECTS: Rotation*

Recovering the 3D location of a point on an object rotating in a given reference frame $R$.

Vectors cannot describe the rotation!

\[ R q^* = ? \]
RECOVERING THE LOCATION OF MOVING OBJECT: \textit{ROTATION} > continued
The general rotation of an object in 3D can be conveniently decomposed into a sequence of three elementary rotations about the axes of the Cartesian reference frame attached to that object, namely:

- \( \text{Rot}(z, \phi) \) a rotation about z-axis, “roll,” by an angle \( \phi \);
- \( \text{Rot}(y, \theta) \) a rotation about y-axis, “pitch,” by an angle \( \theta \);
- \( \text{Rot}(x, \psi) \) a rotation about x-axis, “yaw,” by an angle \( \psi \).
RECOVERING THE LOCATION OF MOVING OBJECT: \textit{ROTATION} > continued

The general rotation in 3D is often known as the \textbf{RPY rotation}, a name reminding of the Roll, Pitch, and Yaw components.
RECOVERING THE LOCATION OF MOVING OBJECT: ROTATION

Roll
Rot( z, \( \phi \) )

Pitch
Rot( y, \( \theta \) )

Yaw
Rot( x, \( \psi \) )

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**RECOVERING THE LOCATION OF MOVING OBJECTS:** Roll, Rot($z, \phi$)
RECOVERING THE LOCATION OF MOVING OBJECTS: $\text{Rot}(z, \phi)$ > continued

\[
\begin{align*}
Q (x,y) & \quad Q^* (x',y') \\
\phi & \\
y & y' \\
x & x' \\
\end{align*}
\]

In a similar way we get:

\[
\begin{align*}
x' &= x \cdot \cos(\phi) - y \cdot \sin(\phi) \\
y' &= y \cdot \cos(\phi) + x \cdot \sin(\phi)
\end{align*}
\]

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
\cos(\phi) & -\sin(\phi) \\
\sin(\phi) & \cos(\phi)
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]
3D Homogeneous Transformation Matrices for Robotics

Homogeneous transformation matrices are general 4-by-4 matrices accounting for both object translation and object rotation in 3D.

\[ T = \begin{pmatrix}
3\text{-by-3 rotation sub-matrix} & 3\text{-by-1 translation sub-matrix} \\
0 & 0 & 0 & 1
\end{pmatrix} \]

The homogeneous transformations matrix for a general 3D translation by a vector \( p_x \cdot \hat{i} + p_y \cdot \hat{j} + p_z \cdot \hat{k} \) is:

\[ \text{Trans} (p_x, p_y, p_z) = \begin{pmatrix}
1 & 0 & 0 & p_x \\
0 & 1 & 0 & p_y \\
0 & 0 & 1 & p_z \\
0 & 0 & 0 & 1
\end{pmatrix} \]
The homogeneous transformations matrices for 3D rotations about the x, y, and z axes:

\[
\text{Rot}(x, \psi) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\psi) & -\sin(\psi) & 0 \\
0 & \sin(\psi) & \cos(\psi) & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\text{Rot}(y, \theta) = \begin{pmatrix}
\cos(\theta) & 0 & \sin(\theta) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(\theta) & 0 & \cos(\theta) & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\text{Rot}(z, \phi) = \begin{pmatrix}
\cos(\phi) & -\sin(\phi) & 0 & 0 \\
\sin(\phi) & \cos(\phi) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Homogeneous transformation matrices allow to calculate the final effect of a sequence of object transforms (translations and/or rotations) by multiplying the homogeneous matrices corresponding to these transforms.