ELG3125 Signal and System Analysis

Frequency Content of Discrete-Time Signal

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Slides and assignments:
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Frequency Content

Fourier Series (periodic signals)

Time Domain		Frequency Domain

Fourier Transform (non-periodic signals)

*Fourier Transform in MATLab: fft()
**There is a relationship between Fourier Series and Fourier Transform
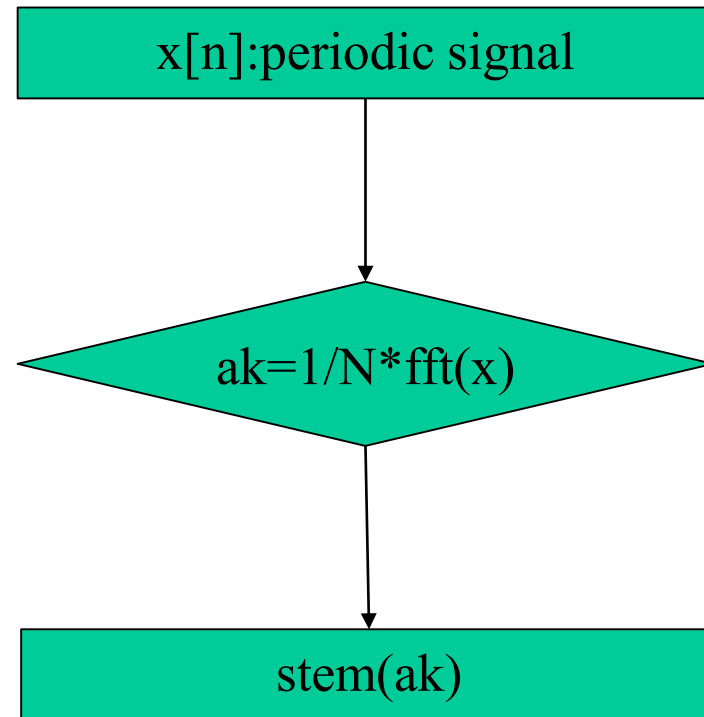
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The relation between Fourier series and Fourier Transform

- In this lab, we learn to investigate the frequency content of the discrete-time signals. For a periodic signal with period N.

Fourier series: $x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}$

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n} \quad = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi}{N}n} = \frac{1}{N} X(k)$$

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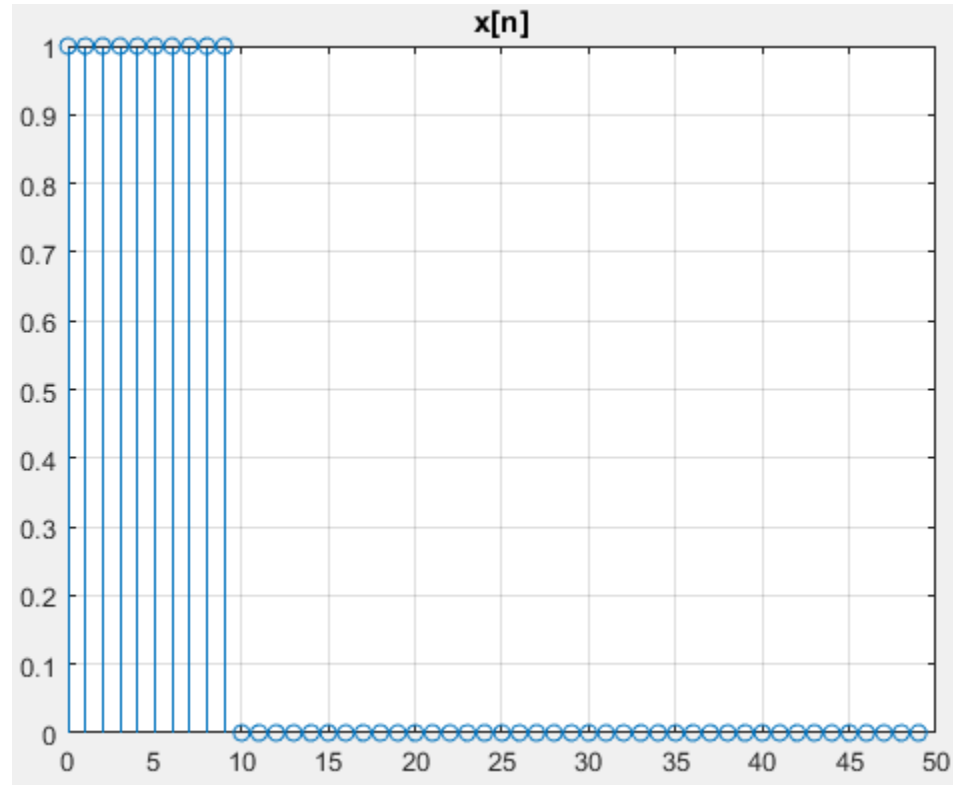
Frequency Contents for Periodic Signal

*We use stem() because output of Fourier Series is always Discrete

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Example 1:

- Periodic signal with period N = 50

$$x[n] = \begin{cases} 1, & 0 \le n < 10 \\ 0, & 10 \le n < 50 \end{cases}$$
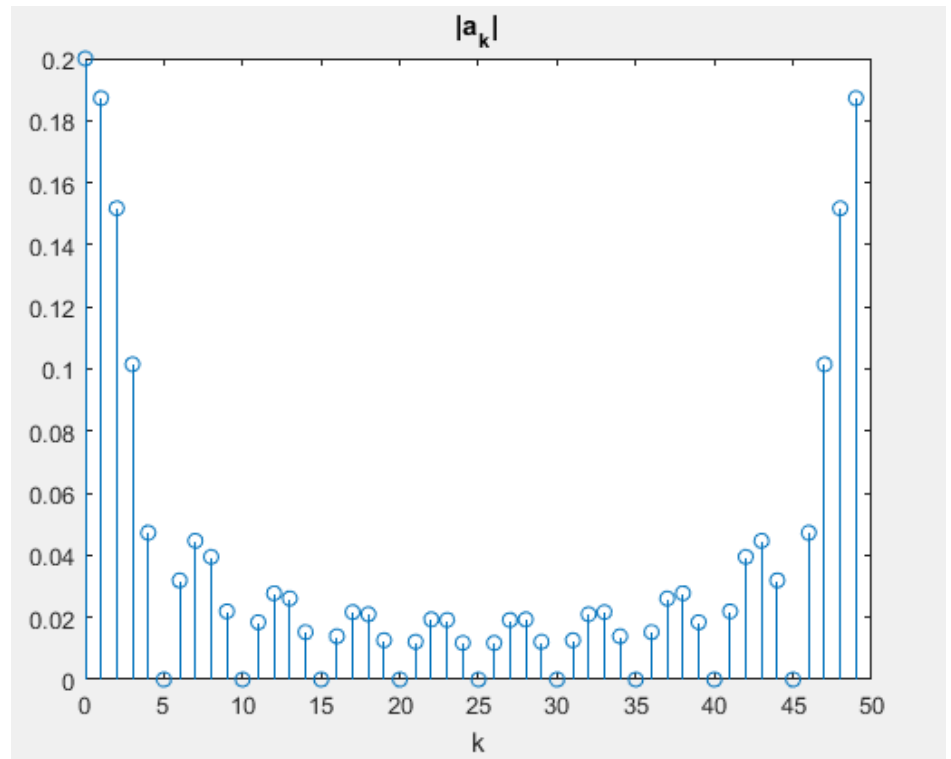


```
N=50;
n=linspace(0,N-1,N);
x=[ones(1,10) zeros(1,N-10)];
stem(n,x);
xlabel('n');
title('x[n]'); grid on;
```

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Example 1 (Cont.)

- Frequency content without shift

```
ak=1/N*fft(x);
stem(n,abs(ak));
xlabel('k');
title('|a{_k}|');
```
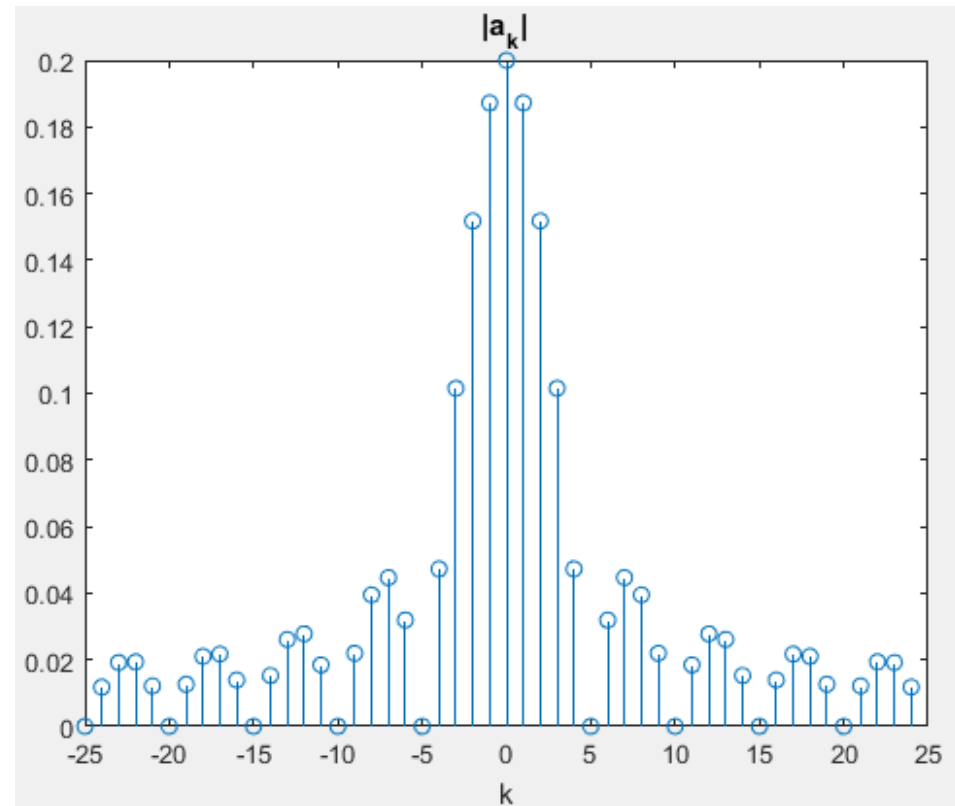
Example 1 (Cont.)

- Frequency content with shift

```
ak=fftshift(ak);
k=n-N/2;
stem(k,abs(ak));
xlabel('k');
title('|a{_k}|');
```

*fftshift() is a function to shift zero frequency to the center.

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Frequency Contents for Non-periodic Signal

$$x[n]: finite\ during\ 0 \leq n < N_x.$$

$$X\left(e^{j\omega}\right) = \sum_{-\infty}^{+\infty} x[n]\, e^{-j\omega n} = \sum_{n=0}^{N_x-1} x[n]\, e^{-j\omega n}, -\pi \leq \omega < \pi$$

$$X\left(e^{jk\Delta\omega}\right) = \sum_{n=0}^{N_x-1} x[n]\, e^{-jk\Delta\omega n} = \sum_{n=0}^{N_x-1} x[n]\, e^{-jk\frac{2\pi}{N_\omega}n}, \Delta\omega = \frac{2\pi}{N_\omega}$$
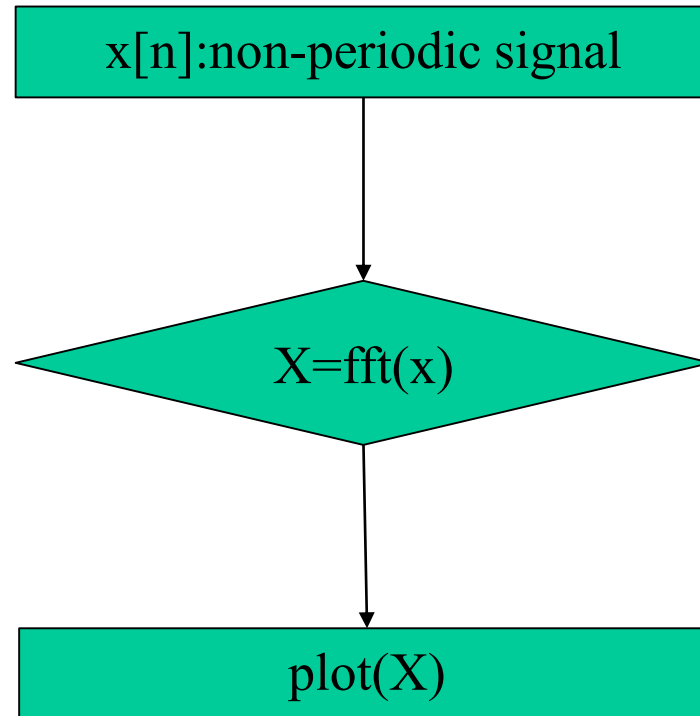
$$X(k) = \sum_{n=0}^{N_x-1} x[n]\, e^{-jk\frac{2\pi}{N_\omega}n}, -\frac{N_\omega}{2} \leq k \leq \frac{N_\omega}{2} - 1$$

If N is chosen such that N=N_x =N_ω,

$$X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-jk\frac{2\pi}{N}n}, -\frac{N}{2} \leq k < \frac{N}{2} - 1$$

Frequency Contents for Non-Periodic Signal

```
┌─────────────────────────────────┐
│   x[n]:non-periodic signal      │
└─────────────────────────────────┘
              │
              ▼
         ◇ X=fft(x) ◇
              │
              ▼
┌─────────────────────────────────┐
│            plot(X)              │
└─────────────────────────────────┘
```

*We use plot() because output of Fourier Transform is always continuous.

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Example 2:

- Finite-length signal with duration of N = 50

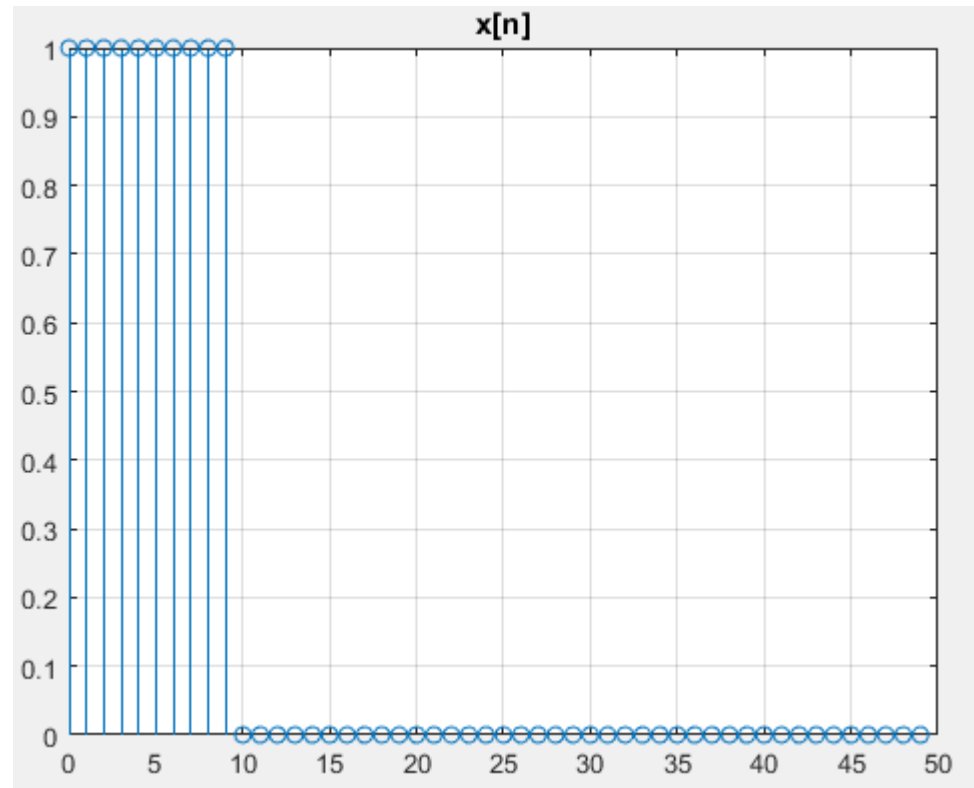$$x[n] = \begin{cases} 1, & 0 \le n < 10 \\ 0, & 10 \le n < 50 \end{cases}$$


```
N=50;
n=linspace(0,N-1,N);
x=[ones(1,10) zeros(1,N-10)];
stem(n,x);
xlabel('n');
title('x[n]'); grid on;
```
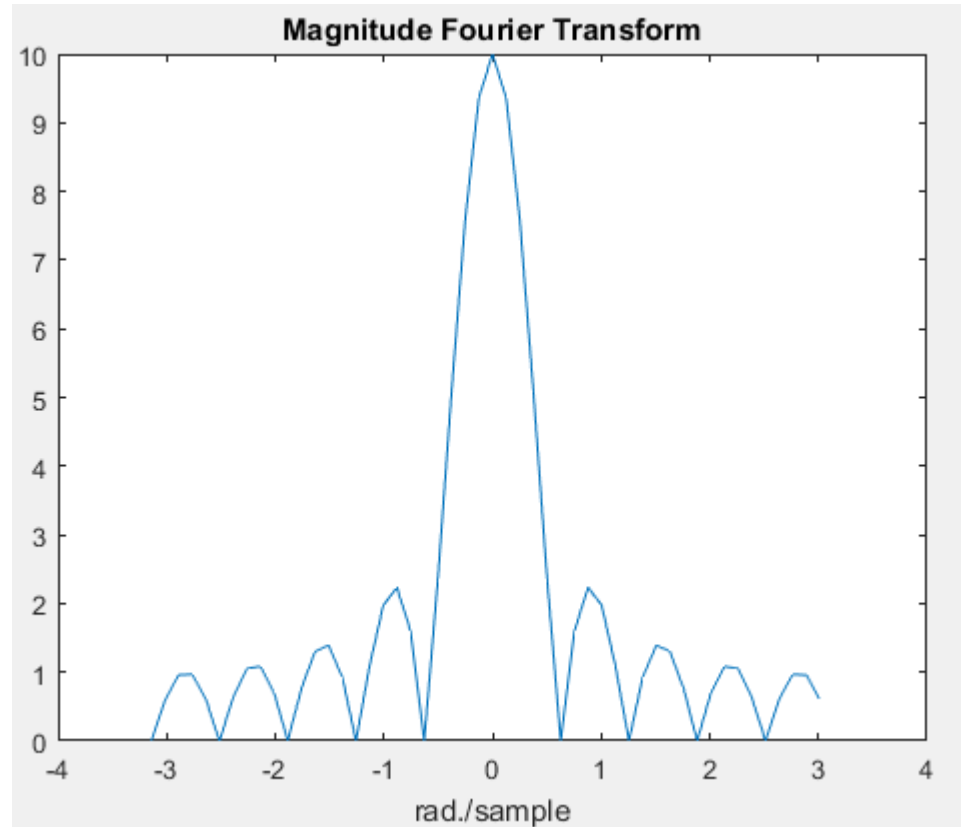
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- Codes to obtain frequency contents(respect to ω): $\omega = k\dfrac{2\pi}{N}$

Magnitude Fourier Transform

```
Xw=fft(x);
Xw=fftshift(Xw);
k=n-N/2;
w=k*2*pi/N;
plot(w,abs(Xw));
xlabel('rad./sample');
title('Magnitude Fourier Transform');
```

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Thank you!

Now, you do the assignments and send codes to msade033@uottawa.ca

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1. MatLab Computation of the Frequency Content of a Periodic Discrete-Time Signal

For a periodic discrete time signal, where the values of x[n], $0 \leq n \leq N-1$ are known over a period N, it is possible to visualize the frequency content using the codes in the example 1 of the slides. Unlike the analysis equation for the Fourier series coefficients a_k, these frequency components are computed numerically using the Matlab function *fft(x)*, where *x* is a vector corresponding to the values of $x[n]$ over one period. The function *fft* uses the Fast Fourier Transform, whose theory will not be covered in the course ELG 3125, but you can read this for fun from such text as "J.G. Proakis and D.G. Manolakis, *Digital Signal Processing*, Prentice Hall, *3/e*, 1996."

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For convenience, the codes to get the frequency contents are systematically written as follows.

```
N=length(x);        %MatLab code to generate Fourier series for periodic signal x[n] with a period N
ak=1/N*fft(x);
ak=fftshift(ak);
k=-(N-1)/2:1:(N-1)/2;    % N odd assumed here in this code
w=k*2*pi/N;
figure(1)
subplot(2,1,1)
stem(w,abs(ak));              % Magnitude
title('magnitude ak coeffs.')
xlabel('rad./sample')
subplot(2,1,2)
stem(w,angle(ak));           % Phase
title('phase ak coeffs.')
xlabel('rad/sample')
```

1) Build a periodic signal $x[n]$ of length 21 samples with a slow variation, and observe the frequency content.

2) Repeat 1) but with a fast variation and, and observe the frequency content.

3) Verify that slow variations and fast variation in the signal correspond to low frequency content and high frequency content, respectively.

2. MatLab Computation of the Frequency Content of a Non-Periodic Discrete-Time Signal

Similarly, for a non-periodic signals having a finite duration (or length) in the interval $0 \le n \le N-1$, it is possible to evaluate the discrete time Fourier transform $X(e^{j\omega})$ at some frequencies by using again the Matlab function *fft*() as done in the following.

```matlab
%code for Fourier transform of a non-periodic signal x[n].
Xw=fft(x,max(1001,length(x)));  % minimum of 1001 values computed,
                                % to provide impression of continuous transform
Xw=fftshift(Xw);
Nfft=length(Xw);
k=-(Nfft-1)/2:1:(Nfft-1)/2;     % with Nfft odd here in this code
w=k*2*pi/Nfft;
figure(1)
subplot(2,1,1)
plot(w,abs(Xw));                        % Magnitude
title('magnitude Fourier transf.')
xlabel('rad./sample')
subplot(2,1,2)
plot(w,angle(Xw));                      % Phase
title('phase Fourier transf.')
xlabel('rad./sample')
```

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1) Build a signal $x[n]$ of finite length 21 samples with a slow variation, and observe the frequency content.

2) Repeat 1) but with a fast variation and, and observe the frequency content.

3) Discuss the similarities and the differences between the frequency contents you observed in this part and those obtained in the last part with the periodic signals.