ELG3125 Signal and System Analysis

Fourier series: Synthesis of signals

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Slides and assignments:
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Outline

1. Continuous-Time Fourier Series and its truncated version

2. Discrete-time Fourier series

Continuous-time Fourier Series …

- Fourier series representation

  Synthesis equation:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

  Analysis equation:

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}$$

Continuous-time Fourier Series …

- Synthesis equation
  The synthesis or reconstruction of signal x(t) from a summation of complex exponential terms (or from cosine terms) weighted by the Fourier Series coefficients can also be written by:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$= a_0 + 2\sum_{k=1}^{+\infty} |a_k| \cos(k\omega_0 t + \angle a_k)$$

Continuous-time Fourier Series (Truncated version)

If instead of using an infinite amount of terms, the summation is truncated to N_a terms (with N_a odd here), we then obtain the following approximation.

$$x(t) \approx \hat{x}(t) = \sum_{k=-(N_a-1)/2}^{(N_a-1)/2} a_k e^{jk\omega_0 t}$$

$$= a_0 + 2 \sum_{k=1}^{(N_a-1)/2} |a_k| \cos(k\omega_0 t + \angle a_k)$$

$* \; N_a \rightarrow \infty \; then \; \hat{x}(t) \rightarrow x(t).$

Discrete-time Fourier Series

- Fourier Series representation

Synthesis equation:

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}$$

Analysis equation:

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n}$$

* No need for truncated version because N is finite already.

Syntax Review

```
x=0;
for k=1:10
    x=x+k;
end
```

$$x = \sum_{k=1}^{10} k$$

```
abs(ak);
```

$$|a_k|$$

```
angle(ak);
```

$$\angle a_k$$

```
exp(-j*k*w0*n);
```

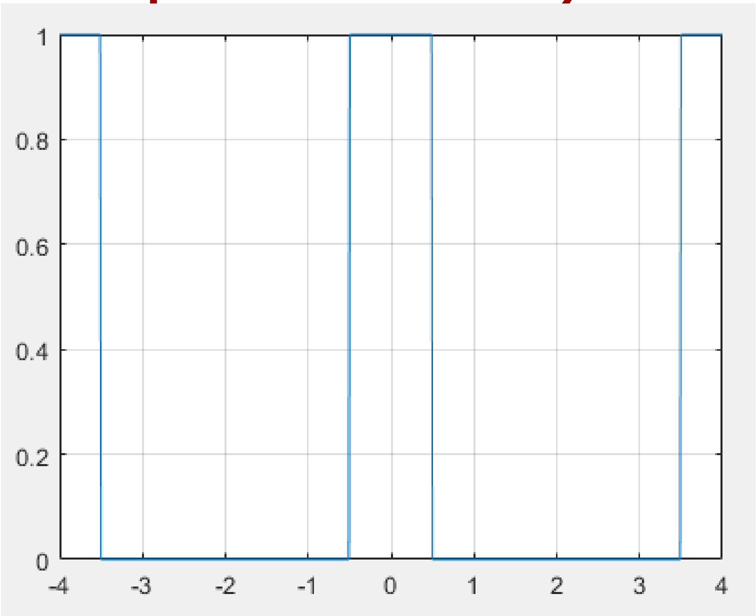
$$e^{-jk\omega_0 n}$$

Example (Fourier series for a square wave)

- Square wave x(t) with period T.

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

$$T_1 = 0.5\,, T = 4$$



- Fourier Series coefficients

$$a_0 = \frac{2T_1}{T} \qquad a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T}, k \neq 0 \qquad \omega_0 = \frac{2\pi}{T}$$

- Now, let's rebuild x(t) from a_0 and a_k.

Example …

So, we implement: $x(t) = a_0 + 2\sum_{k=1}^{(N_a-1)/2} |a_k|\cos(k\omega_0 t + \angle a_k)$

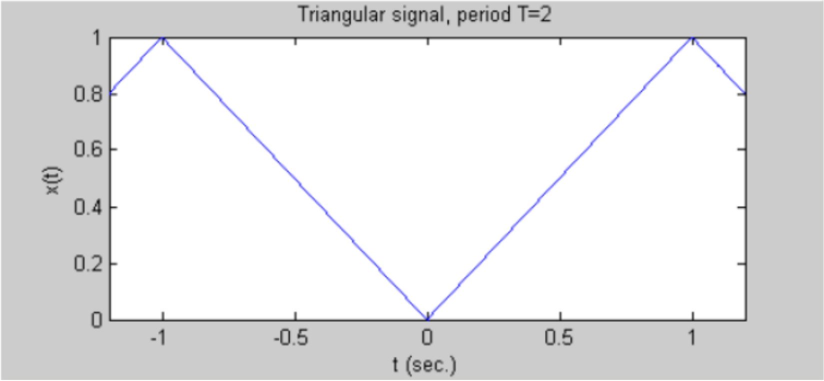
```
T=4;T1=0.5; w0=2*pi/T;
t=-4:0.01:4; a0=2*T1/T;
x_approx=ones(1,length(t))*a0;
Na=9; % Na=27 or 271

for k=1:(Na-1)/2
    ak=sin(k*w0*T1)/(k*pi);
    x_approx=x_approx+2*abs(ak)*...
        cos(k*w0.*t+angle(ak));
end
plot(t,x_approx); grid on;
```

Now, you do the assignment 5.
Submit the final code to msade033@uottawa.ca before Quiz.

Nov 06	Lab Quiz # 2
Nov 27	Lab Quiz # 3

1. Continuous-time Fourier series

For the following continuous-time triangular wave periodic signal:



the coefficients of the Fourier series are:

$$a_k = \frac{2\sin(k\pi/2)}{j(k\pi)^2} e^{-jk\pi/2} \quad k \neq 0$$

$$a_0 = \frac{1}{2}$$

Referring to the truncated version introduced in the slides for the continuous-time Fourier series, write a Matlab script which allows to visualize the approximation $\hat{x}(t)$ obtained for *Na*=7, *Na*=21, *Na*=201, i.e., as a growing but finite number of complex exponentials (or cosines) are used. Your observations should allow to verify that it is only for $N_a \to \infty$ that the synthesized signal would become completely equal to the signal *x(t)* .

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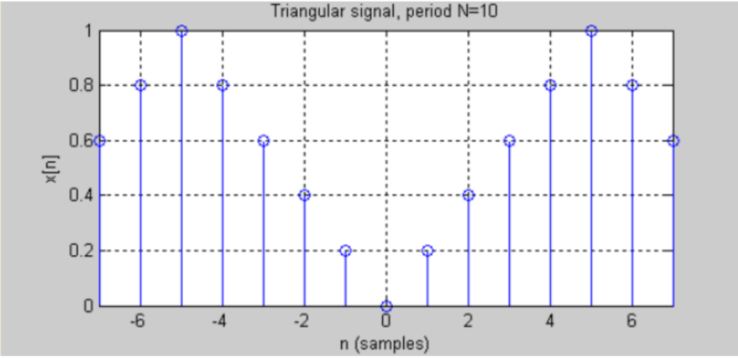
2. Discrete-time Fourier series

For the following discrete-time triangular wave periodic signal:



the Fourier series coefficients are:

$a_0 = 0.5,\ \ a_1 = a_{-1} = -0.20944,\ \ a_2 = a_{-2} = 0,\ \ a_3 = a_{-3} = -0.03056,\ a_4 = a_{-4} = 0,\ a_5 = -0.02$.

For the discrete time Fourier series, the coefficients are periodic (with period N=10 here). The synthesis or reconstruction of signal $x[n]$ from a sum of complex exponentials (or cosines) weighted by the coefficients of the Fourier series can be written as:

$$x[n] = \sum_{k=<N>} a_k e^{jk\frac{2\pi}{N}n} = \sum_{k=-4}^{5} a_k e^{jk\frac{2\pi}{10}n} = \sum_{k=0}^{9} a_k e^{jk\frac{2\pi}{10}n}.$$

Write a Matlab script which allows to perform a synthesis of $x[n]$ from the sum of complex exponential signals, and verify that unlike in the previous case (for continuous time Fourier series coefficients) here it is not required to use an infinite amount of exponentials, i.e. N=10 exponentials are enough.

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