ELG3125 Signal and System Analysis

Property and Combination of LTI Systems

Presented by: Mohammad Sadeghi

Slides and assignments:
http://www.site.uottawa.ca/~msade033/signal/

School of Information Technology and Engineering

msade033@uOttawa.ca

uOttawa

Outline

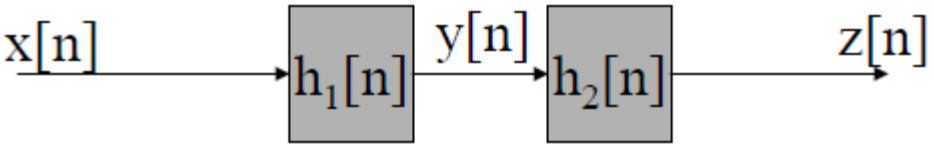
1. Combination of LTI Systems

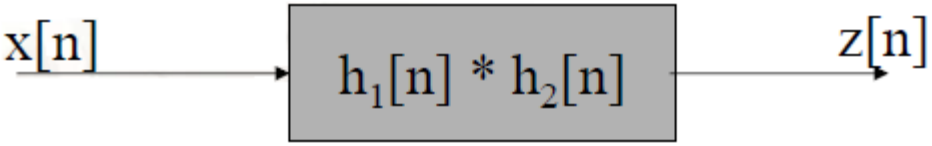
2. Stability Property

3. Invertibility Property

Combination of LTI Systems

- Combination of two LTI systems

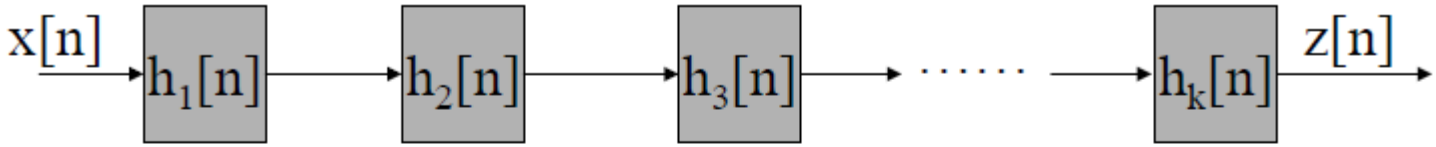


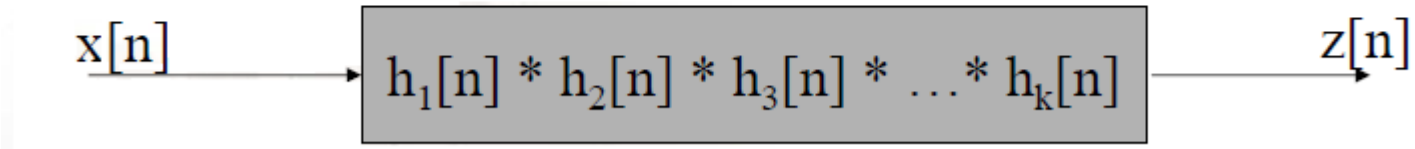


**Also applicable for continuous-time systems.*

Combination of LTI Systems

- Combination of multiple LTI systems





**Also applicable for continuous-time systems.*

Assignment – Question 1

1. A LTI discrete-time system has an input $x[n]$ and an output $y[n]$:

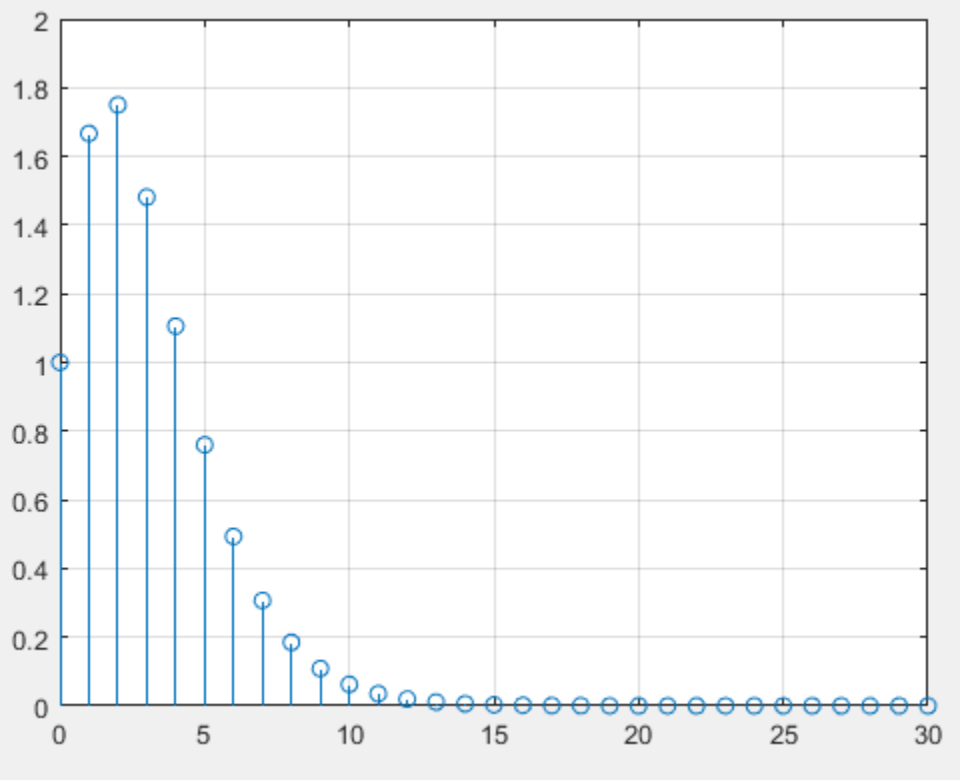
$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$, system is initially at rest.

The output $y[n]$ of this first system then becomes the input of a second system: $z[n]+\frac{9}{20}z[n-1]+\frac{1}{20}z[n-2]=y[n]$, system is initially at rest.

Compute and display the impulse response of the equivalent system between $x[n]$ and $z[n]$.

Assignment – Question 1 …

```
n=0:100;
%Impluse reponse of system 1
B1=[1 0 0];
A1=[1 -5/6 1/6];
h1=impz(B1,A1,n);
%Impluse reponse of system 2
B2=[1 0 0];
A2=[1 -5/6 1/6];
h2=impz(B2,A2,n);
%combination of two systems
h=conv(h1,h2);
%plot
nh=0:length(h)-1;
stem(nh,h),grid;
axis([0,30,0,2]);
```



System Stability

- For a discrete-time system, a sufficient condition to guarantee its stability is: if the impulse response is absolutely summable, i.e.,

$$\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$$

* Then, if the input x[n] is bounded, the output y[n] is bounded.

System Stability …

- For a continuous-time system, a sufficient condition to guarantee its stability is: if the impulse response is absolutely integrable, i.e.,

$$\int_{\tau=-\infty}^{+\infty} |h(\tau)| < \infty$$

* Then, if the input x(t) is bounded the output y(t) is bounded.

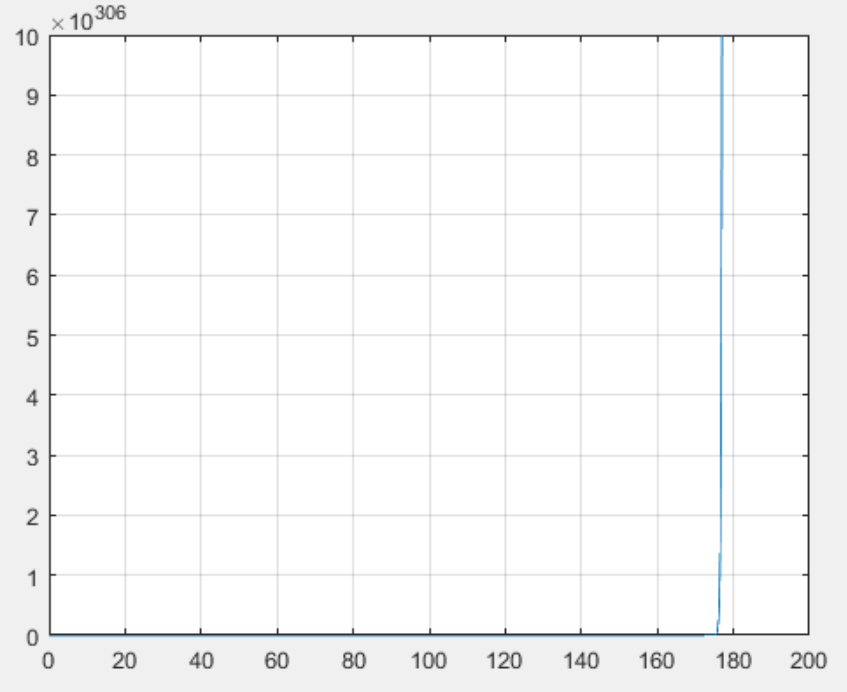
Assignment – Question 2

Show by experiment if the following LTI systems are stable or not:

(1) $\frac{d^2y(t)}{dt^2} - 5\frac{dy(t)}{dt} + 4y(t) = x(t)$, system initially at rest.

```
t=0:0.1:200;
B1=[0 0 1];
A1=[1 -5 4];
h1=impulse(B1,A1,t);
plot(t,h1), grid on;
axis([0 200 0 10^307])
```

Not stable!



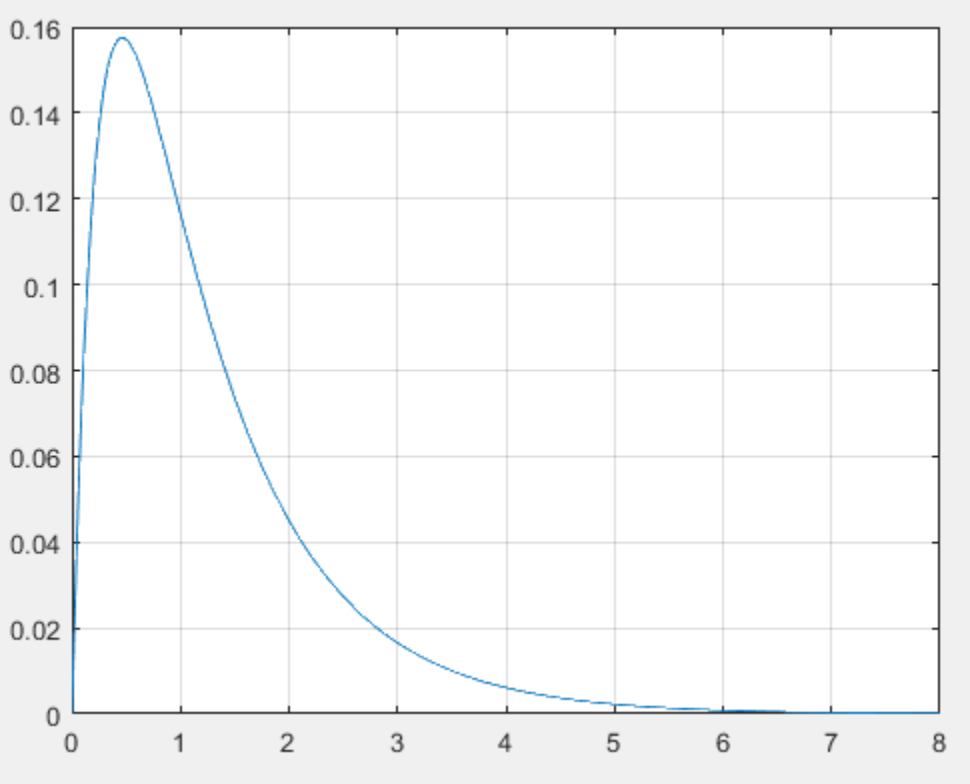
Assignment – Question 2 …

Show by experiment if the following LTI systems are stable or not:

(2) $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = x(t)$, system initially at rest.

```
t=0:0.01:8;
B1=[0 0 1];
A1=[1 +5 4];
h1=impulse(B1,A1,t);
plot(t,h1), grid on;
```

Stable!

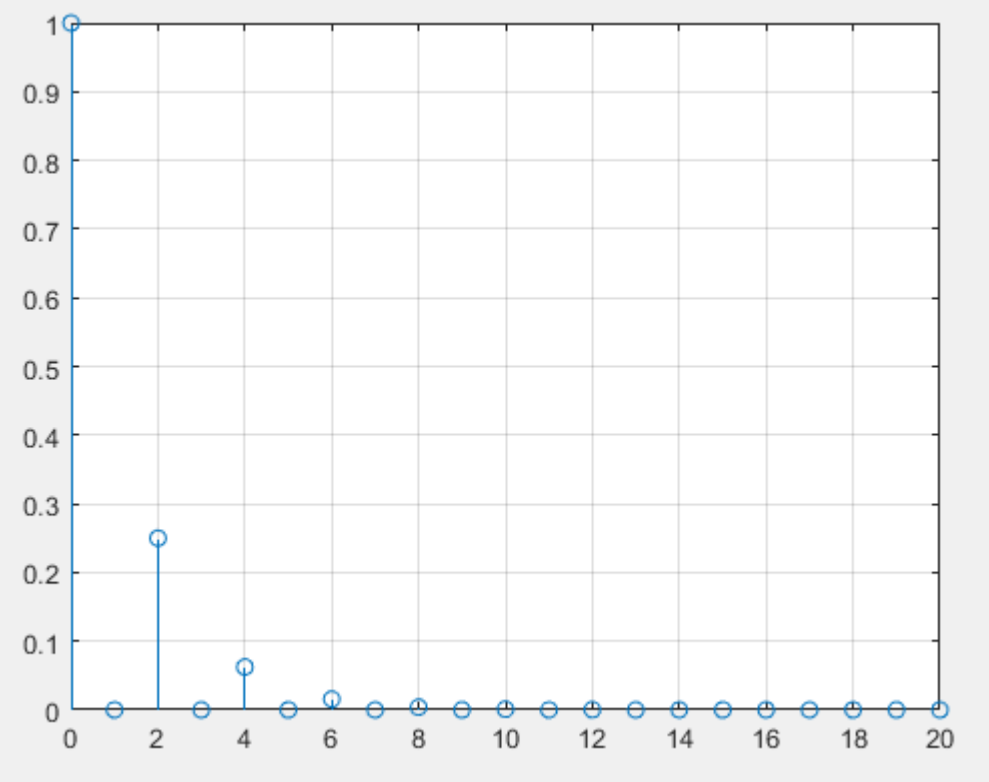


Assignment – Question 2 …

Show by experiment if the following LTI systems are stable or not:

(3) $y[n]-\frac{1}{4}y[n-2]=x[n]$, system initially at rest.

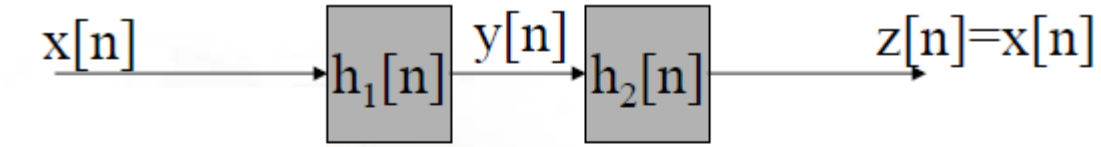
```
n=0:20;
B1=[1  0  0];
A1=[1  0  -0.25];
h1=impz(B1,A1,n);
stem(n,h1), grid on;
```

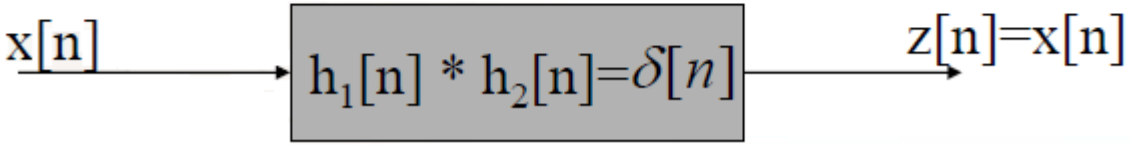


Stable!

Invertibility of LTI Systems

- To see if a LTI system is the inverse of the other LTI system, check if the convolution of their impulse response is a delta function.





Assignment – Question 3

Show by experiment which of the systems listed below (a, b, or c) is the inverse of the following LTI discrete time system:

$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$, system initially at rest.

(1) $y[n]=\frac{1}{6}x[n]-\frac{5}{6}x[n-1]+x[n-2]$, system initially at rest.

(2) $y[n]=x[n]-\frac{5}{6}x[n-1]+\frac{1}{6}x[n-2]$, system initially at rest.

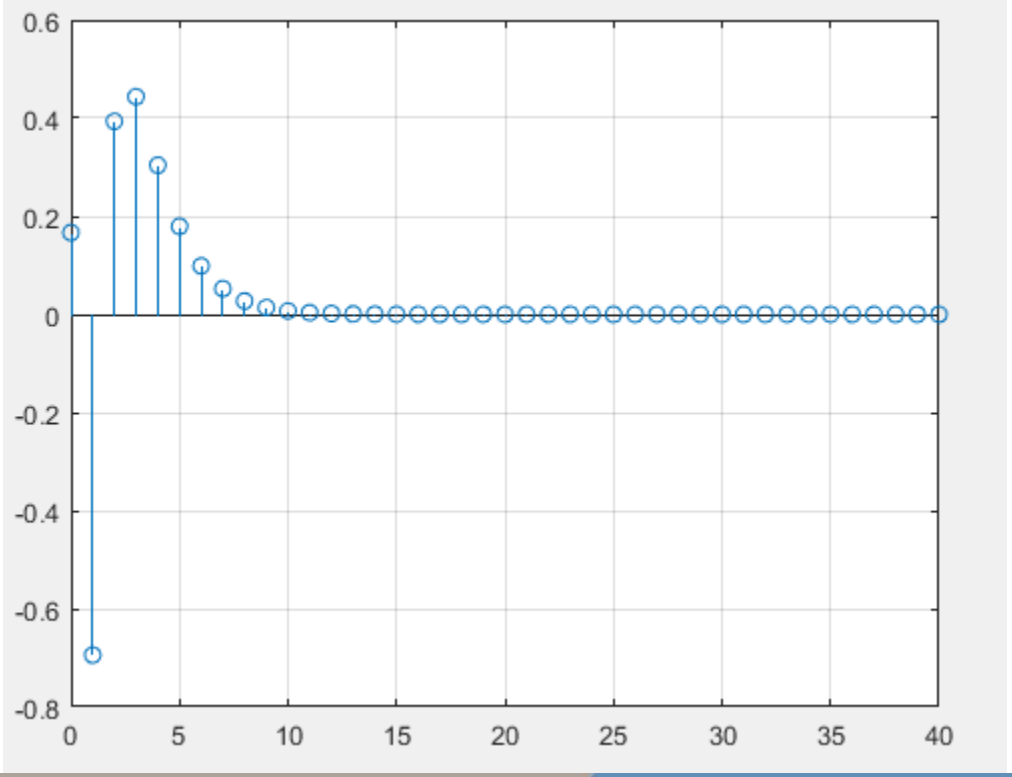
(3) $\frac{1}{6}y[n]-\frac{5}{6}y[n-1]+y[n-2]=x[n]$, system initially at rest.

Assignment – Question 3 …

- To consider if $y[n]=\frac{1}{6}x[n]-\frac{5}{6}x[n-1]+x[n-2]$ is reverse system

of: $y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$

```
n=0:20;
A=[1 -5/6 1/6];
B=[1 0 0];
h=impz(B,A,n);
A1=[1 0 0];
B1=[1/6 -5/6 1];
h1=impz(B1,A1,n);
ht=conv(h,h1);
nt=0:length(ht)-1;
stem(nt,ht), grid on;
```



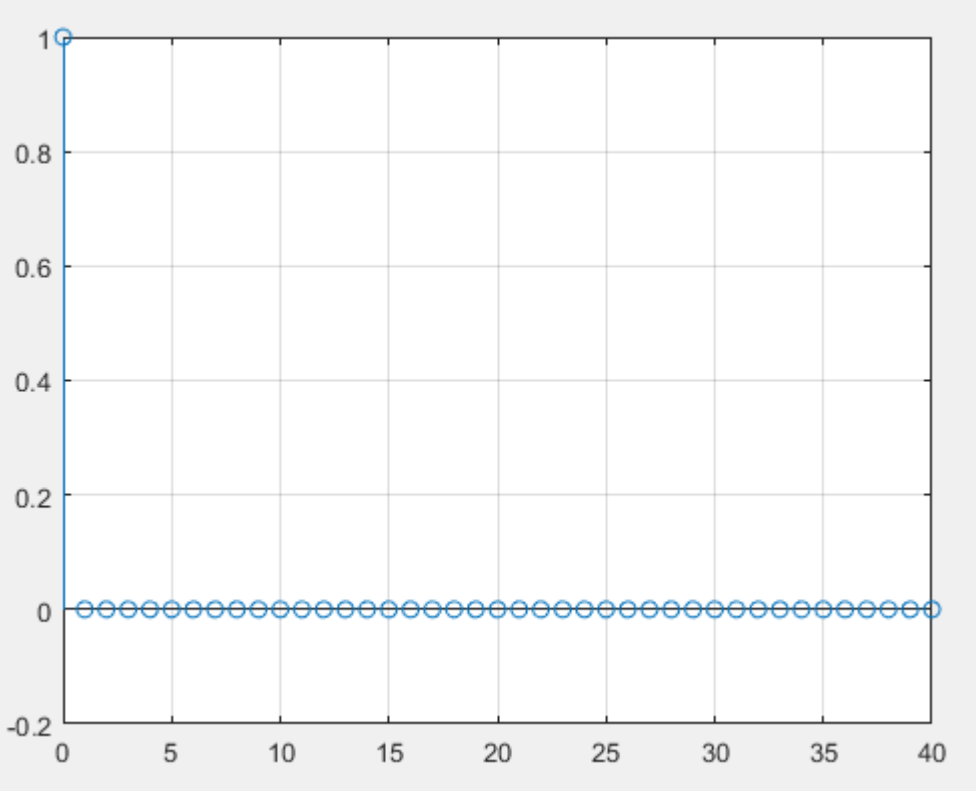
No, it is not.

Assignment – Question 3 …

- To consider if $y[n]=x[n]-\frac{5}{6}x[n-1]+\frac{1}{6}x[n-2]$ is reverse system

of: $y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$

```
n=0:20;
A=[1 -5/6 1/6];
B=[1 0 0];
h=impz(B,A,n);
A1=[1 0 0];
B1=[1 -5/6 1/6];
h1=impz(B1,A1,n);
ht=conv(h,h1);
nt=0:length(ht)-1;
stem(nt,ht), grid on;
```



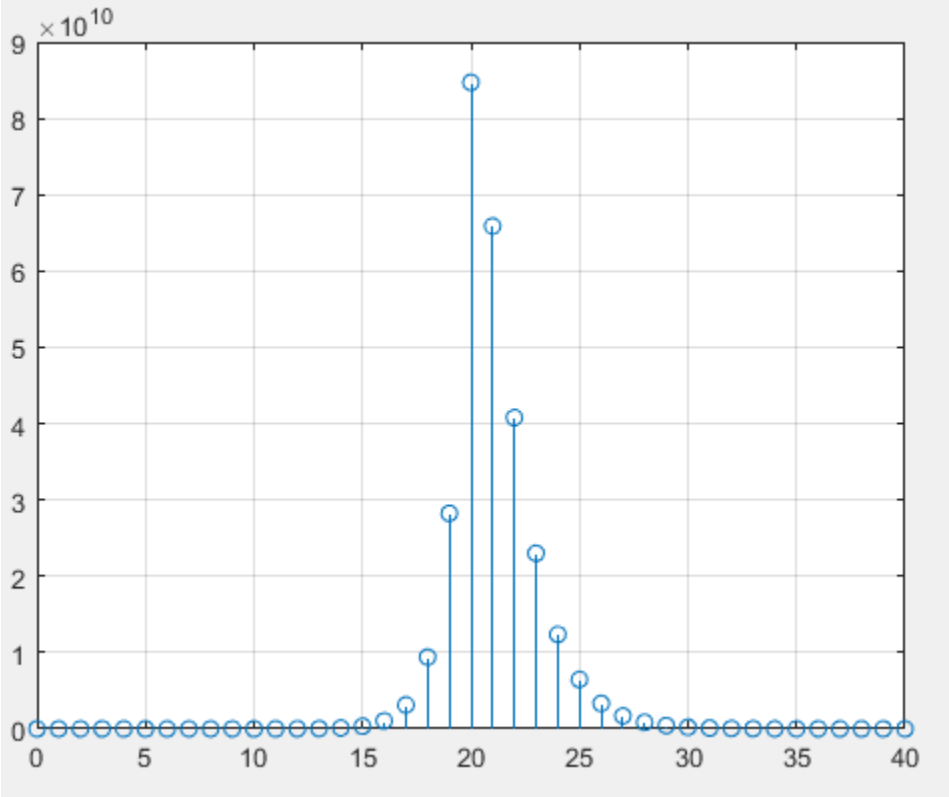
Yes, it is.

Assignment – Question 3 …

- To consider if $\frac{1}{6}y[n]-\frac{5}{6}y[n-1]+y[n-2]=x[n]$ is reverse system

of: $y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$

```
n=0:20;
A=[1 -5/6 1/6];
B=[1 0 0];
h=impz(B,A,n);
A1=[1/6 -5/6 1];
B1=[1 0 0];
h1=impz(B1,A1,n);
ht=conv(h,h1);
nt=0:length(ht)-1;
stem(nt,ht), grid on;
```



No, it is not.

Thank you!

Now, you do the last question of the assignment 4.