ELG3125 Signal and System Analysis

LTI Systems and Convolution Sum

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Slides and assignments:
http://www.site.uottawa.ca/~msade033/signal/

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Outline

1. Differential equations
2. Difference equations
3. Convolution sums

*To learn to simulate LTI systems in continuous time and discrete time, represented by differential equations, difference equations, or impulse response.

Continuous-Time LTI System

- N-th order linear constant-coefficient differential equation:

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

- N-th order linear constant-coefficient differential equation:

$$A = [a_N, a_{N-1}, \ldots, a_0], \; B = [b_M, b_{M-1}, \ldots, b_0]$$

- To find the impulse response h(t) of the system:
  impulse(B,A,t)
- To compute the output given the input x(t):
  lsim(B,A,x,t)

Example: Continuous-Time LTI System

- Second order LTI system

$$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 6x(t)$$

$$A = [1\ \ 2\ \ 3] \quad B = [0\ 1\ 6]$$

- Coefficients are highest-order first from left to right in the vector.

- The size of A and B should be the same.

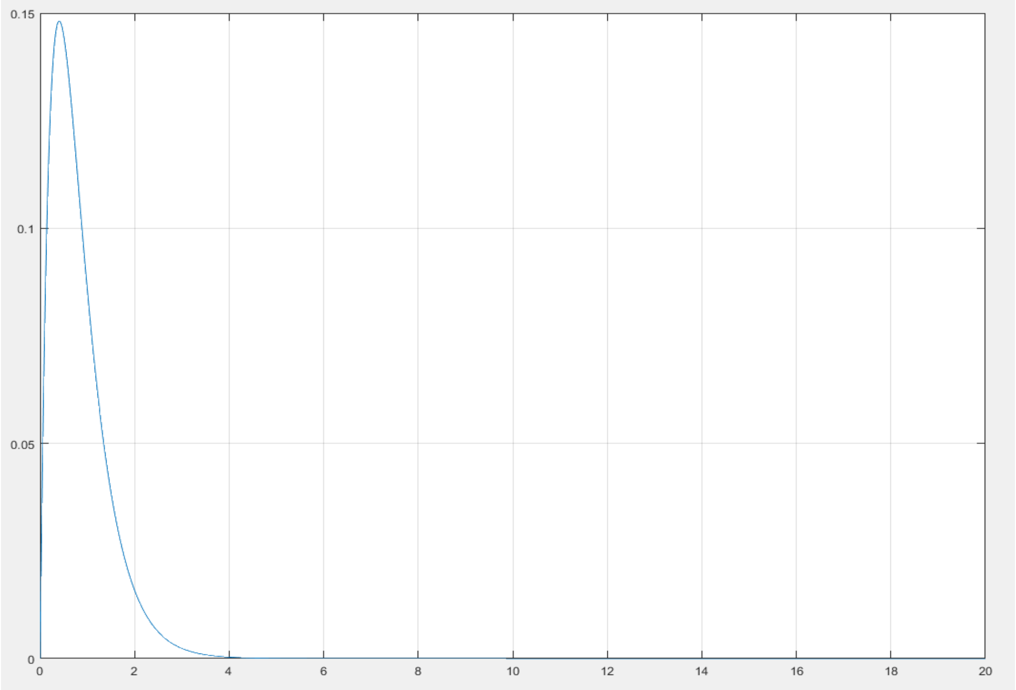
Assignment – Question 1

1. For the following continuous time LTI system:

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t)$$ (system initially at rest)

a) Find and plot using Matlab the impulse response h(t) of the system, using the function *impulse(b,a,t)* where *b* and *a* correspond to the coefficients in the differential equation (i.e. a=[1, 5, 6] and b=[0 0 1]).

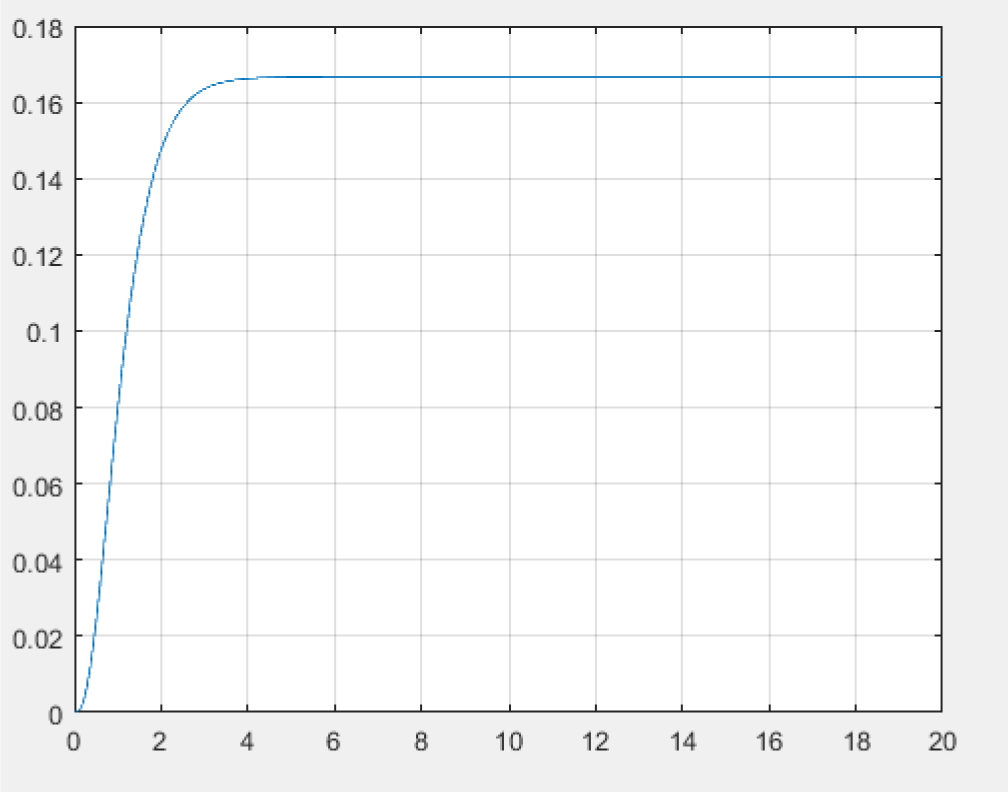
```
a=[1 5 6];
b=[0 0 1];
impulse(b,a,t);
grid on;
```



Assignment …

b). For an input signal defined as: $x(t) = (1 - e^{-3t})u(t)$
compute the output y(t)of that system, using the function *lsim(b,a,x,t)*.
Use a time range long enough to see the y(t) signal stabilize.

```
a=[1,5,6];
b=[0,0,1];
x=1-exp(-3*t);
c=lsim(b,a,x,t);
plot(t,c);
grid on;
```



Discrete-Time LTI System

- *N*th-order linear constant-coefficient difference equation

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

- Describe the system in Matlab

$$A = [a_0, a_1, \ldots, a_N], \ B = [b_0, b_1, \ldots, b_M]$$

- To find the impulse response *h[n]* of the system:

  impz(B,A,n)
- To compute the output given the input *x[n]:*

  filter(B,A,x)

Example: Discrete-Time LTI System

- 3rd order LTI system

$$2y[n]+5y[n-1]+y[n-2]+8y[n-3]=x[n]$$

- Coefficients are most present time first from left to right in the vector

A=[2,5,1,8]   B=[1,0,0,0]=[1,zeros(1,3)]

Convolution sum

- Mathematical formula

$$y[n] = \sum_{-\infty}^{+\infty} x[n]h[n-k] = x[n] * h[n] = h[n] * x[n]$$

- Matlab function: y=conv(x,h)

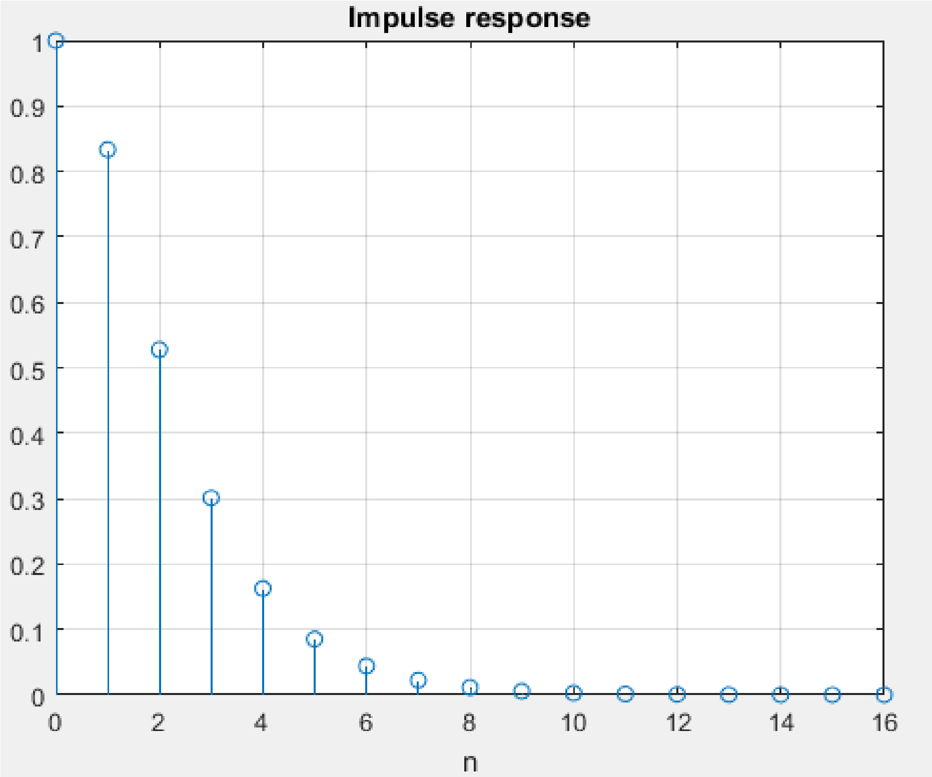
- Length of signal x[n]: length(x)

Assignment – Question 2

2. For the following discrete time LTI system:

$$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n]$$ (system initially at rest)

```
a=[1,-5/6,1/6];
b=[1 0 0];
n=0:16;
h=impz(b,a,n);
figure(1);
stem(n,h),grid
xlabel('n');
ylabel('h[n]');
title('Impulse response');
```



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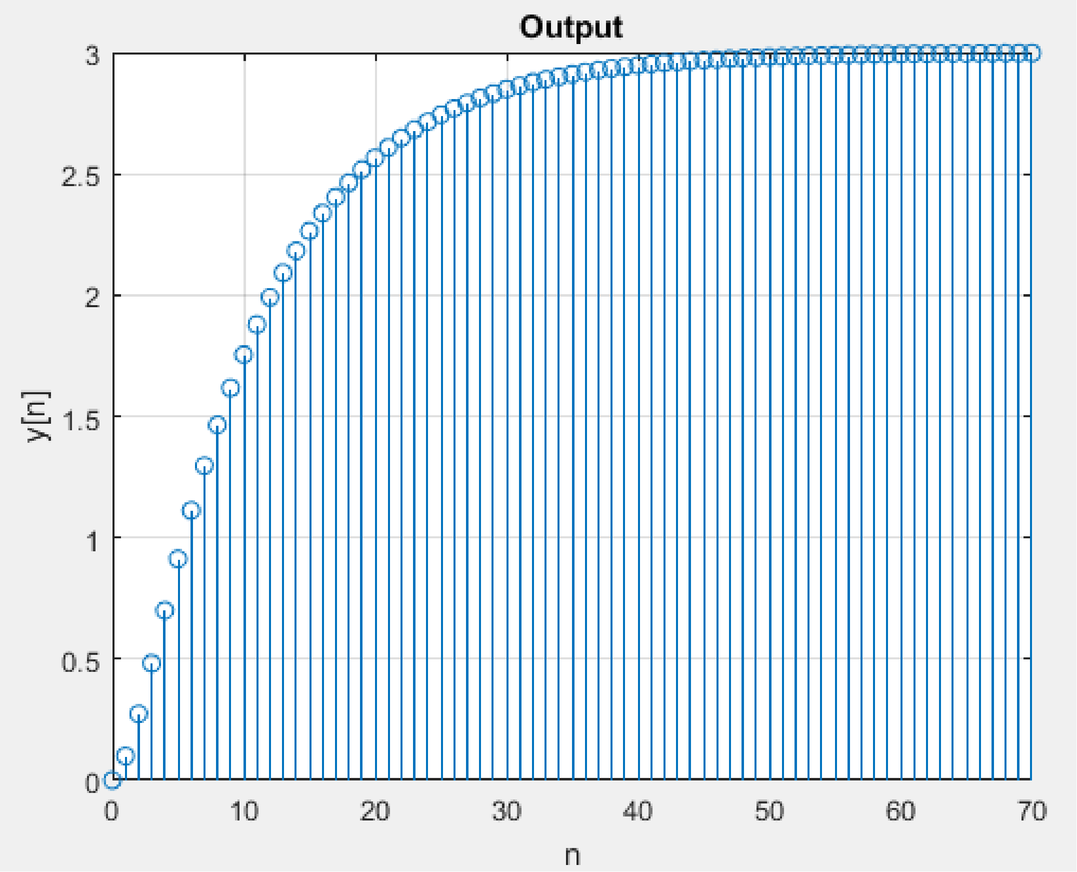
Assignment …

b) For an input signal defined as: $x[n] = (1 - 0.9^n)u[n]$

compute the output y[n] of that system, using the function filter. Use a time range long enough to see the y[n] signal stabilize.

```
a=[1,-5/6,1/6];
b=[1 0 0];
n=0:70;
x=1-power(0.9,n);
y=filter(b,a,x);
stem(n,y),grid;
xlabel('n');
ylabel('y[n]');
title('Output');
```



Learn How to Operate Music Files

- Read a music file in Matlab: *audioread()*
- Listen to the music: *Sound()*

Assignment – Question 3

- First obtain a discrete time input signal x[n] by reading the file "Audio1.wav" from the course's website. This signal has a duration (length) of 190912 samples, with a sampling frequency of 16000 samples/sec. Visualize the resulting signal and listen to it.

- The page: http://www.site.uottawa.ca/~msade033/signal/

```
x=audioread('Audio1.wav');
sound(x); % Please don't execute the
syntax unless you have headphones!
```

Assignment …

$$h[n] = 0.1 \times (0.99)^n, 0 \le n \le 1000$$
$$y[n] = x[n] * h[n]$$

c) Use the function *conv( )* to compute y[n]. Observe the size of the result y[n], plot y[n] and listen to it. What do you observe when directly comparing with x[n]? Does it sound different ?

```
x=audioread('Audio1.wav');
n=0:1000;
h=0.1*(0.99).^n;
y=conv(x,h);
stem(y);
length(y) % has to be equal to
length(x)+length(h)-1
```

Assignment …

- d) Now use the function filter to compute y[n], where the parameter b in the function filter corresponds to h[n] and the parameter a is set to 1.0. Like the function *conv( )*, the function filter also computes the result of a discrete time convolution in this case, but not over the same interval of output y[n] values. Explain the difference.

```
x=audioread('Audio1.wav');
n=0:1000;
h=0.1*(0.99).^n;
y=filter(h,1,x);
stem(y);
length(y) % has to be equal to length(x)
```

Thank you!