ELG 4172 Digital signal processing

Professor: Miodrag Bolic

Final Exam

This exam is **180 minutes** long.
- Simple calculators are allowed
- notes and textbooks are allowed (open book exam)

Last name:

First name:

Student #:

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<th>Score</th>
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Question 1  Downsampling and Upsampling

Consider the following system:

\[
\begin{array}{c}
\Downarrow \frac{T}{3} \quad 2 \\
x(t) \rightarrow A/D \rightarrow r[n] \uparrow 3 \rightarrow x[n] \rightarrow H(e^{j\omega}) \rightarrow y[n] \downarrow 2 \rightarrow s[n]
\end{array}
\]

For the following specifications:

\[X(j\Omega) = 1 \quad |\Omega| < 1000\pi\]
\[X(j\Omega) = 0 \quad |\Omega| > 1000\pi\]
\[T = 1/2000 \text{ sec.}\]
\[H(e^{j\omega}) = 1 \quad |\omega| < \pi/3\]
\[H(e^{j\omega}) = 0 \quad |\omega| > \pi/3\]

draw the spectrum of the different discrete time signals (i.e. \(R(e^{j\omega}), X(e^{j\omega}), Y(e^{j\omega}), S(e^{j\omega})\).
Question 2  Finite Word Effects

Consider an IIR filter shown in figure below. Multiplier coefficients are \( a \), \( b_0 \) and \( b_1 \). Assume that the multiplications are performed in fixed point arithmetics using two's complement number format (normalized between -1 and +1), and using rounding after each multiplication operation. Assume that 10 bits are used for numbers, out of which 1 bit is used for sign and 9 bits for the magnitude.

a) Find the transfer function \( H(z) \) between the input and the output of the filter shown in Figure below.

b) Find the total noise PSD \( P_y(z) \) found at the output \( y(n) \), caused by the rounding operations on the multiplications.

c) Compute the output roundoff noise variance (output power).

\[
x(n) \xrightarrow{b_0} \xrightarrow{z^{-1}} \xrightarrow{a} y(n)
\]
Question 3  Quick Theory Questions

• name an advantage of decomposing and processing a signal in $M$ sub-bands

• when decomposing a signal into $M$ sub-bands, explain why the overall complexity is not increased, even though there are then $M$ signals to process instead of only one.

• is the filter described by the following zeros in the figure below a linear phase filter? Explain why.

<table>
<thead>
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<th>Im{z}</th>
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<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Re{z}</td>
</tr>
</tbody>
</table>

• explain which product quantization method (rounding, truncation) is normally preferred, and why

• what is the best window to use if we need to window a block of measured data to detect two components closely located in frequency (like two sinusoidal components)?

• what are the main benefits (name two of them) of the Chebyshev/Parks-McLellan design method for linear phase FIR filters, over the use of a simple window-based method?

• discuss the main advantages and drawbacks (name one of each) of the floating point number representation over the fixed point number representation.
Question 4  Filter Structures

For the following transfer function:

\[
H(z) = \frac{\left(1 - \frac{1}{5} z^{-1}\right)\left(1 + z^{-1} + \frac{1}{2} z^{-2}\right)}{\left(1 - \frac{1}{3} z^{-1}\right)\left(1 - z^{-1} + \frac{1}{2} z^{-2}\right)}
\]

draw the resulting a) canonical direct form (i.e. direct form II), b) the cascade form and c) the IIR lattice (i.e. lattice-ladder) form.
The following filter is a low-pass Butterworth filter of order 3, with a normalized passband cutoff frequency $\Omega_p' = 1.0$:

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Design a stopband discrete time filter $H(z)$ with cutoff frequencies $\omega_{p1} = \frac{\pi}{4}$ and $\omega_{p2} = \frac{\pi}{2}$. For the conversion from continuous time to discrete time, use the bilinear transform with $T = 2$. In each step, you do not need to find the numerical values of the coefficients and simplifications at each step are not necessary.
Question 6  FIR design

Consider the following specifications for a linear phase FIR bandpass filter with real coefficients:

\[
\begin{align*}
|H(e^{j\omega})| & \leq 0.01 & |\omega| < 0.2\pi \\
0.95 \leq |H(e^{j\omega})| & \leq 1.05 & 0.3\pi < |\omega| < 0.7\pi \\
|H(e^{j\omega})| & \leq 0.02 & |\omega| > 0.8\pi
\end{align*}
\]

specified over the interval 0 to \( \pi \).

a) Use the basic windowing method to design the filter (i.e. computing the impulse response of an ideal filter, then windowing it). Use a Hanning window and compute the order \( M \) (i.e. \( M + 1 \) samples) required for the window. Use the following table:

<table>
<thead>
<tr>
<th>Type of window</th>
<th>Approximate transition width of main lobe</th>
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</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>( \frac{4\pi}{M+1} )</td>
</tr>
<tr>
<td>Bartlett</td>
<td>( \frac{8\pi}{M+1} )</td>
</tr>
<tr>
<td>Hanning</td>
<td>( \frac{8\pi}{M+1} )</td>
</tr>
<tr>
<td>Hamming</td>
<td>( \frac{8\pi}{M+1} )</td>
</tr>
<tr>
<td>Blackman</td>
<td>( \frac{12\pi}{M+1} )</td>
</tr>
</tbody>
</table>
b) For the same specifications, now use the procedure of the Kaiser window, based on the requirements for the ripple levels.