ITI 1121. Introduction to Computing II *

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Abstract

- Recursive list processing (part I)

*These lecture notes are meant to be looked at on a computer screen. Do not print them unless it is necessary.

“To iterate is human, to recurse divine.”
(L. Peter Deutsch)

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What problems have you solved using recursion? Calculating the factorial? Locating a value in array?

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What problems have you solved using recursion? Calculating the factorial? Locating a value in array? Here recursion is used to traverse (singly) linked lists.

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Similarly, except that the interval is smaller by one position.

What size of problem can be solved trivially? An interval of size 1.

Problem (continued)

```
private static int sum( int[] t, int k ) {

    int s, result, length = t.length - k;

    if ( length == 1 ) { // Base case

        result = t[ k ];

    } else { // General case

       int k1 = k+1;
       s = sum( t, k1 );
       result = t[ k ] + s;

    }
    return result;
}
```

What would be the initial call?

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```
public static int sum( int[] t ) {
    return sum( t, 0 );
}
```

```
public class Sum {
    private static int sum( int[] t, int k ) {
        if ( k == ( t.length - 1 ) ) {
            return t[ k ];
        }
        return t[ k ] + sum( t, k+1 );
    }
    public static int sum( int[] t ) {
        if ( t.length == 0 ) {
            throw new IllegalArgumentException();
        }
        return sum( t, 0 );
    }
    public static void main( String[] args ) {
        int[] t = { 1, 2, 3, 4, 5 };
        System.out.println( sum( t ) );
    }
}
```

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Programming languages such as **Lisp**, **Prolog** and **Haskell** have no loop control-structures and therefore recursion is the only way to create iterations!

Pattern (pseudo-code)

```
type method( parameters ) {

    type result;

    if ( test parameters for base case ) { // base case

        // calculate the result without recursion, recursion stop here

    } else { // general case

        // pre-processing: partitioning the data for example

        result = method( sub-set of the data ); // recursive call

        // pos-processing: combining the results for example

    }
    return result;
}
```

Factorial

```
public static int factorial( int n ) {
    int s, result;
    if ( n<=1 ) { // base case
        result = 1;
    } else { // general case
        int n1 = n-1;
        s = factorial( n1 );
        result = n * s;
    }
    return result;
}
```

The factorial method corresponds to the general pattern, the base case is tested first, and the result is computed directly without the need for a recursive call (recursion stops here!), the general case is always making the value of the parameter smaller so that eventually the base case will be applied.

Factorial

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    if ( n<=1 ) {
        return 1;
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    return n * factorial( n-1 );
}
```

A **return** statement returns the control to the caller, it stops the execution of the method, no other statements of the call will be executed.

Binary Search

```
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    return binSearch( value, array, 0, array.length-1 );
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Base case: an interval of size zero, no recursive call or the value was found at the position **middle**.

General case: creates smaller and smaller intervals of values for the search.

```java
private static int binSearch( int value, int[] array, int lo, int hi ) {

    // Base case: value not found
    if ( lo > hi ) {
        return -1;
    }
    // Base case: value was found
    int middle = ( lo + hi ) / 2;
    if ( value == array[ middle ] ) {
        return middle;
    }
    // General case:
    int newLo, newHi;
    if ( value < array[ middle ] ) {
        newLo = lo;
        newHi = middle - 1;
    } else {
        newLo = middle + 1;
        newHi = hi;
    }
    return binarySearch( value, array, newLo, newHi );
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```

Recursive list processing — “head + tail” strategy

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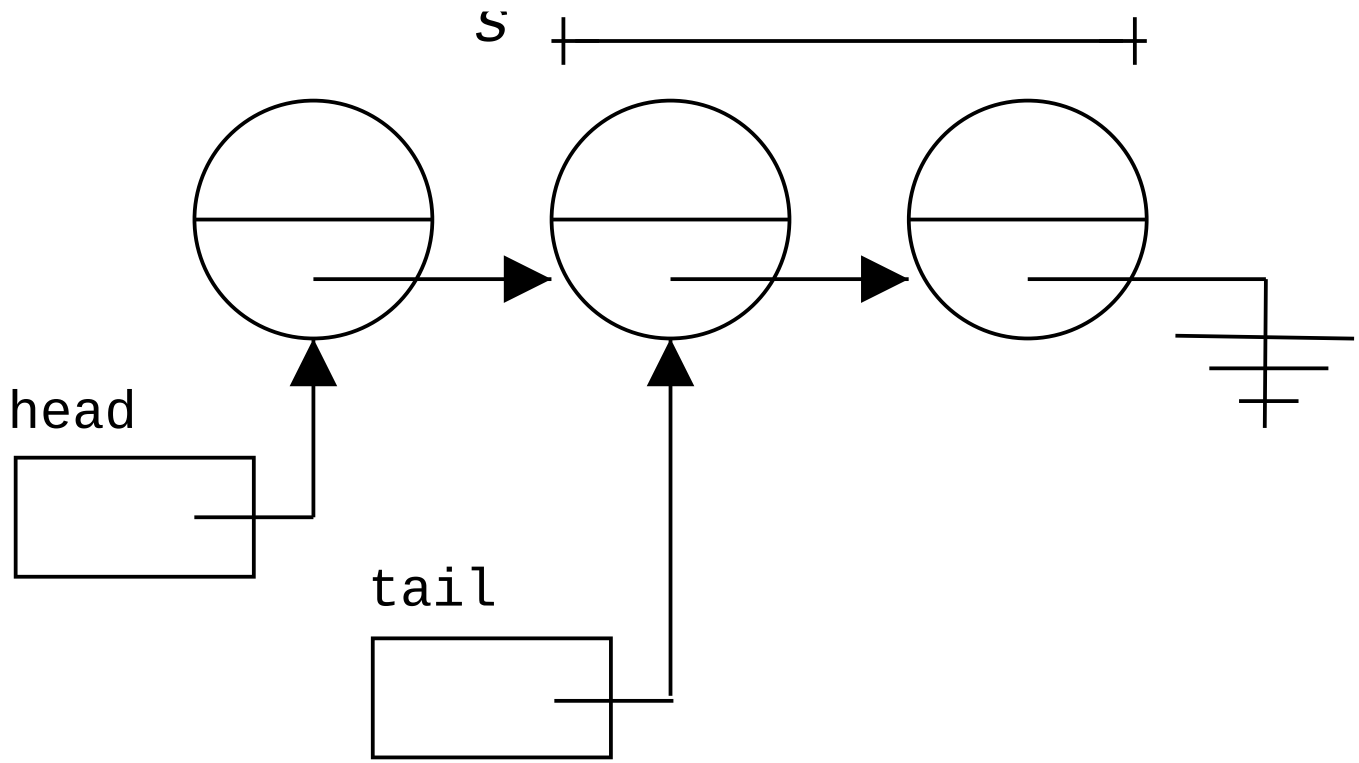
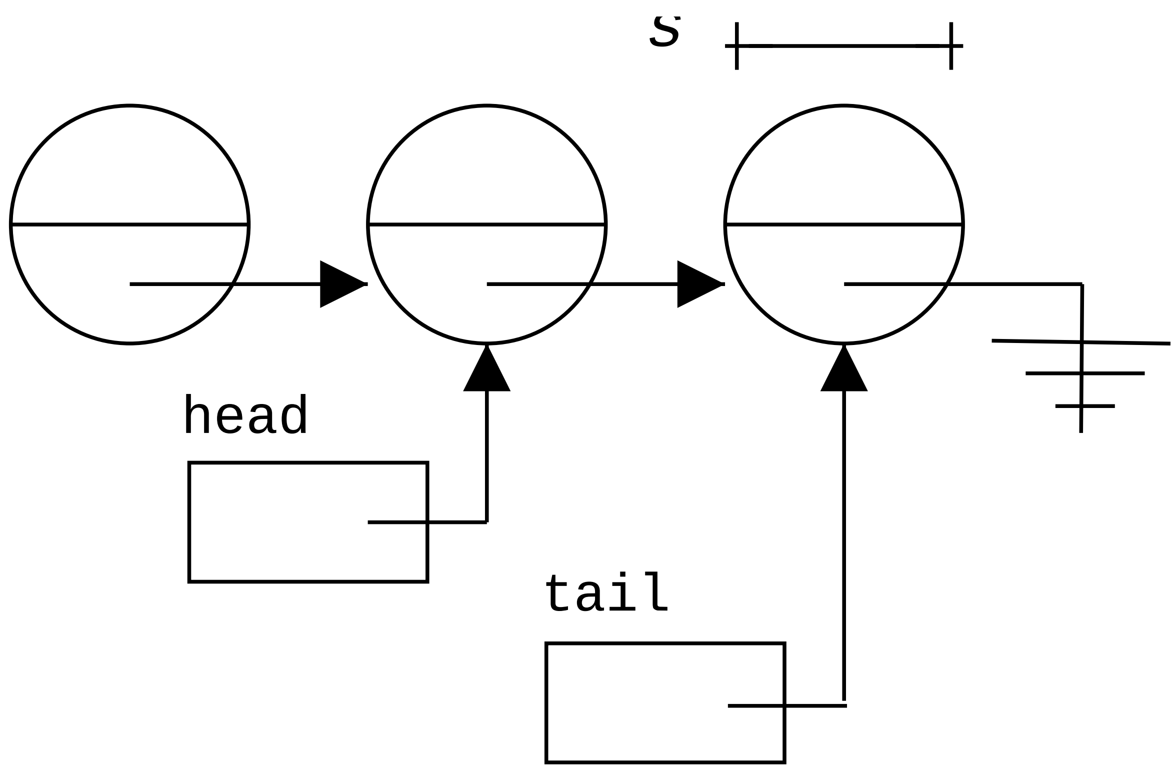
How to obtain s? Well, s is the size of a list, and, furthermore, **it's a shorter one**, we can apply (recursively) the method that we are developing.



$\Rightarrow$ The total length of the list will $s + 1$. **Note: here, head and tail are referring to the current node and remaining nodes**.



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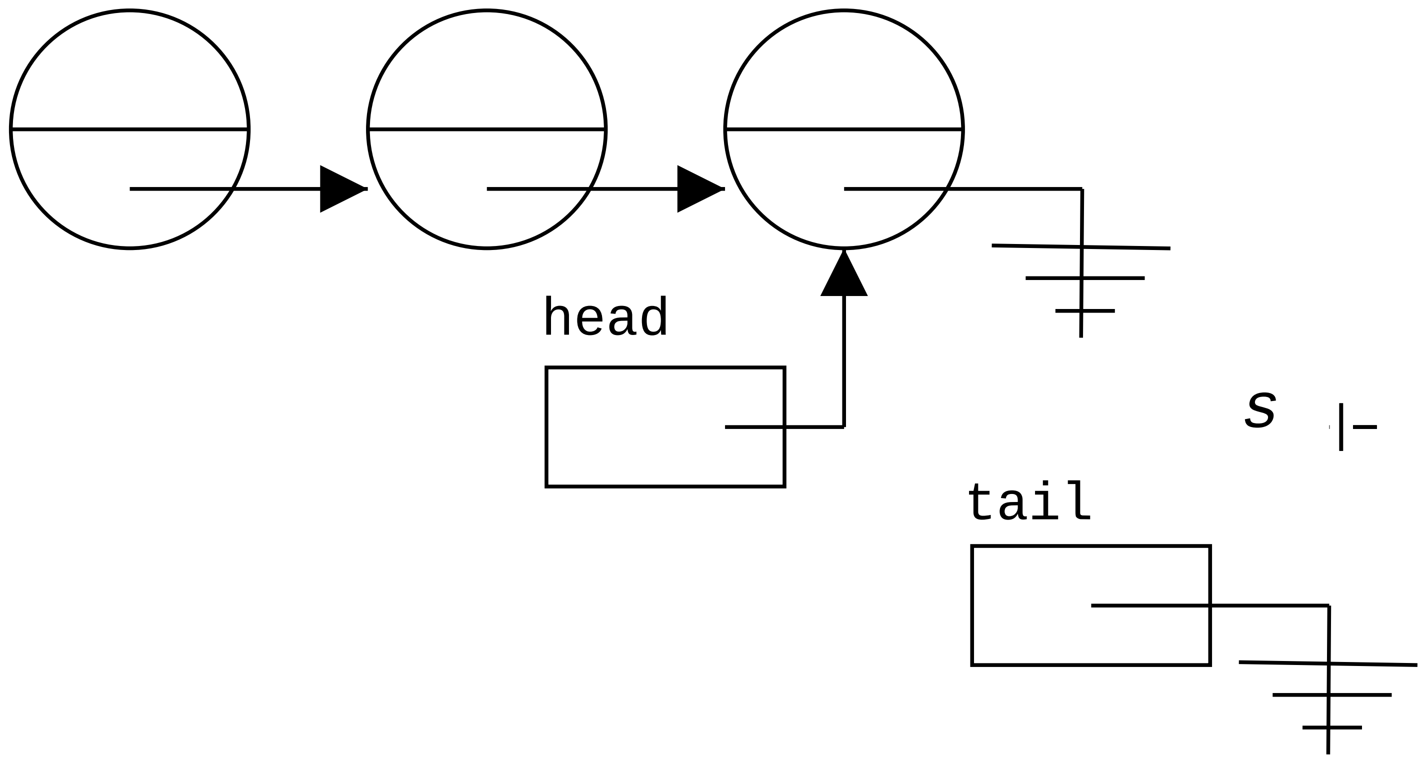

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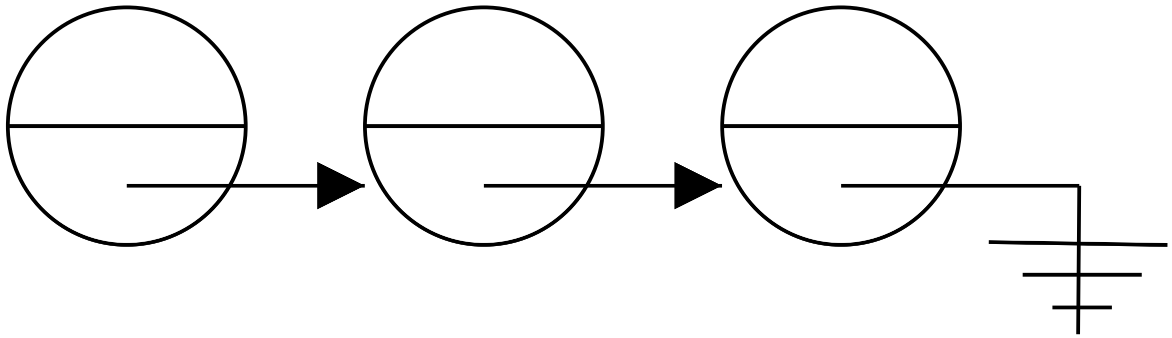

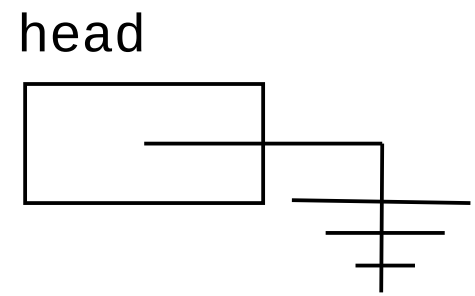

$\Rightarrow$ The length of the list designated by **head** is 0.

Implementation

The methods presented in this lecture are instance methods of a class defined as follows.

```
public class OrderedList< E extends Comparable<E> > {
    private static class Node<E> {
        private E value;
        private Node<E> next;
        Node( E value, Node next ) {
            this.value = value;
            this.next = next;
        }
    }
    private Node<E> first; // <---
    public OrderedList () {
        first = null;
    }
    // other methods ...
}
```

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The empty list!

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The empty list is a valid case, its length is zero, however, it has to be a base case because it has no tail!

Because of that, for all methods that apply the “head + tail” strategy the empty list cannot be part of the general case.

```
int size( Node<E> p ) {
    int s, length;

    if ( p == null ) { // Base case

        length = 0;

    } else { // General case

        s = size( p.next );
        length = 1 + s;

    }
    return length;
}
```

Or simply,

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⇒ Can this method be called from outside of the class? What is the first (initial) call?

No, it cannot be called from outside of the class because it needs to be given a reference to a **Node** (which is an implementation detail and should be **private**).

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We need an auxiliary method that initiates the process starting with the first element of the list:

```
public int size() {
    return size( first );
}
```

Because, the recursive method cannot and should not be used from outside of the class, it should be declared private:

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public int size() {
    return size( first );
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```

All our recursive methods will obey this pattern. They will all have a **public** part that calls the **recursive** methods, **which is private**. The public method initiates the first call to the recursive method with a reference to the first node.

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Given our list implementation, the "head + strategy" strategy is simpler to apply.

The strategy that consists in dividing the list into two sub-lists of approximately the same size can lead to more efficient algorithms, but that's a subject left for CSI 2114 (data-structures), here, we'll use the "head + tail" strategy.

“Head + tail”

```
if ( ... ) { // base case
   calculate results
} else { // general case
  // pre-processing
  s = method( p.next ); // recursion
  // post-processing
}
```

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How can you use it to build the solution for the list starting at **p**?

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What is the smallest valid list? What is the result?

E findMax()

Write a recursive method that finds the maximum value for a list of $<$ **E extends Comparable**$<$**E**$>$ $>$ objects.

E findMax()

Write a recursive method that finds the maximum value for a list of < **E extends Comparable**<**E**> > objects.

Let's take care of the **public non-recursive** part first.

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What are we going to do if the list is empty? Throwing an exception.

Where should the recursion start? It'll start with the **first** element of the list.

E findMax()

```
public E findMax() {
    if ( first == null ) {
        throw new NoSuchElementException();
    }
    return findMax( first );
}
```

Remark: the recursive method will always have at least one more parameter than the non-recursive method.

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Remark: the recursive method will always have at least one more parameter than the non-recursive method. Why? The parameter designates the current element for this step.

E findMax(Node<E> p)

Let's apply the "head+tail" strategy and consider the general case first,

```
E result = findMax( p.next );
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What does **result** represents?

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What should be done next?

Compare **result** to the value of the current **Node**.

```
if ( result.compareTo( p.value ) > 0 ) {
    return result;
} else {
    return p.value;
}
```

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    if ( first == null ) {
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    }
    return findMax( first );
}
private E findMax( Node<E> p ) {
    if ( p.next == null ) {
        return p.value;
    }
    E result = findMax( p.next );
    if ( result.compareTo( p.value ) > 0 ) {
        return result;
    } else {
        return p.value;
    }
}
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