ITI 1121. Introduction to Computing II *

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Abstract

- Abstract data type: Stack
  - Stack-based algorithms

*These lecture notes are meant to be looked at on a computer screen. Do not print them unless it is necessary.
Evaluating arithmetic expressions

Stack-based algorithms are used for syntactical analysis (*parsing*).
Evaluating arithmetic expressions

Stack-based algorithms are used for syntactical analysis (parsing).

For example to evaluate the following expression:

\[ 1 + 2 \times 3 - 4 \]
Evaluating arithmetic expressions

Stack-based algorithms are used for syntactical analysis (parsing).

For example to evaluate the following expression:

\[ 1 + 2 \times 3 - 4 \]

Compilers use similar algorithms to check the syntax of your programs and generate machine instructions (executable).

To verify that parentheses are balanced: '([[]])' is ok, but not '([[]])' or ')((()(()('.
The first two steps of the analysis of a source program by a compiler are the **lexical analysis** and the **syntactical analysis**.
The first two steps of the analysis of a source program by a compiler are the **lexical analysis** and the **syntactical analysis**.

During the **lexical analysis** (*scanning*) the source code is read from left to right and the characters are regrouped into **tokens**, which are successive characters that constitute numbers or identifiers. One of the tasks of the lexical analyser is to remove spaces from the input.

E.g.:

```
·10 ·+ ·2 ·+ ·300
```

where “·” represent blank spaces, is transformed into the following list of tokens:

```
[10,+2,+300]
```
The next step is the syntactical analysis (\textit{parsing}) and consists in regrouping the tokens into grammatical units, for example the sub-expressions of RPN expressions (seen in class this week).

In the next slides, we look at simple examples of lexical and syntactical analysis.
public class Test {

    public static void scan( String expression ) {

        Reader reader = new Reader( expression );

        while ( reader.hasMoreTokens() ) {
            System.out.println( reader.nextToken() );
        }
    }

    public static void main( String[] args ) {
        scan( " 3 + 4 * 567 " );
    }
}

    // > java Test
    // INTEGER: 3
    // SYMBOL: +
    // INTEGER: 4
    // SYMBOL: *
    // INTEGER: 567
public class Token {
    private static final int INTEGER = 1;
    private static final int SYMBOL = 2;
    private int iValue;
    private String sValue;
    private int type;

    public Token( int iValue ) {
        this.iValue = iValue;
        type = INTEGER;
    }
    public Token( String sValue ) {
        this.sValue = sValue;
        type = SYMBOL;
    }

    public int iValue() { ... }
    public String sValue() { ... }
    
    public boolean isInteger() { return type == INTEGER; }
    public boolean isSymbol() { return type == SYMBOL; }
}
public static int execute( String expression ) {
    Token op = null; int l = 0, r = 0;

    Reader reader = new Reader( expression );
    l = reader.nextToken().iValue();

    while ( reader.hasMoreTokens() ) {
        op = reader.nextToken();
        r = reader.nextToken().iValue();
        l = eval( op, l, r );
    }
    return l;
}
public static int eval( Token op, int l, int r ) {

    int result = 0;

    if ( op.sValue().equals( "+" ) )
        result = l + r;
    else if ( op.sValue().equals( "-" ) )
        result = l - r;
    else if ( op.sValue().equals( "*" ) )
        result = l * r;
    else if ( op.sValue().equals( "/" ) )
        result = l / r;
    else
        System.err.println( "not a valid symbol" );

    return result;
}
Evaluating an arithmetic expression: LR Scan

Left-to-right algorithm:

Declare L, R and OP

Read L
While not end-of-expression
do:
  Read OP
  Read R
  Evaluate L OP R
  Store result in L

At the end of the loop the result can be found in L.
$3 \times 8 - 10$
3 + 4 - 5
~

L = 3
OP =
R =

> Read L
While not end-of-expression
do:
  Read OP
  Read R
  Evaluate L OP R
  Store result in L
3 + 4 - 5

L = 3
OP = +
R =

Read L
While not end-of-expression
do:
> Read OP
Read R
Evaluate L OP R
Store result in L
3 + 4 - 5

L = 3
OP = +
R = 4

Read L
While not end-of-expression
do:
    Read OP
> Read R
    Evaluate L OP R
    Store result in L
$3 + 4 - 5$

$L = 3$
$OP = +$
$R = 4$

Read L
While not end-of-expression
do:
  Read OP
  Read R
>  Evaluate L OP R (7)
Store result in L
3 + 4 − 5

L = 7
OP = +
R = 4

Read L
While not end-of-expression
do:
    Read OP
    Read R
    Evaluate L OP R
>
    Store result in L
\(3 + 4 - 5\)

\(^{\sim}\)

\[L = 7\]
\[OP = -\]
\[R = 4\]

Read L
While not end-of-expression
do:
  >    Read OP
    Read R
    Evaluate L OP R
    Store result in L
3 + 4 - 5

L = 7
OP = -
R = 5

Read L
While not end-of-expression
do:
  Read OP
  Read R
Evaluate L OP R
Store result in L
3 + 4 - 5

L = 7
OP = -
R = 5

Read L
While not end-of-expression
do:
   Read OP
   Read R
> Evaluate L OP R (2)
  Store result in L
3 + 4 - 5

L = 2
OP = -
R = 5

Read L
While not end-of-expression
do:
   Read OP
   Read R
   Evaluate L OP R
> Store result in L
3 + 4 - 5

\[ L = 2 \]
\[ \text{OP} = - \]
\[ R = 5 \]

Read \( L \)

While not end-of-expression

\[ \text{do:} \]
\[ \text{Read OP} \]
\[ \text{Read R} \]
\[ \text{Evaluate } L \ \text{OP} \ R \]
\[ \text{Store result in } L \]

\[ \Rightarrow \text{end of expression, exit the loop, } L \text{ contains the result.} \]
What do you think?
What do you think?

Without *parentheses* the following expression cannot be evaluated correctly:

\[ 7 - (3 - 2) \]
What do you think?

Without **parentheses** the following expression cannot be evaluated correctly:

\[ 7 - (3 - 2) \]

Because the result of the left-to-right analysis corresponds to:

\[ (7 - 3) - 2 \]
What do you think?

Without **parentheses** the following expression cannot be evaluated correctly:

\[
\Rightarrow 7 - (3 - 2)
\]

Because the result of the left-to-right analysis corresponds to:

\[
\Rightarrow (7 - 3) - 2
\]

Similarly the following expression cannot be evaluated by our simple algorithm:

\[
\Rightarrow 7 - 3 * 2
\]
What do you think?

Without _parentheses_ the following expression cannot be evaluated correctly:

⇒ 7 - (3 - 2)

Because the result of the left-to-right analysis corresponds to:

⇒ (7 - 3) - 2

Similarly the following expression cannot be evaluated by our simple algorithm:

⇒ 7 - 3 * 2

Since the left-to-right analysis corresponds to:

⇒ (7 - 3) * 2
What do you think?

Without *parentheses* the following expression cannot be evaluated correctly:

$$\Rightarrow 7 - (3 - 2)$$

Because the result of the left-to-right analysis corresponds to:

$$\Rightarrow (7 - 3) - 2$$

Similarly the following expression cannot be evaluated by our simple algorithm:

$$\Rightarrow 7 - 3 \times 2$$

Since the left-to-right analysis corresponds to:

$$\Rightarrow (7 - 3) \times 2$$

But according to the *operator precedences*, the evaluation should have proceeded as follows:

$$\Rightarrow 7 - (3 \times 2)$$
Remarks

The left-to-right algorithm:

• Does not handle parentheses;
• Nor precedence.
Remarks

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- Does not handle parentheses;
- Nor precedence.

Solutions:
Remarks

The left-to-right algorithm:

- Does not handle parentheses;
- Nor precedence.

Solutions:

1. Use a different notation;
2. Develop more complex algorithms.
Remarks

The left-to-right algorithm:

- Does not handle parentheses;
- Nor precedence.

Solutions:

1. Use a different notation;
2. Develop more complex algorithms.

⇒ Both solutions involve stacks!
Notations

There are 3 ways to represent the following expression: $L \text{ OP } R$. 
There are 3 ways to represent the following expression: \texttt{L OP R}.

\textbf{infix}: this is the usual notation, the operator is sandwiched in between its operands: \texttt{L OP R};
Notations

There are 3 ways to represent the following expression: $L \ OP \ R$.

**infix:** this is the usual notation, the operator is sandwiched in between its operands: $L \ OP \ R$;

**postfix:** in postfix notation, the operands are placed before the operator, $L \ R \ OP$. This notation is also called *Reverse Polish Notation* or *RPN*, it’s the notation used by certain scientific calculators (such as the HP-35 from Hewlett-Packard or the Texas Instruments TI-89 using the RPN Interface by Lars Frederiksen\(^1\)) or PostScript programming language.

\[
7 \ - \ (3 \ - \ 2) \ = \ 7 \ 3 \ 2 \ - \ - \\
(7 \ - \ 3) \ - \ 2 \ = \ 7 \ 3 \ - \ 2 \ -
\]
Notations

There are 3 ways to represent the following expression: $L \ OP \ R$.

**infix:** this is the usual notation, the operator is sandwiched in between its operands: $L \ OP \ R$;

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$$7 - (3 - 2) = 7 \ 3 \ 2 - -$$

$$\ (7 - 3) - 2 = 7 \ 3 - 2 -$$

**prefix:** the third notation consists in placing the operator before the operands, $OP \ L \ R$. The programming language Lisp uses a combination of parentheses and prefix notation: $(- \ 7 \ (* \ 3 \ 2))$.\(^1\)
Notations

There are 3 ways to represent the following expression: $L \ OP \ R$.

**infix:** this is the usual notation, the operator is sandwiched in between its operands: $L \ OP \ R$;

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$$7 - (3 - 2) = 7 \ 3 \ 2 - -$$

$$ (7 - 3) - 2 = 7 \ 3 - 2 -$$

**prefix:** the third notation consists in placing the operator before the operands, $OP \ L \ R$. The programming language Lisp uses a combination of parentheses and prefix notation: $(\ - \ 7 \ (\ * \ 3 \ 2))$.

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\(^1\)[www.calculator.org/rpn.html](http://www.calculator.org/rpn.html)
Infix $\rightarrow$ postfix (mentally)

Successively transform, one by one, all the sub-expressions following the same order of evaluation that you would normally follow to evaluate an infix expression.
Infix $\rightarrow$ postfix (mentally)

Successively transform, one by one, all the sub-expressions following the same order of evaluation that you would normally follow to evaluate an infix expression.

An infix expression $l \diamond r$ becomes $l r \diamond$, where $l$ and $r$ are sub-expressions and $\diamond$ is an operator.
\[ \frac{9}{(2 \times 4 - 5)} \]
\[
\frac{9}{(2 \times 4 - 5)}
\]
\[ \frac{9}{(2 \times 4 - 5)} \]

\[ \frac{9}{(\begin{array}{c} 2 \\ l \end{array} \times \begin{array}{c} 4 \\ r \end{array} - 5)} \]

\[ \frac{9}{(\begin{array}{c} 2 \\ l \end{array} \times \begin{array}{c} 4 \\ r \end{array} - 5)} \]
\[
9 / ( 2 \times 4 - 5 )
\]

\[
9 / ( \begin{array}{c} 2 \\ \text{l} \\ \text{r} \end{array} \times 4 - 5 )
\]

\[
9 / ( \begin{array}{c} 2 \\ \text{l} \\ \text{r} \end{array} \times 4 - 5 )
\]

\[
9 / ( [ 2 4 \times ] - 5 )
\]
\[ 9 / ( 2 \times 4 - 5 ) \]

\[ 9 / ( \underbrace{2 \quad \times \quad 4}_{l \quad r} - 5 ) \]

\[ 9 / ( \underbrace{2 \quad \times}_{l} \quad 4 \quad -}_{r} 5 ) \]

\[ 9 / ( \underbrace{[ 2 4 \times ]}_{l} -_{r} 5 ) \]
\[ \frac{9}{(2 \times 4 - 5)} \]

\[
\frac{9}{(\frac{2}{l} \times \frac{4}{r} - 5)}
\]

\[
\frac{9}{(\frac{2}{l} \times \frac{4}{r} - 5)}
\]

\[
\frac{9}{([2 4 \times] - 5)}
\]

\[
\frac{9}{([2 4 \times] - 5)}
\]

\[
\frac{9}{([2 4 \times] - 5)}
\]
\[ \frac{9}{(2 \times 4 - 5)} \]

\[ \frac{9}{(\frac{2}{l} \times \frac{4}{r} - 5)} \]

\[ \frac{9}{(\frac{2}{l} \times \frac{4}{r} - 5)} \]

\[ \frac{9}{([2 \times 4] - 5)} \]

\[ \frac{9}{([2 \times 4] - 5)} \]

\[ \frac{9}{([2 \times 5] - [2 \times 5 - ])} \]

\[ \frac{9}{([2 \times 5] - [2 \times 5 - ])} \]
\[
\frac{9}{(2 \times 4 - 5)}
\]

\[
\frac{9}{(\left(\begin{array}{cc}
2 & \times \\
\_l & \_r
\end{array}\right) - 5)}
\]

\[
\frac{9}{(\left(\begin{array}{ccc}
2 & 4 & \times \\
\_l & \_r & \_l
\end{array}\right) - 5)}
\]

\[
\frac{9}{([2 4 \times] - 5)}
\]

\[
\frac{9}{([2 4 \times] - 5)}
\]

\[
\frac{9}{([2 4 \times] - 5)}
\]

\[
\frac{9}{([2 4 \times 5 - ])}
\]

\[
\frac{9}{([2 4 \times 5 - ])}
\]
\[ 9 \div (2 \times 4 - 5) \]

\[ 9 \div (\frac{2}{l} \times \frac{4}{r} - 5) \]

\[ 9 \div (\frac{2}{l} \times \frac{4}{r} - 5) \]

\[ 9 \div (\frac{2}{l} \times \frac{4}{r} - 5) \]

\[ 9 \div (\frac{2}{l} \times \frac{4}{r} - 5) \]

\[ 9 \div (\frac{2}{l} \times \frac{4}{r} - 5) \]

\[ 9 \div (\frac{2}{l} \times \frac{4}{r} - 5) \]
\[ \frac{9}{(2 \times 4 - 5)} \]

\[ \frac{9}{(2 \times 4 - 5)} \]

\[ \frac{9}{((2 \times 4 - 5)} \]

\[ \frac{9}{([2 4 \times] - 5)} \]

\[ \frac{9}{([2 4 \times] - 5)} \]

\[ \frac{9}{([2 4 \times] - 5)} \]

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\[ \frac{9}{([2 4 \times] - 5)} \]

\[ \frac{9}{([2 4 \times] - 5)} \]
Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace $l \, r \, \diamond$ by the result of the evaluation of $l \, \diamond \, r$.

\[ 9 \, 2 \, 4 \times 5 - / \]
Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace \( l \circ r \) by the result of the evaluation of \( l \circ r \).

\[
9 \ 2 \ 4 \times 5 - /
\]

\[
9 \overset{2}{l} \overset{4}{r} \times 5 - /
\]
Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace $l \diamond r$ by the result of the evaluation of $l \diamond r$.

$$9 \ 2 \ 4 \times \ 5 \ 9 \ \ 2 \ 4 \times \ 5$$

$$9 \ 2 \ 4 \times \ 5 \ 8 \ 9 \ 8 \ 5 \ 9 \ 8 \ 5$$
Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace $l \ r \diamond$ by the result of the evaluation of $l \diamond r$.

\[ \begin{align*}
9 & 2 4 \times 5 \ - \ / \\
9 & \underbrace{2}_{l} \ 4_{r} \times 5 \ - \ / \\
9 & \underbrace{8}_{l} \ 5_{r} \ - \ / \\
9 & \underbrace{8}_{l} \ 5_{r} \ - \ /
\end{align*} \]
Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace \( l \circ r \) by the result of the evaluation of \( l \circ r \).

\[
9 2 4 \times 5 - / \\
\text{9 \ (2 \ (4 \times 5) - /)} \\
\text{9 \ (8 \ - /)} \\
\text{9 \ (3 \ /)}
\]
Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace \( l \circ r \) by the result of the evaluation of \( l \diamond r \).

\[
9 \ 2 \ 4 \times 5 \ - \\
9 \ 8 \ 5 \ - \\
9 \ 3 \ /
\]
Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace $l \ r \ ◊$ by the result of the evaluation of $l \ ◊ \ r$.

$$9 \ 2 \ 4 \ \times \ 5 \ − \ /$$

$9 \ \begin{array}{c}
\ 2 \\
\ l
\end{array} \ \begin{array}{c}
\ 4 \\
\ r
\end{array} \ \begin{array}{c}
\ \times \\
\ ◊
\end{array} \ 5 \ − \ /$

$9 \ \begin{array}{c}
\ 8 \\
\ l
\end{array} \ \begin{array}{c}
\ 5 \\
\ r
\end{array} \ − \ /$

$9 \ \begin{array}{c}
\ 3 \\
\ l
\end{array} \ \begin{array}{c}
\ / \\
\ r
\end{array}$
Evaluating a postfix expression

Until the end of the expression has been reached:

1. From left to right until the first operator;
2. Apply the operator to the two preceding operands;
3. Replace the operator and its operands by the result.

At the end we have result.
9 2 4 * 5 - /
Remarks: infix vs postfix

The order of the operands is the same for both notations, however operators are inserted at different places:

\[ 2 + (3 \times 4) = 2 3 4 * + \]

\[ (2 + 3) \times 4 = 2 3 + 4 * \]
Remarks: infix vs postfix

The order of the operands is the same for both notations, however operators are inserted at different places:

\[ 2 + (3 \times 4) = 2 3 4 \times + \]

\[ (2 + 3) \times 4 = 2 3 + 4 \times \]

Evaluating an infix expression involves handling operators precedence and parenthesis — in the case of the postfix notation, those two concepts are embedded in the expression, i.e. the order of the operands and operators.
Algorithm: Eval Infix

What role will the stack be playing?
Algorithm: Eval Infix

What role will the stack be playing?

operands = new stack;

while ( "has more tokens" ) {
    t = next token;
    if ( "t is an operand" ) {
        operands.push( "the integer value of t" );
    } else { // this is an operator
        op = "operator value of t";
        r = operands.pop();
        l = operands.pop();
        operands.push( "eval( l, op, r )" );
    }
}
return operands.pop();
Evaluating a postfix expression

The algorithm requires a stack (Numbers), a variable that contains the last element that was read (X) and two more variables, L and R, whose purpose is the same as before.

Numbers = [

While not end-of-expression
  do:
    Read X
    If X isNumber, PUSH X onto Numbers
    If X isOperator,
      R = POP Numbers (right before left?!)
      L = POP Numbers
      Evaluate L X R; PUSH result onto Numbers

To obtain the final result: POP Numbers.
9 - 3 / 2
\[
\frac{9}{((2 \times 4) - 5)} = \frac{9}{2 \times 4 \times 5 - /}
\]

\[
> \quad \text{Numbers} = [
\begin{array}{l}
X = \\
L = \\
R = 
\end{array}
\]

While not end-of-expression
do:
    Read X
    If X isNumber, PUSH X onto Numbers
    If X isOperator,
        R = POP Numbers
        L = POP Numbers
        Evaluate L X R; PUSH result onto Numbers

⇒ Create a new stack
\[
\frac{9}{(2 \times 4) - 5} = 9 \div 2 \times 4 - 5
\]

Numbers = [
X = 9
L =
R =

While not end-of-expression
do:
> Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ Read X
9 / ((2 * 4) - 5) = 9 2 4 * 5 - /

Numbers = [9
X = 9
L =
R =

While not end-of-expression
do:
  Read X
>    If X isNumber, PUSH X onto Numbers
    If X isOperator,
      R = POP Numbers
      L = POP Numbers
      Evaluate L X R; PUSH result onto Numbers

⇒ Push X
\[
9 / ((2 * 4) - 5) = 9 \ 2 \ 4 \ * \ 5 - / \\
\]

\[
\text{Numbers} = [9 \\
X = 2 \\
L = \\
R = \\
\]

While not end-of-expression
do:
>  Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ Read X
\[
9 / ((2 \times 4) - 5) = 9 2 4 \times 5 - / \\
\]

Numbers = [9 2
X = 2
L =
R =

While not end-of-expression
do:
  Read X
>  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ Push X
\[
9 / ((2 \times 4) - 5) = 9 \ 2 \ 4 \times 5 - / \\
\]

Numbers = [9 2
X = 4
L =
R =

While not end-of-expression
do:
>  Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ Read X
\[
9 \div ((2 \times 4) - 5) = 9 \ 2 \ 4 \times 5 - /
\]

Numbers = [9 2 4
X = 4
L =
R =

While not end-of-expression
do:
    Read X
  >   If X isNumber, PUSH X onto Numbers
      If X isOperator,
          R = POP Numbers
          L = POP Numbers
          Evaluate L X R; PUSH result onto Numbers

⇒ Push X
9 / ((2 * 4) - 5) = 9 2 4 * 5 - / ~

Numbers = [9 2 4
X = *
L =
R =

While not end-of-expression
do:
> Read X
   If X isNumber, PUSH X onto Numbers
   If X isOperator,
      R = POP Numbers
      L = POP Numbers
      Evaluate L X R; PUSH result onto Numbers

⇒ Read X
\[ 9 / ((2 * 4) - 5) = 9 \ 2 \ 4 \ * \ 5 \ - \ / \]

\[
\text{Numbers} = [9 \ 2  \\
X = *  \\
L =  \\
R = 4
\]

While not end-of-expression
do:
    Read X
    If X isNumber, PUSH X onto Numbers
    If X isOperator,
        > R = POP Numbers
        L = POP Numbers
        Evaluate L X R; PUSH result onto Numbers

⇒ Ah! X is an operator, pop the top element save into R
9 / ((2 * 4) - 5) = 9 2 4 * 5 - /

Numbers = [9
X = *
L = 2
R = 4

While not end-of-expression
do:
  Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    > L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ Top element is removed and saved into L
\[ 9 / ((2 \times 4) - 5) = 9 \ 2 \ 4 \ * \ 5 \ - \ / \]

Numbers = [9 8
X = *
L = 2
R = 4

While not end-of-expression
do:
Read X
If X isNumber, PUSH X onto Numbers
If X isOperator,
R = POP Numbers
L = POP Numbers
> Evaluate L X R; PUSH result onto Numbers

⇒ Push the result of L X R, 2 \times 4 = 8, onto the stack
\[ 9 / ((2 \times 4) - 5) = 9 \quad 2 \quad 4 \quad \times \quad 5 \quad - \quad / \]

Numbers = [9 8
X = 5
L = 2
R = 4

While not end-of-expression
do:
> Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ Read X
\[
9 \div ((2 \times 4) - 5) = 9 \ 2 \ 4 \times 5 - / \\
\]

Numbers = [9 8 5
X = 5
L = 2
R = 4

While not end-of-expression
do:
    Read X
>
    If X isNumber, PUSH X onto Numbers
    If X isOperator,
        R = POP Numbers
        L = POP Numbers
        Evaluate L X R; PUSH result onto Numbers

⇒ Push X
\[
\frac{9}{(2 \times 4) - 5} = 9 \times 2 \times 4 + 5 - /
\]

Numbers = [9 8 5

X = -

L = 2

R = 4

While not end-of-expression
do:
> Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ Read X
$9 \div ((2 \times 4) - 5) = 9 \ 2 \ 4 \times 5 - \div$

Numbers = [9 8
X = -
L = 2
R = 5

While not end-of-expression
do:
   Read X
   If X isNumber, PUSH X onto Numbers
   If X isOperator,
   >   R = POP Numbers
   >   L = POP Numbers
   >   Evaluate L X R; PUSH result onto Numbers

⇒ Remove the top element and save it into R
\[
9 / ((2 \times 4) - 5) = 9 2 4 \times 5 - / \\
\]

Numbers = [9
X = -
L = 8
R = 5

While not end-of-expression
  do:
    Read X
    If X isNumber, PUSH X onto Numbers
    If X isOperator,
      R = POP Numbers
      >
      L = POP Numbers
      Evaluate L X R; PUSH result onto Numbers

⇒ Remove the top element and save it into L
\[
9 / ((2 * 4) - 5) = 9 \ 2 \ 4 * 5 - /
\]

Numbers = [9 3
X = -
L = 8
R = 5

While not end-of-expression
do:

Read X
If X isNumber, PUSH X onto Numbers
If X isOperator,
  R = POP Numbers
  L = POP Numbers
>
Evaluate L X R; PUSH result onto Numbers

⇒ Push the result of L X R, 8 - 5 = 3, onto the stack
\[
9 \div ((2 \times 4) - 5) = 9 \begin{array}{c}2 \times 4 \div 5 \end{array}
\]

Numbers = [9 3
X = /
L = 8
R = 5

While not end-of-expression
do:
> Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ Read X
\[
9 / ((2 \times 4) - 5) = 9 2 4 \times 5 - /
\]

Numbers = [9
X = /
L = 8
R = 3

While not end-of-expression
do:
  Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    > R = POP Numbers
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers

⇒ R = POP Numbers.
\[
9 \div ((2 \times 4) - 5) = 9 \quad 2 \quad 4 \times 5 - \div
\]

\[
\text{Numbers} = [\text{X} = \div, \text{L} = 9, \text{R} = 3]
\]

While not end-of-expression do:
\[
\text{Read X}
\]
\[
\text{If X isNumber, PUSH X onto Numbers}
\]
\[
\text{If X isOperator,}
\]
\[
\text{R} = \text{POP Numbers}
\]
\[
> \quad \text{L} = \text{POP Numbers}
\]
\[
\text{Evaluate L X R; PUSH result onto Numbers}
\]

\[
\Rightarrow \text{L} = \text{POP Numbers}.
\]
9 \div ((2 \times 4) - 5) = 9 2 4 \times 5 - /

Numbers = [3
X = /
L = 9
R = 3

While not end-of-expression
do:
  Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
  > Evaluate L X R; PUSH result onto Numbers

⇒ Push L X R, 9 \div 3 = 3, onto the stack sur la pile.
\[ 9 \div ((2 \times 4) - 5) = 9 \frac{2}{4} \times 5 - / \]

Numbers = [3
X = /
L = 9
R = 3

While not end-of-expression
  do:
    Read X
    If X isNumber, PUSH X onto Numbers
    If X isOperator,
      R = POP Numbers
      L = POP Numbers
    > Evaluate L X R; PUSH result onto Numbers

⇒ End-of-expression
\[
9 / ((2 * 4) - 5) = 9 \quad 2 \quad 4 \quad * \quad 5 \quad - \quad / \\
\]

Numbers = [
X = /
L = 9
R = 3

While not end-of-expression
do:
Read X
If X isNumber, PUSH X onto Numbers
If X isOperator,
   R = POP Numbers
   L = POP Numbers
   Evaluate L X R; PUSH result onto Numbers
>
⇒ The result is “POP Numbers = 3”; the stack is now empty
Problem

Rather than evaluating an RPN expression, we would like to convert an RPN expression to infix (usual notation).
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Hum?
Problem

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Hum?

Do we need a new algorithm?
Problem

Rather than evaluating an RPN expression, we would like to convert an RPN expression to infix (usual notation).

Hum?

Do we need a new algorithm?

No, a simple modification will do, replace “Evaluate L OP R” by “Concatenate (L OP R)”.

Note: parentheses are essential (not all of them but some are).

This time the stack does not contain numbers but character strings that represent sub-expressions.
String rpnToInfix(String[] tokens)

Numbers = [
X =
L =
R =

While not end-of-expression
do:
    Read X
    If X isNumber, PUSH X onto Numbers
    If X isOperator,
        R = POP Numbers
        L = POP Numbers
        Concatenate ( L X R ); PUSH result onto Numbers
While not end-of-expression
  do:
    Read X
    If X isNumber, PUSH X onto Numbers
    If X isOperator,
      R = POP Numbers
      L = POP Numbers
      Process L X R; PUSH result onto Numbers

We’ve seen an example where 'Process == Evaluate', then one where 'Process == Concatenate', but Process could also produce assembly code (i.e. machine instructions).

This shows how programs are compiled or translated.
⇒ Schematic and simplified representation of the memory during the execution of a Java program.
Method call

The Java Virtual Machine (JVM) must:
Method call

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1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
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Method call

The Java Virtual Machine (JVM) must:

1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
Method call

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1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
4. Copy the values of the effective parameters into the designated area of the current activation record;
Method call

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1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
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3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
4. Copy the values of the effective parameters into the designated area of the current activation record;
5. Initial the local variables;
Method call

The Java Virtual Machine (JVM) must:

1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
4. Copy the values of the effective parameters into the designated area of the current activation record;
5. Initial the local variables;
6. Start executing the instruction designated by the program counter.
Method call

The Java Virtual Machine (JVM) must:

1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
4. Copy the values of the effective parameters into the designated area of the current activation record;
5. Initial the local variables;
6. Start executing the instruction designated by the program counter.

⇒ activation block = stack frame, call frame or activation record.
When the method ends

The JVM must:
When the method ends

The JVM must:

1. Save the return value (at the designated space)
When the method ends

The JVM must:

1. Save the return value (at the designated space)
2. Return the control to the calling method, i.e. set the program counter and basePtr back to their previous value;
When the method ends

The JVM must:

1. Save the return value (at the designated space)
2. Return the control to the calling method, i.e. set the program counter and basePtr back to their previous value;
3. Remove the current block;
When the method ends

The JVM must:

1. Save the return value (at the designated space)
2. Return the control to the calling method, i.e. set the program counter and basePtr back to their previous value;
3. Remove the current block;
4. Execute instruction designated by the current value of the program counter.
public class Calls {
    public static int c( int v ) {
        int n;
        n = v + 1;
        return n;
    }
    public static int b( int v ) {
        int m,n;
        m = v + 1;
        n = c( m );
        return n;
    }
    public static int a( int v ) {
        int m,n;
        m = v + 1;
        n = b( m );
        return n;
    }
}
public static void main( String[] args ) {
    int m,n;
    m = 1;
    n = a( m );
    System.out.println( n );
}
Example 1 (simplified)
Example 1 (simplified)
Example 1 (simplified)

```
Example 1 (simplified)
```
Example 1 (simplified)
Example 1 (simplified)
Example 1 (simplified)
Example 1: summary

c:  
\[
\begin{array}{c}
v \\ \ \ \ \ \ \ 3 \\
4 \\
4 \\
\end{array}
\]

b:  
\[
\begin{array}{c}
v \\ \ \ \ \ \ \ 2 \\
4 \\
4 \\
\end{array}
\]

a:  
\[
\begin{array}{c}
v \\ \ \ \ \ \ \ 1 \\
4 \\
4 \\
\end{array}
\]

main:  
\[
\begin{array}{c}
args \\
m \\ \ \ \ \ \ \ 1 \\
n \\ \ \ \ \ \ \ 4 \\
\end{array}
\]
Example 2 (with a program counter)

```java
01 public class Fact {
02     public static int fact( int n ) {
03         // pre-condition: n >= 0
04         int a, r;
05         if ( n == 0 || n == 1 ) {
06             a = 1;
07         } else {
08             r = fact( n-1 );
09             a = n * r;
10         }
11         return a;
12     }
13     public static void main( String[] args ) {
14         int a, n;
15         n = 3;
16         a = fact( n );
17     }
18 }
```
<table>
<thead>
<tr>
<th>Function</th>
<th>Args</th>
<th>Program Counter</th>
</tr>
</thead>
<tbody>
<tr>
<td>main</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
main   args
n      3
a
program counter  16
return address

3

args

program counter 16

fact

n

a

r

return address

main

args

n

3

a

program counter 16

fact(3)
<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>fact</td>
<td>n</td>
</tr>
<tr>
<td>args</td>
<td>3</td>
</tr>
<tr>
<td>main</td>
<td></td>
</tr>
<tr>
<td>program counter</td>
<td>16</td>
</tr>
</tbody>
</table>

fact(3)
```
return address

3

main

args

3

program counter

16

fact

n

3

ar

r

return address

16

fact(3)

```
fact
n
3
a
r
return address
16

main
args
n
3
a

program counter
8

fact(3)
return address
main
args
program counter

fact
n
a
r
fact
n
a
r

return address
16

fact
3

fact
2

fact
3

program counter
8
return address 16

main args n 3 a

program counter 8

fact n 3 a r

fact n 2 a r

fact(2)
main

args

n

3

program counter

8

return address

8

fact

n

3

return address

16

fact

n

2

fact(3)

fact(2)
main

args

n 3

n 3

n 2

n 3

program counter 5

return address 8

return address 16

fact(3)

fact(2)
function fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)

main:
args = 3

fact(3)
```c
void fact(int n) {
    if (n > 1) {
        fact(n-1);
    }
    printf("%d\n", n);
}

int main() {
    int args[] = {3, 2, 1};
    int n = args[0];
    printf("Enter the number: ");
    scanf("%d", &n);
    fact(n);
    return 0;
}
```
main

args

3

program counter

8

fact

n
a
r

return address

n
a
r

return address

n
a
r

return address

fact

n
a
r

return address

fact

n
a
r

return address

main

args

3

program counter

8
Main

Arguments

\texttt{fact(3)}

\texttt{fact(2)}

\texttt{fact(1)}

Program Counter

Return Address

\texttt{8}
<table>
<thead>
<tr>
<th>fact</th>
<th>n</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>return address</td>
<td>8</td>
<td>fact(1)</td>
</tr>
<tr>
<td>fact</td>
<td>n</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>return address</td>
<td>8</td>
<td>fact(2)</td>
</tr>
<tr>
<td>fact</td>
<td>n</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>return address</td>
<td>16</td>
<td>fact(3)</td>
</tr>
<tr>
<td>main</td>
<td>args</td>
<td></td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>program counter</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
main

args

n

3

program counter

6

fact

n

1

return address

8

fact

n

2

return address

8

fact

n

3

return address

16

fact(1)
fact(2)
fact(3)

main

fact

n

3

return address

8

program counter

6
```
main
  args
    n  3
    a
  return address
    8
  program counter
    11
  fact
    n  1
    a  1
    return address
    8
  fact
    n  2
    a
    return address
    8
  fact
    n  3
    a
    return address
    16
```

```java
fact(3)
fact(2)
fact(1)
```
main

args

n

3

program counter

11

return address

fact

n

1

n

1

fact(1)

n

2

fact(2)

n

3

fact(3)

return address

16

return address

8

return address

8

fact

n

1

fact

n

2
program counter  8
main  args  n  3
    return address  16
fact  n  3
    return address  8
fact( 3 )
    return address  8
fact  n  2
    return address  8
fact( 2 )
fact( 1 )
    return address  8
return address  8
return address  8

fact  n  1
    return address  8
fact( 1 )

main  args  n  3

fact

program counter  8

fact

program counter  8

fact

program counter  8

fact

program counter  8

fact

program counter  8

fact

program counter  8

fact
main

fact

fact(3)

fact(2)

program counter 8

args

n 3

return address 16

return address 8

return address 8

fact n 3

fact n 2

1
```c
main(argc, argv)
{
    int n = 3;
    int fact(int n)
    {
        if (n == 0 || n == 1)
            return 1;
        else
            return n * fact(n - 1);
    }
    printf("%d\n", fact(n));
}
```
1
fact  n  2
    a  2
    r  1
return address  8

fact  n  3
    a
    r
return address  16

main  args
    n  3
    a

program counter  11

fact(2)
fact(3)
main

fact

fact(3)

fact(2)

return address

program counter

n

3

return address

n

3

args

n

2

2

1

8

11
2

fact

return address 16

main args

program counter 8

fact(3)
return address

3

2

fact

n
3

a

2

r

16

return address

fact(3)

main

args

n
3

a

program counter

9
return address: 16

main

args

n: 3
a:

program counter: 11

fact

n: 3
a: 6
r: 2

fact(3)
program counter

main

args

return address

n

3

2

r

a

n

3

fact

2

6

fact(3)
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>main</td>
<td>args</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>program counter</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
6

main    args
n   3
a   6

program counter  17