CSI 5165 Combinatorial Algorithms | Fall 2011
Computer Science | University of Ottawa

Homework Assignment #1 (100 points, weight 15%)
Due: October 12 at 10:00 a.m. (in lecture)

Generating elementary combinatorial objects

1. (20 points) **Simple practice with combinatorial generation algorithms**
   Calculate the result for the following operations. Show your work.

   - Subsets:
     Give the SUCCESSOR and the RANK of 11010110 in the Gray code G^8.
   - k-subsets:
     Give RANK of $\{3, 6, 7, 9\}$ considered as a 4-subset of $\{1, \dots, 13\}$ in lexicographic and revolving-door order. What is the SUCCESSOR in each of these orders?
   - Permutations:
     Find the rank and successor of the permutation $[2, 4, 6, 7, 5, 3, 1]$ in lexicographic and Trotter-Johnson order.

     UNRANK the rank $r = 54$ as a permutation of $\{1, 2, 3, 4, 5\}$, using the lexicographic and Trotter-Johnson order.

2. (20 points) **Gray codes and revolving door ordering**

   Suppose $1 \le k \le n$, and we delete all vectors in the binary reflected Gray code G^n that do not correspond to subsets of cardinality k. Prove that the vectors that remain comprise the vectors in the revolving door ordering $A^{n,k}$.

3. (30 points) **Generating all variations of a multiset**
   The variations of a multiset are the permutations of all its sub-multisets.
   Example: the variations of multiset $\{1, 2, 2, 3\}$ are:

   ϵ, 1, 12, 122, 1223, 123, 1232, 13, 132, 1322
   2, 21, 212, 2123, 213, 2132, 22, 221, 2213, 223, 2231, 23, 231, 2312, 232, 2321
   3, 31, 312, 3122, 32, 321, 3212, 322, 3221

   Give an algorithm (pseudocode) to generate all variations of a multiset.
   To verify that your algorithm works, you may implement it and test it; however the implementation is not required for the assignment.

   Hint: You may use a similar idea as the one for lexicographical order of permutations.

4. (30 points) **Generalized Gray codes**

   (a) Let $m_0, m_1, \dots m_{n-1}$ be integer numbers greater than or equal 2. In this exercise we want to generate all n-tuples $(a_{n-1}, a_{n-2}, \dots, a_1, a_0)$ where $0 \le a_j < m_j$ for all j, $0 \le j < n$, according to the following minimal change ordering: two successive tuples differ in exactly one component with the absolute value of their difference equals to 1 (i.e. the component is either incremented or decremented by 1). Adapt the binary reflected Gray code successor algorithm to the case of this generalized Gray code. Give your algorithm in pseudocode form.

   (b) Given the prime factorization of a number $p_1^{e_1} p_2^{e_2} \cdots p_t^{e_t}$, give an algorithm to run through all divisors of the number, by repeatedly multiplying or dividing by a single prime at each step.
   Hint: Use the algorithm developed in part 1.