1. (20 points) Let IndependentSet denote the decision problem that asks, given a graph $G = (V, E)$ and an integer $k$, whether $G$ has an independent set of cardinality at least $k$. (Recall the definition of independent sets given in class and in the textbook).

Prove that if IndependentSet $\in \mathcal{P}$, then the problem of listing the vertices of an independent set of maximum cardinality (FindMaxIndependentSet) can be solved in polynomial time.

*Hint: Design an algorithm for FindMaxIndependentSet that performs a polynomial number of steps plus makes a polynomial number of calls to the algorithm that solves IndependentSet. Justify that your algorithm does the promised job (i.e. is correct and runs in polytime).*

2. (20 marks) For each of the following statements about problems and class of problems discussed in lecture, specify whether the statement is “true” or “false”; briefly justify your answer. You may use any known results.

(a) For all problems $X \in \mathcal{NP}$, we have $X \leq_p 3D-Matching$.

(b) Given the current knowledge on the theory of NP-completeness, it is still possible that $\text{HamCycle} \in \mathcal{P}$ and $\text{VertexCover} \notin \mathcal{P}$.

(c) If $\mathcal{P} = \mathcal{NP}$ then $\text{ShortestPath}$ is NP-complete.

(d) If $\text{VertexCover} \in \mathcal{P}$ then $\text{HamCycle} \in \mathcal{P}$.

(e) (Requires extra research) Suppose that $\mathcal{P} \neq \mathcal{NP}$. Then for any problem $X \in \mathcal{NP}$ then, either $X \in \mathcal{P}$ or $X$ is NP-complete.

3. (30 points) Textbook chapter 8 exercise 6: Show that MSFTV is NP-complete (the acronym stand for Monotone Satisfiability with Few True Values). See details of the exercise in page 507.

*Hint: Remember to justify why the problem is in $\mathcal{NP}$, and then prove that an NP-complete problem $X$ of your choice polynomial-time reduces to MSFTV ($X \leq_p \text{MSFTV}$).*

4. (30 points) Textbook chapter 8 exercise 21: Show that FullyCompatibleConfiguration is NP-complete. See details of the exercise in page 516-517.

*Hint: Remember to justify why the problem is in $\mathcal{NP}$, and then prove that an NP-complete problem $X$ of your choice polynomial-time reduces to FullyCompatibleConfiguration ($X \leq_p \text{FullyCompatibleConfiguration}$).*