CSI 4105 Design and Analysis of Algorithms II — Winter 2011
Computer Science — University of Ottawa

Homework Assignment #1 (100 points, weight 5%)
Due: Wednesday Feb 2, at 1:00 p.m. (in lecture)

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1. (20 points) Let IndependentSet denote the decision problem that asks, given a graph $G = (V, E)$ and an integer k, whether G has an independent set of cardinality at least k. (Recall the definition of independent sets given in class and in the textbook).

   Prove that if IndependentSet $\in \mathcal{P}$, then the problem of listing the vertices of an independent set of maximum cardinality (FindMaxIndependentSet) can be solved in polynomial time.

   Hint: Design an algorithm for FindMaxIndependentSet *that performs a polynomial number of steps plus makes a polynomial number of calls to the algorithm that solves* IndependentSet. *Justify that your algorithm does the promised job (i.e. is correct and runs in polytime).*

2. (20 marks) For each of the following statements about problems and class of problems discussed in lecture, specify whether the statement is "true" or "false"; **briefly justify your answer**. You may use any known results.

   (a) For all problems $X \in \mathcal{NP}$, we have $X \leq_P$ 3D-Matching.

   (b) Given the current knowledge on the theory of NP-completeness, it is still possible that HamCycle $\in P$ and VertexCover $\notin P$.

   (c) If $\mathcal{P} = \mathcal{NP}$ then ShortestPath is NP-complete.

   (d) If VertexCover $\in \mathcal{P}$ then HamCycle $\in \mathcal{P}$.

   (e) (Requires extra research) Suppose that P $\neq$ NP. Then for any problem $X \in \mathcal{NP}$ then, either $X \in P$ or X is NP-complete.

3. (30 points) Textbook chapter 8 exercise 6: Show that MSFTV is NP-complete (the acronym stand for Monotone Satisfiability with Few True Values). See details of the exercise in page 507.

   Hint: Remember to justify why the problem is in $\mathcal{NP}$, and then prove that an NP-complete problem X of your choice polynomial-time reduces to MSFTV *($X \leq_P$* MSFTV*).*

4. (30 points) Textbook chapter 8 exercise 21: Show that FullyCompatibleConfiguration is NP-complete. See details of the exercise in page 516-517.

   Hint: Remember to justify why the problem is in $\mathcal{NP}$, and then prove that an NP-complete problem X of your choice polynomial-time reduces to FullyCompatibleConfiguration *($X \leq_P$* FullyCompatibleConfiguration*).*