CSI 4105 Design and Analysis of Algorithms II — Winter 2010
Computer Science — University of Ottawa

Homework Assignment #1 (100 points, weight 5%)
Due: Wednesday Jan 27, at 1:30 p.m. (in lecture)

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1. (20 points) Let Ham-Cycle denote the problem that establishes whether a given undirected graph $G = (V, E)$ has a hamiltonian cycle. Recall that a hamiltonian cycle is a simple cycle that contains every vertex in V.

   Prove that if Ham-Cycle $\in \mathcal{P}$, then the problem of listing (in order) the vertices of a hamiltonian cycle (if one exists), Ham-Cycle-Search, can be solved in polynomial time.

   Hint: Design an algorithm for Ham-Cycle-Search *that performs a polynomial number of steps plus makes a polynomial number of calls to the algorithm that solves/decides* Ham-Cycle.

2. (20 points) Let X be an arbitrary (decision) problem in $\mathcal{NP}$. Show that X can be solved/decided by an algorithm that runs in time at most $2^{p(n)}$ for an instance of size n, where $p(n) \in O(n^k)$, for some constant k.

   Hint: Use the definition of $\mathcal{NP}$.

3. (35=5+15+15 points) (Based on textbook chapter 8, exercise 1)
   Let us define the decision version of the Interval Scheduling Problem from Chapter 4 as follows.
   Interval Scheduling: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k.

   (a) Read section 4.1, and briefly describe how the algorithm that solves the Interval Scheduling Problem (maximization version) can be adapted to solve Interval Scheduling. What is the running time of the resulting algorithm ?

   For each of the two questions below, decide whether the answer is (i) "Yes", (ii) "No", or (iii) "Unknown, because it would resolve the question whether $\mathcal{P} = \mathcal{NP}$".

   (b) Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

   (c) Is it the case that Independent Set $\leq_P$ Interval Scheduling ?

4. (25 points)Textbook chapter 8 exercise 2: Show that Diverse Subset is NP-complete. See details of the exercises in page 505.

   Hint: Remember to justify why the problem is in $\mathcal{NP}$*, and then prove that an NP-complete problem* X *of your choice polynomial-time reduces to* Diverse Subset *(*$X \leq_P$ Diverse Subset*).*