1. (20 points) **DNF-SAT is polynomial-time solvable**
   A formula is in disjunctive normal form (DNF) if it is a disjunction (or’s) of clauses that are conjunctions (and’s) of literals. Note that no assumption is made about the number of variables in each clause.
   Example: $\Phi = (x_1 \land x_2 \land \neg x_1 \land x_3) \lor (x_2 \land x_3 \land \neg x_4) \lor (x_1 \land \neg x_1)$ is satisfiable.
   Prove that the problem of determining the satisfiability of a boolean formula in disjunctive normal form is polynomial-time solvable. (To prove that you have to provide an algorithm to solve the problem and show that it runs in polynomial time).

2. (30 points) **0-1 Integer Programming Problem is NP-complete**
   Given an integer $m \times n$ matrix $A$ and an integer $m$-vector $b$, the **0-1 integer programming problem** asks whether there is an integer $n$-vector $x$ with elements in the set $\{0, 1\}$ such that $Ax \geq b$.
   Note that the problem is simply asking whether the following system of equations have a solution with each $x_j \in \{0, 1\}$, $1 \leq i \leq n$:
   
   $ \begin{align*}
a_{1,1}x_1 + a_{1,2}x_2 + \ldots a_{1,n}x_n & \geq b_1, \\
a_{2,1}x_1 + a_{2,2}x_2 + \ldots a_{2,n}x_n & \geq b_2, \\
\vdots & \vdots \\
a_{m,1}x_1 + a_{m,2}x_2 + \ldots a_{m,n}x_n & \geq b_m.
\end{align*} $ 

1. Define the language associated to the 0-1 integer programming problem.
2. Prove that the 0-1 integer programming problem is NP-complete.
   Hint: reduce from 3-CNF-Sat

3. (30 points) **HamPath is NP-complete**
   A **hamiltonian path** in a graph is a simple path that visits every vertex exactly once. Let $\text{HamPath}=\{<G,u,v>: \text{there is a hamiltonian path from } u \text{ to } v\}$. Show that $\text{HamPath}$ is NP-complete. Hint: reduce from $\text{HAMCYCLE}$.

4. (20 points) **HamPath for directed acyclic graphs is polynomial-time solvable**
   A directed graph is called **acyclic** if it does not contain a directed cycle. Show that the hamiltonian path problem can be solved in polynomial time on directed acyclic graphs. This means that you need to show that the language $\text{HamPathDirAcycl}=\{<G,u,v>: G \text{ is a directed acyclic graph and there is a hamiltonian path from } u \text{ to } v\}$ is in $P$.
   Give an efficient algorithm for the decision problem and analyse its complexity.
   Hint: Review Dijkstra’s algorithm for shortest path and note that it works with negative weights for directed acyclic graphs.