CSI 4105 Midterm 2001 (Format was compressed to fit 2 pages)

1. Turing Machines (9 points)

Consider a Turing machine M with input alphabet $\Sigma = \{0, 1\}$ and machine alphabet $\Gamma = \{0, 1, \sqcup\}$, and transition function depicted in the following diagram:

Part A (6 points) Give L, the language *accepted* by M. **Part B** (3 points) Does M decide L? Justify.

2. Short answers (21 points) For each of the statements below choose TRUE or FALSE and briefly justify your answer. Note: You may use any result shown in class or in homework without proving it.

- NP \cap co-NP $= \emptyset$
- if Ham-Cycle $\in \mathbf{P}$ then Vertex-Cover $\in \mathbf{P}$
- if SAT $\notin \mathbf{P}$ then CLIQUE $\notin \mathbf{P}$
- Shortest-Path $\in \mathbf{P}$
- if SHORTEST-PATH \in NPC then P = NP.
- It is possible that there exists a RAM program that decides a language L_1 in polynomial time, but every Turing machine that decides L_1 runs in exponential time.
- If there exists a polynomial-time algorithm that **accepts** a language L_2 , then there exists a polynomial-time algorithm that **decides** L_2 .

3. Polynomial-time reducibility (10 points)

Prove that \leq_P relation is a transitive relation on languages. That is, prove that: if $L_1 \leq_P L_2$ and $L_2 \leq_P L_3$ then $L_1 \leq_P L_3$. Hint: Use the definition of \leq_P .

4. NP-completeness reductions (20 points)

In this question, you are asked to apply some reductions discussed in class to the examples given. For language L_1 and L_2 shown in each part below, we have seen a reduction algorithm that shows that $L_1 \leq_P L_2$; you should apply the algorithms discussed in class.

Part A (5 points) CIRCUIT-SAT \leq_P SAT Consider the following instance for the circuit satisfiability problem (CIRCUIT-SAT): circuit C_1 :

• Give the corresponding instance for the formula satisfiability problem (SAT), in accordance with the reduction algorithm for CIRCUIT-SAT \leq_P SAT.

• Does $< C_1 > \in$ CIRCUIT-SAT ? Justify.

Part B (5 points) HAM-CYCLE \leq_P TSP

Consider the following instance for the hamiltonian cycle problem (HAM-CYCLE):

 G_1 :

- Give the corresponding instance for the traveling salesman problem (TSP), in accordance with the reduction algorithm for HAM-CYCLE \leq_P TSP.
- Does $\langle G_1 \rangle \in$ HAM-CYCLE ? Justify.

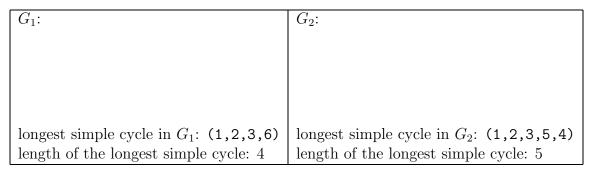
Part C. (10 points) VERTEX-COVER \leq_P HAM-CYCLE Consider the following instance for the vertex-cover problem (VERTEX-COVER):

$$k=2,$$
 G_2 :

- Give the corresponding instance for the hamiltonian cycle problem (HAM-CYCLE), in accordance with the reduction algorithm for VERTEX-COVER \leq_P HAM-CYCLE.
- Outline the hamiltonian cycle (on the graph you have drawn above) that corresponds to the vertex cover $C = \{v_1, v_2\}$ of G_2 .

5.(40 points) Longest simple cycle is in NPC

The **longest-simple-cycle** problem is the problem of determining a simple cycle (no repeated vertices) of **maximum** length in a graph. For example:



Part A.(2) State the **decision problem** associated with the longest-simple-cycle problem: **Part B.** (3) Give the language corresponding to the decision problem given in part A: LSC =**Part C.** (35) Prove that LSC is NP-complete, by proving that:

- (10) LSC \in NP.
- (25) LSC is NP-hard. (Hint: reduce from HAM-CYCLE)