CSI 2101 Discrete Structures | Winter 2012
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Quiz #9

1. For what values $n \geq 3$ do the following graphs contain Euler circuits and Hamilton circuits?

   (a) K_n:

   Euler Since the graph contains all possible edges, a vertex is connected to all the others, so it has degree $n-1$. Thus, if n is even, $n-1$ is odd, so there is no Euler circuit in this case. However, if n is odd, then $n-1$ is even, so for n even, we can always find a Euler circuit.

   Ham There is always a Hamilton cycle. In fact, any arbitrary permutation of the vertices is a Hamilton cycle since every pair of vertices form an edge.

   (b) C_n:

   Euler Regardless of n, every vertex has degree 2, so the graph has an Euler citcuit.

   Ham For all n, we have a Hamilton circuit: the graph is essentially a Hamilton circuit: just listing the vertices in order (starting at any vertex) will give a Hamilton cycle.

   (c) W_n:

   Euler No such graph contains an Euler circuit, since every vertex other than the hub (central) vertex will always have degree 3.

   Ham The graphs always contain a Hamilton circuit. Start at the hub, pick any vertex on the rim and then traverse all vertices along the rim in order. Then we can return to the hub, which gives a Hamilton circuit.