1. Use a loop invariant to prove that the following program segment for computing the $n$th power (where $n$ is a positive integer) of a real number $x$ is correct.

```
power = 1
i = 1
while $i \leq n$ do
    power = power $\times$ $x$
    $i = i + 1$
end while
```

Let $p$ be the assertion “$i \leq n + 1$ and $power = x^{i-1}$”. We must first show that $p$ is a loop invariant: to do so, we have to show that if $p$ is true at the beginning of an execution of the loop, then $p$ is still true after the execution of the loop.

Suppose that, at the beginning of one execution of the while loop, $p$ is true and the condition of the while loop holds: in other words, we assume that $power = i^{n-1}$ and that $i \leq n$. The new values $i_{new}$ and $power_{new}$ are:

\[
\begin{align*}
    i_{new} &= i + 1 \\
    power_{new} &= power \times x = x^{i-1} \times x = x^i = x^{i_{new}-1}
\end{align*}
\]

Because $i \leq n$, we also have that $i_{new} \leq n + 1$. Thus, $p$ is still true at the end of the execution of the loop.

We now must show that $p$ was true before the loop executed. In the program header, we set $i = 1$. Since, by assumption, $n$ is a positive integer, then $n \geq 1$, so we have that $i \leq n + 1$. We also have $power = 1 = x^0 = x^{i-1}$. Thus, $p$ is true before the loop is executed.

Because $p$ is a loop invariant, we have that when the loop terminates, it terminates with $p$ true and the condition of the loop, $i \leq n$ false. Thus, $i = n + 1$, and so $power = x^{(n+1)-i} = x^n$, which is the result that we want, so the program calculates correctly.

Finally, we need to check that the loop actually does terminate. The value of $i$ is initialized to 1, and with every execution of the loop, is incremented by 1. Thus, the loop will terminate after $n$ iterations.

Thus, the program is correct.