CSI 2101 Discrete Structures — Winter 2012
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Quiz #6

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1. **S4.3, Exercise 43:** For a full binary tree T, use structural induction to show that $n(T) \geq 2h(T) + 1$.

   Basis step: Let R be the full binary tree consisting solely of a root node. We have that $h(R) = 0$. Thus $2h(R) + 1 = 2 \cdot 0 + 1 = 1$, and as $n(R) = 1 \geq 1$, the claim holds for R.

   Inductive hypothesis: Let T_1 and T_2 be full binary trees and assume $n(T_1) \geq 2h(T_1) + 1$ and $n(T_2) \geq 2h(T_2) + 1$.

   Inductive step: Let $T = T_1 \cdot T_2$ be a full binary tree. We must show that the expression holds for T. We have that:

$$
\begin{aligned}
n(T) &= n(T_1) + n(T_2) + 1 \\
&\geq (2h(T_1) + 1) + (2h(T_2) + 1) + 1 && \text{by the inductive hypothesis} \\
&= 2(h(T_1) + h(T_2) + 1) + 1
\end{aligned}
$$

   For any two nonnegative integers x and y, we have that $x + y \geq \max(x, y)$. Thus, we have:

$$
\begin{aligned}
n(T) &\geq 2(\max(h(T_1), h(T_2)) + 1) + 1 \\
&= 2h(T) + 1 && \text{since } h(T) = \max(h(T_1), h(T_2)) + 1
\end{aligned}
$$

   Thus, the claim holds, as shown by structural induction.