1. **S4.1, Exercise 31:** Use induction to show that $2|(n^2 + n)$ for all positive integers $n$.

Let $P(n)$ be the statement: $2|(n^2 + n)$.

**Basis step:** We show that $P(1)$ is true: $(1)^2 + 1 = 2$, and $2|2$.

**Inductive hypothesis:** Let $k$ be a positive integer and assume $P(k)$ is true, i.e. $2|(k^2 + k)$. This means that there exists an integer $j$ such that $k^2 + k = 2j$.

**Inductive step:** Show that $P(k+1)$ is true, i.e. $2|((k+1)^2 + (k+1))$. We have that:

\[
(k + 1)^2 + (k + 1) = k^2 + 2k + 1 + k + 1 \\
= (k^2 + k) + 2k + 2 \\
= 2j + 2k + 2 \quad \text{by the inductive hypothesis} \\
= 2(j + k + 1)
\]

Hence, $2|((k + 1)^2 + (k + 1))$.

Thus, for all positive integers $n$, $2|(n^2 + n)$ as shown by induction.