Homework Assignment #3 (100 points, weight 6.25%)
Due: March 29 at 10:00 a.m. (in tutorial)

**Induction and Recursion**: Your best 4 questions will be used to calculate your mark.

1. • (10 points) Exercise 32, page 280 (induction to prove divisibility facts).
   • (15 points) Prove that this is the following recursive algorithm correctly computes $2 - \left(\frac{1}{2}\right)^n$, for all $n \geq 0$.
   
   ```
   procedure $P(n$:nonnegative integer
   if $n = 0$ then return 1
   else return $1 + \frac{1}{2}P(n - 1)$
   ```


3. (25 points) Exercise 64, page 282 (celebrity identification). Show the statement for $n \geq 1$. Note that finding the celebrity with $x$ questions really means doing so with at most $x$ questions.

4. (25 points) Exercise 32 page 309 (structural induction for strings).
   Hint: Use definition 2 (strings) and definition 3 (concatenation of strings). The structural induction can be done based on the definition of strings applied to string $t$.

5. (25 points) (Program Verification) Consider the following iterative program that computes the $n$th Fibonacci number.
   ```
   procedure iterativeFibonacci($n$:nonnegative integer)
   if $n = 0$ then return 0
   else begin
   $x \leftarrow 0$
   $y \leftarrow 1$
   $i \leftarrow 1$
   while $i \leq n - 1$ do begin
   $z \leftarrow x + y$
   $x \leftarrow y$
   $y \leftarrow z$
   $i \leftarrow i + 1$
   end
   return $y$
   end
   ```
   Use program verification techniques (Hoare triples, loop invariants) to prove that the above algorithm correctly computes $f_n$, the $n$th Fibonacci number, for $n \geq 0$. 