Homework Assignment #1 (100 points, weight 6.25%)  
Due: Monday Feb 1, at 10:00 p.m. (in tutorial)

Propositional Logic

1. (12 points) Use logical equivalences to show that \([-p \land (p \lor q)] \rightarrow q\) is a tautology.

2. (12 points) Recall that a collection of logical operators is functionally complete if every compound proposition is logically equivalent to a compound proposition using only these logical operators. The logical operator NOR, denoted by \(\downarrow\), is true when both \(p\) or \(q\) are false, and is false otherwise. Show that \(\{\downarrow\}\) is a functionally complete collection of operators, by proving the following steps.

(a) Use truth tables to show that \(p \downarrow p\) is logically equivalent to \(-p\).

(b) Use truth tables to show that \((p \downarrow q) \downarrow (p \downarrow q)\) is logically equivalent to \(p \lor q\).

(c) Complete the argument by using the fact that \(\{\neg, \lor\}\) is functionally complete.

Predicate Logic

3. (12 points) Exercise 34, page 48. (English-predicates-negate-English)

4. (12 points) Exercise 36, page 49. (Counter examples to universally quantified statements)

5. (16 points) Exercise 32, page 61. Note: show your steps. (Negations of nested quantified statements)

Inference Rules

6. (14 points) Exercise 12, page 73. (Formal proofs)

7. (10 points) Exercise 20, page 74. Please justify a “yes” by referring to a rule of inference applied and a “no” by pointing out the fallacy. (Validity of arguments)

Proof Methods

8. (12 points) Use a proof by contraposition to prove the following:
Let \(x, y\) and \(a\) be real numbers. If \(x + y \geq a\) then \(x \geq a/2\) or \(y \geq a/2\).