CSI2101-2009 - ASSIGNMENT#3

Due date: Thursday March 19 at 12:30 (up to max 24hs late with 10% off)
Hand in method: You may hand in to the TA at the tutorial immediately BEFORE the due date (i.e. tutorial of March 16); or otherwise at the dropoff box at SITE 1st floor.

(1) (26 points) **Correctness of recursive algorithms**
Prove that Algorithm 6 (recursive binary search algorithm) in page 314 (Section 4.4) is correct, as follows. Consider the following statement:

> $P(k)$: " If n is an integer and $a_1, a_2, \ldots, a_n$ are integers in increasing order, and i, j, x are integers such that $1 \leq i \leq n, 1 \leq j \leq n$ and $j - i = k$, then procedure $binarysearch(i, j, x)$ calculates $location$, where $location = 0$ if there exists no l, $i \leq l \leq j$, with $a_l = x$, or $location = m$ and $a_m = x$ with $i \leq m \leq j$, otherwise."

Use induction to prove that $P(k)$ is true for all $k \geq 0$.

(2) (25 points) **Program correctness and verification with loop invariant**
Section 4.5, Exercise 12, page 328. In addition to the partial correctness asked in this exercise, show termination as well.

(3) (25 points) **Structural induction for recursively defined full binary trees.**
Section 4.3, Exercise 44, page 310. For this question, you need to refer to definition 6 (full binary trees) in page 303, and the definitions of leaves and internal vertices of these trees, right above exercise 44.

(4) **Big-Oh, big-Omega and big-Theta notation for function growth**

(a) (12 points, 2 marks per part) Section 3.2, exercise 2, page 191. For each part, answer yes or no. Justify the "yes" by giving the witnesses C and k, as explained at the beginning of this exercise section. You don't need to justify the "no".

(b) (12 points, 2 marks per part) Section 3.2, exercise 22, page 191. For each part and for each of the big-Omega and big-Theta relationships, answer yes or no. You don't need to justify.