CSI2101-2009 - ASSIGNMENT#3

Due date: Thursday March 19 at 12:30 (up to max 24hs late with 10% off)
Hand in method: You may hand in to the TA at the tutorial immediately BEFORE the
due date (i.e. tutorial of March 16); or otherwise at the dropoff box at SITE 1st floor.

(1) (26 points) Correctness of recursive algorithms

Prove that Algorithm 6 (recursive binary search algorithm) in page 314 (Section
4.4) is correct, as follows. Consider the following statement:

\[ P(k) : \text{“If } n \text{ is an integer and } a_1, a_2, \ldots, a_n \text{ are integers in increasing order,}
\text{and } i, j, x \text{ are integers such that } 1 \leq i \leq n, 1 \leq j \leq n \text{ and } j - i = k, \text{ then}
\text{procedure } \text{binarysearch}(i, j, x) \text{ calculates location, where location } = 0 \text{ if}
\text{there exists no } l, i \leq l \leq j, \text{ with } a_l = x, \text{ or location } = m \text{ and } a_m = x
\text{with } i \leq m \leq j, \text{ otherwise.”} \]

Use induction to prove that \( P(k) \) is true for all \( k \geq 0 \).

(2) (25 points) Program correctness and verification with loop invariant

Section 4.5, Exercise 12, page 328. In addition to the partial correctness asked in
this exercise, show termination as well.

(3) (25 points) Structural induction for recursively defined full binary trees.

Section 4.3, Exercise 44, page 310. For this question, you need to refer to definition
6 (full binary trees) in page 303, and the definitions of leaves and internal vertices
of these trees, right above exercise 44.

(4) Big-Oh, big-Omega and big-Theta notation for function growth

(a) (12 points, 2 marks per part) Section 3.2, exercise 2, page 191. For each
part, answer yes or no. Justify the “yes” by giving the witnesses \( C \) and \( k \), as
explained at the beginning of this exercise section. You don’t need to justify
the “no”.

(b) (12 points, 2 marks per part) Section 3.2, exercise 22, page 191. For each part
and for each of the big-Omega and big-Theta relationships, answer yes or no.
You don’t need to justify.