CSI2101-2009 - ASSIGNMENT #1

STUDENT ID: .......................... NAME: ...................................................

Hand in at the assignment drop box for this course at SITE 1st floor by the due dates:
Part 1. Propositional logic: Wednesday, January 28 at 12:30pm
Part 2. Predicate logic: Wednesday, February 4 at 12:30pm.

1. Propositional Logic (30/100 marks)

Instructions for Part 1. Answer these questions in a separate piece of paper, in order.
Put your name and student id in all pages and staple them. (-3 points, if not followed)

(1) (Ex 24 and 26, p.29; 2 marks) Use logical equivalences, to show that:
   • \((p \rightarrow q) \lor (p \rightarrow r)\) and \(p \rightarrow (q \lor r)\) are logically equivalent.
   • \(\neg p \rightarrow (q \rightarrow r)\) and \(q \rightarrow (p \lor r)\) are logically equivalent.

(2) (Ex. 33, p 29, 2 marks) Show that \((p \rightarrow q) \rightarrow (r \rightarrow s)\) and \((p \rightarrow r) \rightarrow (q \rightarrow s)\) are not logically equivalent.

(3) (Ex. 56, p 30, 4 marks) Show that if \(p, q,\) and \(r\) are compound propositions such that \(p\) and \(q\) are logically equivalent and \(q\) and \(r\) are logically equivalent, then \(p\) and \(r\) are logically equivalent.

(4) (Ex. 52, p 29, 8 marks) A collection of logical operators is functionally complete if every compound proposition is logically equivalent to a compound proposition using only these logical operators. You have learned that \(\{\neg, \land\}\) is a functionally complete collection of logical operators. The same is true for \(\{\neg, \lor\}\). The logical operator NAND, denoted by \(|\), is true when either \(p\) or \(q\), or both, are false. Show that \(\{|\}\) is a functionally complete collection of operators.
   Hint: check the steps used for Exercise 50. Note I suspect there is a typo in 50-c, where “Exercise 49” should read “Exercise 45”.

(5) (Ex 60, p 30, 6 marks) Which of these compound propositions are satisfiable/why?
   (a) \((p \lor q \lor \neg r) \land (p \lor \neg q \lor \neg s) \land (p \lor \neg r \lor \neg s) \land (\neg p \lor \neg q \lor \neg s) \land (p \lor q \lor \neg s)\)
   (b) \((\neg p \lor \neg q \lor \neg r) \land (\neg p \lor q \lor \neg s) \land (p \lor \neg q \lor \neg s) \land (\neg p \lor \neg q \lor \neg s) \land (p \lor q \lor \neg r) \land (p \lor \neg r \lor \neg s)\)
   (c) \((p \lor q \lor r) \land (p \lor \neg q \lor \neg s) \land (q \lor \neg r \lor s) \land (\neg p \lor r \lor s) \land (\neg p \lor q \lor \neg s) \land (p \lor \neg q \lor \neg r) \land (\neg p \lor \neg q \lor \neg s) \land (\neg p \lor \neg r \lor \neg s)\)

(6) (8 marks) Consider the boolean function Agreement : \(\{0,1\}^3 \rightarrow \{0,1\}\), which is the function that has value 1 if all three inputs are identical (all are 0 or all are 1).
   (a) Write a truth table for a proposition that represents the boolean function Agreement\((x, y, z)\).
   (b) Find a compound proposition in DNF (disjunctive normal form) that represents the boolean function Agreement\((x, y, z)\).
   (c) Find a compound proposition in CNF (conjunctive normal form) that represents the boolean function Agreement\((x, y, z)\).
   (d) Draw a circuit that computes the boolean function Agreement\((x, y, x)\).
2. Predicate Logic (75/100, due February 4, 12:30pm)

Ex.(7)-(14) are to be solved in the given space in these pages, while 15 and 16 can be handed in a separate page (staple all!) (-7 if not followed)

(7) (Ex. 10, p. 47, 5 marks) Let $C(x)$ be the statement “$x$ has a cat”, let $D(x)$ be the statement “$x$ has a dog” and let $F(x)$ be the statement “$x$ has a ferret”. Express each of the following statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers and logical connectives. Let the domain consist of all students in your class.
(a) A student in your class has a cat, a dog and a ferret.
A:

(b) All students in your class have a cat, a dog, or a ferret.
A:

(c) Some student in your class has a cat and a ferret, but not a dog.
A:

(d) No student in your class has a cat, a dog and a ferret.
A:

(e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet.
A:

(8) (Ex. 12, p. 47, 4*1+2+2+2=10 marks) Let $Q(x)$ be the statement “$x + 1 > 2x$”. If the domain consists of all integers, what are these truth values?
(a) $Q(0)$ true false (circle one)

(b) $Q(-1)$ true false

(c) $Q(1)$ true false

(d) $\exists x Q(x)$ true false
   Justify:

(e) $\forall x Q(x)$ true false
   Justify:

(f) $\exists x \neg Q(x)$ true false
   Justify:

(g) $\forall x \neg Q(x)$ true false
   Justify:
(9) (Ex. 20, p. 47, 1+1+2+2+2=8 marks) Suppose the domain of the propositional function $P(x)$ consists of $-5, -3, -1, 1, 3,$ and $5$. Express these statements without using quantifiers, instead using only negations, disjunctions and conjunctions.

(a) $\exists x P(x)$

(b) $\forall x P(x)$

(c) $\forall x ((x \neq 1) \rightarrow P(x))$

(d) $\exists x ((x \geq 0) \land P(x))$

(e) $\exists x (\neg P(x)) \land \forall x ((x < 0) \rightarrow P(x))$

(10) (Ex. 38, p. 49, 5 marks) Translate these system specifications into English where the predicate $S(x, y)$ is “$x$ is in state $y$” and where the domain for $x$ and $y$ consists of all systems and all possible states, respectively.

(a) $\exists S(x, \text{open})$

(b) $\forall x (S(x, \text{malfunctioning}) \lor S(x, \text{diagnostic}))$

(c) $\exists x S(x, \text{open}) \lor \exists x S(x, \text{diagnostic})$

(d) $\exists x \neg S(x, \text{available})$

(e) $\forall x \neg S(x, \text{working})$

(11) (Ex. 58, p.50, 3 marks) Suppose that Prolog facts are used to define predicates $\text{mother}(M, Y)$ and $\text{father}(F, X)$, which represent that “$M$ is the mother of $Y$” and “$F$ is the father of $X$”, respectively. Give a Prolog rule to define the predicate $\text{grandfather}(X, Y)$, which represents that $X$ is the grandfather of $Y$. [Hint: You can write a disjunction in Prolog either by using a semicolon or by putting these predicates on separate lines.]
(12) (Ex. 12, p. 59, $1+1+2+2+2+2=10$ marks) Let $I(x)$ be the statement “$x$ has an internet connection” and $C(x, y)$ be the statement “$x$ and $y$ have chatted over the internet”, where the domain for the variables $x$ and $y$ consists of all students in your class. Use quantifiers to express each of these statements:

(c) Jan and Sharon never chatted over the internet.

d) No one in the class has chatted with Bob.

e) Sanjay has chatted with everyone except Joseph.

j) Everyone in your class with an internet connection has chatted over the internet with at least one other student in your class.

k) Someone in your class has an internet connection but has not chatted with anyone else in your class.

n) There are at least two students in your class who have not chatted with the same person in your class.

(13) (Ex. 40, p. 62, 6 marks) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

(a) $\forall x \exists y (x = 1/y)$

(b) $\forall x \exists y (y^2 - x < 100)$

(c) $\forall x \forall y (x^2 \neq y^3)$

(14) (Ex. 46, p. 62, 6 marks) Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain of the variables consists of

(a) the positive real numbers.
   Justification:

(b) the integers.
   Justification:

(c) the nonzero real numbers.
   Justification:
(15) (Ex. 32, p.61, 3+3+3+3=12 marks) Express the negations of each of these statements so that all negation symbols immediately precede predicates (that is, no negation is outside a quantifier or an expression involving logical connectives). Show all the steps in your derivation.

(a) $\exists z \forall y \forall x T(x, y, z)$

(b) $\exists x \exists y P(x, y) \land \forall x \forall y Q(x, y)$

(c) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$

(d) $\forall y \exists x \exists z (T(x, y, z) \lor Q(x, y))$

(16) (Ex. 50, p. 50, 5 marks) Show that $\forall x P(x) \lor \forall x Q(x)$ and $\forall x (P(x) \lor Q(x))$ are not logically equivalent.

Hint: In order to do this, it is enough to show an “interpretation” (a choice for domain and meaning for $P(x)$ and $Q(x)$) for which these two statement have a different value (one is false and one is true).