Ottawa-Carleton Discrete Math Day 2006

Plenary Talks

William Cook, Georgia Institute of Technology
Exact Solution of Linear and Integer Programming Problems
Most linear-programming (LP) applications can be modeled with linear inequalities described by rational data. Typical software for these models uses floating-point arithmetic and inexact linear algebra to obtain approximate solutions. In this talk we treat the problem of finding exact rational solutions to LP models using the simplex algorithm. We describe computational results for LP benchmark instances, mixed-integer programming, coding-theory bounds, factoring integers, and the traveling salesman problem. This talk is based on joint work with David Applegate, Sanjeeb Dash, and Daniel Espinoza.

Anthony B. Evans, Wright State University
Latin squares, orthogonal mates, and groups
Given a latin square, does it have an orthogonal mate? We will discuss answers to this question when L is the multiplication table of a group or has a group-like structure.

Johnathan Jedwab, Simon Fraser University
Written on a torus or on a cylinder? An elementary proof of the Barker array conjecture
The existence pattern for Barker sequences arose as a problem in digital sequence design in the 1950s. Although deceptively easy to state, the problem continues to resist solution. It has stimulated the development of a large body of theory, including the merit factor problem in communications engineering and the algebraic study of difference sets in discrete mathematics.

In 1989 Alquaddoomi and Scholtz proposed a generalisation of Barker sequences to two dimensions, conjecturing that no non-trivial examples exist except of size 2×2. I shall present a recent proof of this conjecture. The proof uses only elementary methods. A key conceptual point in the proof is whether to regard the array as being written on a torus or on a cylinder.

The talk will not assume any prior knowledge.

(Joint work with J.A. Davis and K.W. Smith)

Pierre Leroux, Université du Québec à Montréal

Characterizations and enumeration of toroidal $K_{3,3}$-subdivision-free graphs

In his 2003 Ph.D thesis at University of Manitoba, Andrei Gagarin has studied graph embeddability on the projective plane and the torus, from an algorithmic point of view, particularly when avoiding $K_{3,3}$-subdivisions. Building on his results, we have been able to determine completely the structure of projective planar and toroidal $K_{3,3}$-subdivision-free graphs and to enumerate them. Their characterization is expressed in terms of substitution of 2-pole planar networks for the edges of canonically defined non-planar graphs called *projective-planar cores* and *toroidal cores* respectively. Their enumeration (both labelled and unlabelled) is achieved by using methods developed by T. Walsh in 1982 for edge substitutions in the context of 3-connected and homeomorphically irreducible 2-connected graphs.

(Joint work with A.V. Gagarin and G. Labelle)

Kieka Mynhardt, University of Victoria

A conjecture on domination in prisms of graphs

Informally, we consider the following problem. Given an arbitrary graph G, form a new graph πG by joining the vertices of two disjoint copies of G by some matching. It is easy to see that the domination number $\gamma(\pi G)$ lies between $\gamma(G)$ and $2\gamma(G)$. Which graphs always satisfy $\gamma(\pi G) = \gamma(G)$, regardless of the matching used to construct πG? More precisely, are there *any* graphs with nonempty edge sets for which this is true?

We conjecture that the edgeless graphs $\overline{K}_n$ are the only graphs with this property and discuss progress on this conjecture.

Contributed Talks

Robert F. Bailey, Queen Mary, University of London

Error-correcting codes from permutation groups

The traditional setting for error-correcting codes is vector spaces over finite fields; in this talk this is replaced by finite permutation groups. I will discuss some analogies between the two settings (such as the distance enumerator polynomial), and also describe a decoding algorithm for these codes, which uses the notion of a base for a permutation group.

Christina Boucher, University of Waterloo
Using Graph Clustering to Find Weak Motifs
We consider a graph-theoretic approach to the weak motif recognition problem in DNA sequences. It is unlikely that a polynomial-time algorithm will be found for motif recognition, since it is a NP-complete problem. In this talk, we describe a graph-theoretic algorithm for finding weak motifs, which uses a graph clustering approach to find cliques indicating motif instances. Our algorithm is reliant on a specific graph construction which models motif consensus as cliques and Restricted Neighbourhood Search Clustering (RNSC), a randomized algorithm that quickly generates clusters that are dense and sparsely inter-connected.

Karel Casteels, University of Waterloo
The Cycle Spaces of an Infinite Graph
The edge space of a graph $G = (V, E)$ over a field F is the set of functions $\{f : E - F\}$, or equivalently the $|E|$-dimensional vector space over F. If G is a finite graph then the cycle space is the subspace generated by the cycles of G and the cut space is the space generated by the cuts of E. The cycle and cut spaces turn out to be orthogonal complements of one another.

Suppose G is now a locally finite (i.e., every vertex has finite degree) infinite graph. Extending the notion of the cycle and bond spaces to such graphs presents some non-trivial problems as many basic results for finite dimensional spaces no longer hold in infinite dimensions.

By first topologizing and then compactifying the graph we define 2 types of cycle spaces and up to (depending on F) 3 cut spaces and determine their algebraic properties, such as all the orthogonality relations between them.

This is joint work with Bruce Richter.

David Loker, University of Waterloo
Graph isomorphism and recognition of self-complementary graphs
The inability to directly classify the Graph Isomorphism (GI) problem into either of the conventional complexity classes P or NP-complete led to the definition of the computational complexity class GI. A problem is said to be GI-complete if it is provably as hard as graph isomorphism; that is, there is a polynomial-time Turing reduction from the graph isomorphism problem. A graph is said to be self-complementary (SC) if it is isomorphic to its complement. The SC recognition problem is determining if an input graph is iso-

morphic to its complement graph. M. J. Colbourn and C. J. Colbourn showed that determining the isomorphism of two general graphs is polynomial-time reducible to recognition of SC graphs. Despite the general SC recognition problem being GI-complete, there are specific graph classes where an algorithm for SC recognition is polynomial-time solvable. We show graph classes for which SC recognition remains GI-complete, and others where it becomes polynomial-time solvable.

Conrado Martinez, Universitat Politecnica de Catalunya
Chunksort: a generalized partial sorting algorithm
We introduce here the problem of generalized partial sorting and *chunksort*, an algorithm closely related to quicksort and quickselect that solves this problem in an elegant and efficient way. In generalized partial sorting we are given an array of n elements and p intervals $I_1 = [\ell_1, u_1]$, $I_2 = [\ell_2, u_2]$, $\ldots$, $I_p = [\ell_p, u_p]$, which define p blocks in the array and $p + 1$ gaps between the blocks. The goal is to sort the blocks and gaps relative to each other, and furthermore sort the elements within each block.

We provide an analysis of the average performance of chunksort and show how it generalizes well known results for quicksort, quickselect, multiple quickselect and partial quicksort.

Craig Sloss, University of Waterloo
Enumeration of Walks on the Induced Subgraph Order
Consider a partially ordered set whose elements are unlabelled graphs, with the order given by $G \leq H$ if and only if G is an induced subgraph of H. Walks on the cover relations of this poset correspond to the possible outcomes of a sequence of vertex additions or deletions from an unlabelled graph. We look at the problem of enumerating walks on this poset, using techniques similar to those used by Stanley and Fomin to examine differential posets and graded graphs, respectively. By giving a natural weight to the cover relations of this poset, we can show that the raising and lowering operators with respect to these weights (U and D, respectively) satisfy $(DU - 2UD)G = 2^n G$ when G has n vertices. We discuss the enumerative consequences this algebraic fact, demonstrating the interplay between algebra and combinatorics.

Gabriel Verret, University of Ottawa
Shifts in Cayley Graphs

An automorphism of a simple graph is called a shift if it maps every vertex to an adjacent one. We consider which Cayley Graphs have shifts and show some of the work towards the classification of groups on which all Cayley Graphs admit a shift.

Timothy Walsh, Université du Québec à Montréal
Counting Unlabeled Planar Two-pole Networks
A planar graph is a connected, undirected graph with neither loops nor multiple edges that can be imbedded on the sphere as a 2-cell imbedding, perhaps in several distinct ways, each of which is a planar map. A strongly planar two-pole network is a planar graph G with two distinguished vertices, called the poles of G, such that G is either 2-connected or can be made 2 connected by adding an edge between the poles (assuming that the poles are not already adjacent) and remains planar after the addition of that edge. An isomorphism from a two-pole network to another one must take poles into poles; so an automorphism of a two-pole network must either preserve both the poles or exchange them. A two-pole network is called pole-symmetric if it has an automorphism that exchanges its poles. An unlabeled strongly planar two-pole network is an isomorphism class of strongly planar two-pole networks. A. Gagarin, G. Labelle and P. Leroux reduced the problem of counting unlabeled toroidal graphs with no homeomorph of $K_{3,3}$ by number of vertices and edges to the problem of counting unlabeled strongly planar two-pole networks and unlabeled pole-symmetric strongly planar two-pole networks by number of vertices and edges (see Leroux' invited talk). We counted unlabeled strongly planar two-pole networks and unlabeled pole-symmetric strongly planar two-pole networks with up to ten vertices by number of vertices and edges, enabling Gagarin, Labelle and Leroux to extend their tables of numbers of unlabeled toroidal graphs with no homeomorph of $K_{3,3}$.