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# A Column Expansion Construction for Optimal and Near-Optimal Mixed Covering Arrays

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Mixed covering array construction
by column expansion.

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Introduction:

Construction for strength 2 mixed covering array from an orthogonal array and one or more ordered designs.

Construction emerged during development of commercial covering test case generator. Important characteristics for practical applications:

- Generates *mixed* arrays of strength 2

  Arrays may have several different alphabet sizes => flexibility

- MCAs have optimal or nearly optimal sizes

  Arrays with optimal sizes ($N = v_1 v_2$) are predictable

  Sizes of near-optimal arrays are bounded ($N < v_1 v_2 + v_{oa}$)

- Construction is not a computational challenge

Mixed covering array construction
by column expansion.

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A few references:


M. A. Chateauneuf and D. Kreher, On the State of Strength-Three Covering Arrays, *J Combin Designs*, 10(4):217-238, 2002.

Charles J. Colbourn, Sosina S. Martirosyan, Gary L. Mullen, Dennis Shasha, George B. Sherwood, Joseph L. Yucas, Products of mixed covering arrays of strength two, *J Combin Designs*, 14(2):124-138, 2006.

Lucia Moura, John Stardom, Brett Stevens, Alan Williams, Covering Arrays with Mixed Alphabet Sizes, *J Combin Designs*, 11(6):413-432, 2003.

Covering Array & Orthogonal Array.
Definitions 1.

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Covering array

CA($N$;$t$,$k$,$v$) size $N$, strength $t$, degree $k$, order $v$

$N$ x $k$ array of $v$ symbols

Every subarray of $t$ distinct columns contains every $t$-tuple of $v$ symbols in at least one row.

$N \geq v^t$

Orthogonal array (index 1)

OA($N_{oa}$;$t$,$k_{oa}$,$v_{oa}$) size $N_{oa}$, strength $t$, degree $k_{oa}$, order $v_{oa}$

$N_{oa}$ x $k_{oa}$ array of $v_{oa}$ symbols

Every subarray of $t$ distinct columns contains every $t$-tuple of $v_{oa}$ symbols in exactly one row.

$N_{oa} = v_{oa}^{\ t}$

Mixed Covering Array & Ordered Design.
Definitions 2.

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Mixed covering array

MCA($N$;$t$,$k$,$\prod v_j^{k_j}$) size $N$, strength $t$, degree $k$

$N$ x $k$ array. Different columns have different alphabet sizes indicated by the product

$v_1^{k_1}\ v_2^{k_2} \ldots v_j^{k_j} \ldots$ in which $k_j$ columns have $v_j$ symbols. $k = \sum k_j$.

Every subarray of $t$ distinct columns contains every $t$-tuple of symbols from the corresponding alphabets in at least one row.

$N \geq$ product of $t$ largest alphabet sizes.

Ordered design

OD($N_{od}$;$t$,$k_{od}$,$v_{od}$) size $N_{od}$, strength $t$, degree $k_{od}$, order $v_{od}$

$N_{od}$ x $k_{od}$ array of $v_{od}$ symbols

Every subarray of $t$ distinct columns contains every $t$-tuple of distinct symbols once.

$N_{od} = \binom{v_{od}}{t}\ t\,!$

Conventions.

Construction for strength 2 mixed covering array:

MCA($N$;2,$k$,$\prod v_j^{k_j}$)

Alphabet sizes $v_1$, $v_2$, … are in decreasing order.

$N \geq v_1^2$ if $k_1 > 1$

$N \geq v_1 v_2$ if $k_1 = 1$

Equality => optimal size

OA($N_{oa}$;2,$k_{oa}$,$v_{oa}$)

$N_{oa} = v_{oa}^2$

OD($N_{od}$;2,$k_{od}$,$v_{od}$)

$N_{od} = v_{od}(v_{od} - 1)$

Symbols for alphabet size $v_j$ are taken from $Z_{v_j}$ specifically {0, 1, …, $v_j$ -1}

Unassigned values are denoted by *

MCA(25;2,8,$5^5 3^3$).
Example 1, part 1 of 2.

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OA(25;2,6,5)

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 2 | 2 | 2 | 2 | 2 |
| 0 | 3 | 3 | 3 | 3 | 3 |
| 0 | 4 | 4 | 4 | 4 | 4 |
| 1 | 0 | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 | 0 |
| 1 | 2 | 3 | 4 | 0 | 1 |
| 1 | 3 | 4 | 0 | 1 | 2 |
| 1 | 4 | 0 | 1 | 2 | 3 |
| 2 | 0 | 2 | 4 | 1 | 3 |
| 2 | 1 | 3 | 0 | 2 | 4 |
| 2 | 2 | 4 | 1 | 3 | 0 |
| 2 | 3 | 0 | 2 | 4 | 1 |
| 2 | 4 | 1 | 3 | 0 | 2 |
| 3 | 0 | 3 | 1 | 4 | 2 |
| 3 | 1 | 4 | 2 | 0 | 3 |
| 3 | 2 | 0 | 3 | 1 | 4 |
| 3 | 3 | 1 | 4 | 2 | 0 |
| 3 | 4 | 2 | 0 | 3 | 1 |
| 4 | 0 | 4 | 3 | 2 | 1 |
| 4 | 1 | 0 | 4 | 3 | 2 |
| 4 | 2 | 1 | 0 | 4 | 3 |
| 4 | 3 | 2 | 1 | 0 | 4 |
| 4 | 4 | 3 | 2 | 1 | 0 |

OD(6;2,3,3)

| | | |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 2 | 0 |
| 2 | 0 | 1 |
| 0 | 2 | 1 |
| 1 | 0 | 2 |
| 2 | 1 | 0 |

MCA(25;2,8,$5^5 3^3$).
Example 1, part 2 of 2.

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| | | | | | expansion column ↓ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 2 | 2 | 2 | 2 | 2 |
| 0 | 3 | 3 | 3 | 3 | * |
| 0 | 4 | 4 | 4 | 4 | * |
| 1 | 0 | 1 | 2 | 3 | * |
| 1 | 1 | 2 | 3 | 4 | 0 |
| 1 | 2 | 3 | 4 | 0 | 1 |
| 1 | 3 | 4 | 0 | 1 | 2 |
| 1 | 4 | 0 | 1 | 2 | * |
| 2 | 0 | 2 | 4 | 1 | * |
| 2 | 1 | 3 | 0 | 2 | * |
| 2 | 2 | 4 | 1 | 3 | 0 |
| 2 | 3 | 0 | 2 | 4 | 1 |
| 2 | 4 | 1 | 3 | 0 | 2 |
| 3 | 0 | 3 | 1 | 4 | 2 |
| 3 | 1 | 4 | 2 | 0 | * |
| 3 | 2 | 0 | 3 | 1 | * |
| 3 | 3 | 1 | 4 | 2 | 0 |
| 3 | 4 | 2 | 0 | 3 | 1 |
| 4 | 0 | 4 | 3 | 2 | 1 |
| 4 | 1 | 0 | 4 | 3 | 2 |
| 4 | 2 | 1 | 0 | 4 | * |
| 4 | 3 | 2 | 1 | 0 | * |
| 4 | 4 | 3 | 2 | 1 | 0 |

ordered design

| | | |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 2 | 0 |
| 2 | 0 | 1 |
| 0 | 2 | 1 |
| 1 | 0 | 2 |
| 2 | 1 | 0 |

| expansion rows | unexpanded columns | | | | | expanded columns | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| → | 0 | 3 | 3 | 3 | 3 | **0** | **1** | **2** |
| → | 0 | 4 | 4 | 4 | 4 | **1** | **2** | **0** |
| → | 1 | 0 | 1 | 2 | 3 | **2** | **0** | **1** |
| | 1 | 1 | 2 | 3 | 4 | 0 | 0 | 0 |
| | 1 | 2 | 3 | 4 | 0 | 1 | 1 | 1 |
| | 1 | 3 | 4 | 0 | 1 | 2 | 2 | 2 |
| → | 1 | 4 | 0 | 1 | 2 | **0** | **2** | **1** |
| → | 2 | 0 | 2 | 4 | 1 | **1** | **0** | **2** |
| → | 2 | 1 | 3 | 0 | 2 | **2** | **1** | **0** |
| | 2 | 2 | 4 | 1 | 3 | 0 | 0 | 0 |
| | 2 | 3 | 0 | 2 | 4 | 1 | 1 | 1 |
| | 2 | 4 | 1 | 3 | 0 | 2 | 2 | 2 |
| | 3 | 0 | 3 | 1 | 4 | 2 | 2 | 2 |
| → | 3 | 1 | 4 | 2 | 0 | * | * | * |
| → | 3 | 2 | 0 | 3 | 1 | * | * | * |
| | 3 | 3 | 1 | 4 | 2 | 0 | 0 | 0 |
| | 3 | 4 | 2 | 0 | 3 | 1 | 1 | 1 |
| | 4 | 0 | 4 | 3 | 2 | 1 | 1 | 1 |
| | 4 | 1 | 0 | 4 | 3 | 2 | 2 | 2 |
| → | 4 | 2 | 1 | 0 | 4 | * | * | * |
| → | 4 | 3 | 2 | 1 | 0 | * | * | * |
| | 4 | 4 | 3 | 2 | 1 | 0 | 0 | 0 |

Single column expansion, $N = N_{oa}$.
Theorem 1, part 1 of 7.

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Given:

$OA(N_{oa};2,k_{oa},v_{oa})$

$OD(N_{od};2,k_{od},v_{od})$

$mN_{od} \le N_{oa} - v_{oa}v_{od}$

*column expansion degree m* is a positive integer.

There is:

$MCA(N_{oa};2,k_{oa}+k_{od}{}^{m}-1,v_{oa}{}^{k_{oa}-1}v_{od}{}^{k_{od}{}^{m}})$

with optimal size when $k_{oa} > 2$.

Single column expansion, $N = N_{oa}$.
Theorem 1, part 2 of 7. Proof.

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Construction (1st expansion):

Choose any column from the OA to expand.

Reduce the alphabet size for this *expansion column* from $v_{oa}$ to $v_{od}$ by keeping only the $v_{oa}v_{od}$ symbols from $Z_{v_{od}}$.

The remaining $N_{oa}$ - $v_{oa}v_{od}$ unassigned positions designate *expansion rows*.

Replace the expansion column with the $N_{oa}$ x $k_{od}$ subarray which repeats the column $k_{od}$ times.

In the subarray replace any $N_{od}$ of the expansion rows with the OD rows.

There are enough expansion rows because $N_{od} \leq N_{oa} - v_{oa}v_{od}$.

Single column expansion, $N = N_{oa}$.
Theorem 1, part 3 of 7. Proof.

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Coverage ($1^{st}$ expansion):

The unchanged OA columns form a covering array with each of the expanded (subarray) columns individually. The only other pairs of columns to consider are pairs within the subarray. Therefore the expanded array is a covering array if the subarray is a covering array.

The subarray is a covering array because

(a) each symbol is paired with itself in a row from the OA, and

(b) each symbol is paired with all the other symbols in an expansion row from the OD.

Single column expansion, $N = N_{oa}$.
Theorem 1, part 4 of 7. Proof.

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Construction (recursion):

If $m > 1$, replace each of the expanded columns with the $N_{oa}$ x $k_{od}$ subarray which repeats the column $k_{od}$ times.

In each subarray replace any $N_{od}$ of the remaining expansion rows with the OD rows.

Repeat this process recursively a total of $m$ times.

There are enough expansion rows because $mN_{od} \leq N_{oa} - v_{oa}v_{od}$.

Coverage (recursion):

In each expansion, a new subarray replaces each of the previously expanded columns. Consider any pair of different columns according to four cases:

(1) 2 columns not expanded from the OA.

(2) 1 not expanded & 1 expanded in a new subarray.

(3) 2 columns from different new subarrays.

(4) 2 columns from the same new subarray.

Pairs for (1) & (2) are included because the unchanged OA columns form a CA with each new subarray column individually. Pairs of columns from different new subarrays (3) cover as follows:

(a) each symbol is paired with itself in a row from the OA, and

(b) each symbol is paired with all the other symbols in an OD row from a previous expansion.

Single column expansion, $N = N_{oa}$.
Theorem 1, part 6 of 7. Proof.

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Coverage (recursion):

The only other pairs of columns to consider are pairs within the same new subarray (4). Therefore the expanded array is a covering array if the new subarrays are covering arrays.

Each new subarray is a covering array because

(a) each symbol is paired with itself in a row from the OA, and

(b) each symbol is paired with all the other symbols in an OD row for the current expansion.

Thus the array resulting from $m$ expansions is a covering array.

Single column expansion, $N = N_{oa}$.
Theorem 1, part 7 of 7. Proof.

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Parameters:

The number of expanded columns is multiplied by $k_{od}$ in each expansion. There are $k_{od}^m$ columns with alphabet size $v_{od}$, and $k_{oa}$ -1 columns with alphabet size $v_{oa}$.

The size of the array is unchanged: $N = N_{oa} = v_{oa}^2$.

For $k_{oa} > 2$, there are at least 2 columns with $v_{oa}$ symbols. Thus the size $N = v_{oa}^2$ is optimal.

□

Single column expansion, $N = N_{oa}$.
Boundaries.

Theorem 1 inequality =>

Maximum column expansion degree: $m \leq [(N_{oa} - v_{oa}v_{od})/N_{od}]$

Approximate maximum $v_{od}$ given $v_{oa}$: $v_{od} \lesssim 0.62v_{oa}+0.28$

$(v_{oa} >> m = 1)$.

Generalize expansion to more columns.

Covering pairs for different expansion columns occur in OA.

Alphabet sizes for different expansion columns can be same or different.

Expansion degree depends on alphabet size for that column only.

Multiple column expansion, $N = N_{oa}$.
Theorem 2, part 1 of 3.

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Given:

OA($N_{oa}$;2,$k_{oa}$,$v_{oa}$)

*column expansion number* $n$, $1 \le n \le k_{oa}$

$n$ ordered designs OD($N_{odj}$;2,$k_{odj}$,$v_{odj}$), $1 \le j \le n$

$m_j N_{odj} \le N_{oa} - v_{oa}v_{odj}$

*column expansion degree* $m_j$ is a positive integer.

There is:

$$\mathrm{MCA}\left(N_{oa};2,k_{oa}-n+\sum_{j=1}^{n} k_{odj}^{m_j}, v_{oa}^{k_{oa}-n}\prod_{j=1}^{n} v_{odj}^{k_{odj}^{m_j}}\right)$$

with optimal size when $n \le k_{oa}-2$.

Multiple column expansion, $N = N_{oa}$.
Theorem 2, part 2 of 3. Proof.

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Construction:

Apply the construction of Theorem 1 to any $n$ columns of the OA.

There are enough expansion rows because $m_j N_{odj} \leq N_{oa} - v_{oa} v_{odj}$.

Coverage:

Consider any pair of different columns according to four cases:

(1) 2 columns not expanded from OA: All symbol pairs remain from OA.

(2) 1 not expanded & 1 expanded: As in Theorem 1, all pairs remain from OA.

(3) 2 columns expanded from the same OA column $j$: As in Theorem 1,

(a) each symbol paired with itself in row from OA, and

(b) each symbol paired with all other symbols in row from OD.

(4) 2 columns expanded from different OA columns: All symbol pairs remain from OA.

These are the only choices for a pair, so the expanded array is a MCA.

Multiple column expansion, $N = N_{oa}$.
Theorem 2, part 3 of 3. Proof.

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Parameters:

For each expansion column $j$, there are $k_{odj}{}^{m_j}$ resulting columns. There are $k_{oa}$ -$n$ unchanged columns from the OA with alphabet size $v_{oa}$. $k = k_{oa} - n + \sum_{j=1}^{n} k_{odj}{}^{m_j}$.

The size of the array is unchanged: $N = N_{oa} = v_{oa}^2$.

For $n \leq k_{oa}$-2, there are at least 2 columns with $v_{oa}$ symbols. Thus the size $N = v_{oa}^2$ is optimal.

□

$MCA(25;2,137,5^3 3^6 2^{128})$.
Example 2.

Theorem 2 => Construction for $MCA(25;2,137,5^3 3^6 2^{128})$.

3 OA columns remain with alphabet size 5.

2 OA columns expanded with alphabet size 3 and expansion degree 1.

1 OA column expanded with alphabet size 2 and expansion degree 7.

Many more arrays possible by symbol fusion.

$MCA(25;2,137,5^3 3^6 2^{128}) \Rightarrow MCA(25;2,137,5^2 4^1 3^4 2^{130})$.

Tabulate examples with 2 alphabet sizes

for largest expansion degree $m$

for $1 \le n \le k_{oa}$.

Size is optimal when $n \le k_{oa}-2$.

Parameters for mixed covering arrays of size $N = v_{oa}^2$ with 2 alphabet sizes. Table 1, part 1 of 2.

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| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 2 | 2 | 1 | 9 | 3 | 4-$n$ | 2 | $2n$ |
| 4 | 5 | 2 | 2 | 4 | 16 | 4 | 5-$n$ | 2 | $2^4n$ |
| 5 | 6 | 2 | 2 | 7 | 25 | 5 | 6-$n$ | 2 | $2^7n$ |
| | | 3 | 3 | 1 | 25 | 5 | 6-$n$ | 3 | $3n$ |
| 6 | 3 | 2 | 2 | 12 | 36 | 6 | 3-$n$ | 2 | $2^{12}n$ |
| | | 3 | 3 | 3 | 36 | 6 | 3-$n$ | 3 | $3^3n$ |
| | | 4 | 4 | 1 | 36 | 6 | 3-$n$ | 4 | $4n$ |
| 7 | 8 | 2 | 2 | 17 | 49 | 7 | 8-$n$ | 2 | $2^{17}n$ |
| | | 3 | 3 | 4 | 49 | 7 | 8-$n$ | 3 | $3^4n$ |
| | | 4 | 4 | 1 | 49 | 7 | 8-$n$ | 4 | $4n$ |

optimal for $n \leq k_{oa}$ -2

Parameters for mixed covering arrays of size $N = v_{oa}^2$ with 2 alphabet sizes. Table 1, part 2 of 2.

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| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ |
|---|---|---|---|---|---|---|---|---|---|
| 8 | 9 | 2 | 2 | 24 | 64 | 8 | 9-$n$ | 2 | $2^{24}n$ |
| | | 3 | 3 | 6 | 64 | 8 | 9-$n$ | 3 | $3^6n$ |
| | | 4 | 4 | 2 | 64 | 8 | 9-$n$ | 4 | $4^2n$ |
| | | 5 | 5 | 1 | 64 | 8 | 9-$n$ | 5 | $5n$ |
| 9 | 10 | 2 | 2 | 31 | 81 | 9 | 10-$n$ | 2 | $2^{31}n$ |
| | | 3 | 3 | 9 | 81 | 9 | 10-$n$ | 3 | $3^9n$ |
| | | 4 | 4 | 3 | 81 | 9 | 10-$n$ | 4 | $4^3n$ |
| | | 5 | 5 | 1 | 81 | 9 | 10-$n$ | 5 | $5n$ |
| 10 | 3 | 2 | 2 | 40 | 100 | 10 | 3-$n$ | 2 | $2^{40}n$ |
| | | 3 | 3 | 11 | 100 | 10 | 3-$n$ | 3 | $3^{11}n$ |
| | | 4 | 4 | 5 | 100 | 10 | 3-$n$ | 4 | $4^5n$ |
| | | 5 | 5 | 2 | 100 | 10 | 3-$n$ | 5 | $5^2n$ |
| | | 6 | 3 | 1 | 100 | 10 | 3-$n$ | 6 | $3n$ |
| optimal for $n \le k_{oa}$ -2 | | | | | | | | | |

Column expansion, $N > N_{oa}$.
Rationale.

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Can construct MCA with $v_{odj} \leq v_{oa}$ and $m_j$ with no upper bound.

The size will grow: $N > N_{oa}$.

Each expansion column will need $N_j = m_j N_{odj} + v_{oa} v_{odj}$ rows.

There is:

$$MCA(N_J; 2, k_{oa} - n + \sum_{j=1}^{n} k_{odj}^{m_j}, v_{oa}^{k_{oa}-n} \prod_{j=1}^{n} v_{odj}^{k_{odj}^{m_j}})$$

with size $N = N_J$.

$J$ designates an expansion column where $N_j$ takes its maximum value.

Typically size is not optimal.

If one column of the OA has its alphabet size increased, there are MCAs for larger values of

$v_{odj}$ and $m_j$ which may be optimal.

Construction requires an OD with a special property…

Latin Ordered Design.
Definitions 3.

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Latin ordered design

LOD($N_{od}$;$t$,$k_{od}$,$v_{od}$) is an ordered design OD($N_{od}$;$t$,$k_{od}$,$v_{od}$) partitioned into $\binom{v_{od}-1}{t-1}(t-1)!$

$v_{od}$ x $k_{od}$ Latin rectangles

Every column of each Latin rectangle contains all $v_{od}$ symbols.

LOD($N_{od}$;2,$k_{od}$,$v_{od}$) has $v_{od}-1$ Latin rectangles.

$v_{od}$ a prime power => LOD($N_{od}$;2,$v_{od}$,$v_{od}$) consisting of Latin squares

Choose element of Latin square $h$ in row $i$ and column $j$ to be $i + h \times j$ with addition

and multiplication from GF($v_{od}$) and $h \neq 0$.

Otherwise => trivial LOD($N_{od}$;2,2,$v_{od}$)

In Latin rectangle $h$ elements of row $i$ are $(i, i + h \bmod v_{od})$, $h \neq 0$.

MCA(32;2,17,$6^1 5^3 4^4 3^9$).
Example 3, part 1 of 2.

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OA(25;2,6,5)

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 2 | 2 | 2 | 2 | 2 |
| 0 | 3 | 3 | 3 | 3 | 3 |
| 0 | 4 | 4 | 4 | 4 | 4 |
| 1 | 0 | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 | 0 |
| 1 | 2 | 3 | 4 | 0 | 1 |
| 1 | 3 | 4 | 0 | 1 | 2 |
| 1 | 4 | 0 | 1 | 2 | 3 |
| 2 | 0 | 2 | 4 | 1 | 3 |
| 2 | 1 | 3 | 0 | 2 | 4 |
| 2 | 2 | 4 | 1 | 3 | 0 |
| 2 | 3 | 0 | 2 | 4 | 1 |
| 2 | 4 | 1 | 3 | 0 | 2 |
| 3 | 0 | 3 | 1 | 4 | 2 |
| 3 | 1 | 4 | 2 | 0 | 3 |
| 3 | 2 | 0 | 3 | 1 | 4 |
| 3 | 3 | 1 | 4 | 2 | 0 |
| 3 | 4 | 2 | 0 | 3 | 1 |
| 4 | 0 | 4 | 3 | 2 | 1 |
| 4 | 1 | 0 | 4 | 3 | 2 |
| 4 | 2 | 1 | 0 | 4 | 3 |
| 4 | 3 | 2 | 1 | 0 | 4 |
| 4 | 4 | 3 | 2 | 1 | 0 |

LOD(12;2,4,4)

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 1 | 0 | 3 | 2 |
| 2 | 3 | 0 | 1 |
| 3 | 2 | 1 | 0 |
| 0 | 2 | 3 | 1 |
| 1 | 3 | 2 | 0 |
| 2 | 0 | 1 | 3 |
| 3 | 1 | 0 | 2 |
| 0 | 3 | 1 | 2 |
| 1 | 2 | 0 | 3 |
| 2 | 1 | 3 | 0 |
| 3 | 0 | 2 | 1 |

LOD(6;2,3,3)

| | | |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 2 | 0 |
| 2 | 0 | 1 |
| 0 | 2 | 1 |
| 1 | 0 | 2 |
| 2 | 1 | 0 |

MCA(32;2,17,$6^1 5^3 4^4 3^9$).
Example 3, part 2 of 2.

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| | incremented column ↓ | unexpanded columns | | | expanded columns | | | | expanded columns | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| orthogonal array rows, including some expansion rows | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | 0 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | **0** | **0** | **0** | **2** | **2** | **2** | **1** | **1** | **1** |
| | 0 | 4 | 4 | 4 | **0** | **2** | **3** | **1** | **1** | **1** | **1** | **0** | **0** | **0** | **2** | **2** | **2** |
| | 1 | 0 | 1 | 2 | 3 | 3 | 3 | 3 | **2** | **2** | **2** | **1** | **1** | **1** | **0** | **0** | **0** |
| | 1 | 1 | 2 | 3 | **1** | **3** | **2** | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 2 | 3 | 4 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 3 | 4 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | 1 | 4 | 0 | 1 | 2 | 2 | 2 | 2 | **0** | **1** | **2** | **0** | **1** | **2** | **0** | **1** | **2** |
| | 2 | 0 | 2 | 4 | 1 | 1 | 1 | 1 | **1** | **2** | **0** | **1** | **2** | **0** | **1** | **2** | **0** |
| | 2 | 1 | 3 | 0 | 2 | 2 | 2 | 2 | **2** | **0** | **1** | **2** | **0** | **1** | **2** | **0** | **1** |
| | 2 | 2 | 4 | 1 | 3 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 3 | 0 | 2 | **2** | **0** | **1** | **3** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 2 | 4 | 1 | 3 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | 3 | 0 | 3 | 1 | **3** | **1** | **0** | **2** | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | 3 | 1 | 4 | 2 | 0 | 0 | 0 | 0 | **0** | **2** | **1** | **0** | **2** | **1** | **0** | **2** | **1** |
| | 3 | 2 | 0 | 3 | 1 | 1 | 1 | 1 | **1** | **0** | **2** | **1** | **0** | **2** | **1** | **0** | **2** |
| | 3 | 3 | 1 | 4 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 4 | 2 | 0 | 3 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 4 | 0 | 4 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 4 | 1 | 0 | 4 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | 4 | 2 | 1 | 0 | **0** | **3** | **1** | **2** | **2** | **1** | **0** | **2** | **1** | **0** | **2** | **1** | **0** |
| | 4 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | * | * | * | * | * | * | * | * | * |
| | 4 | 4 | 3 | 2 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| pairing subarray | 5 | 0 | 0 | 0 | **0** | **1** | **2** | **3** | **0** | **0** | **0** | **1** | **1** | **1** | **2** | **2** | **2** |
| | 5 | 1 | 1 | 1 | **1** | **0** | **3** | **2** | **1** | **1** | **1** | **2** | **2** | **2** | **0** | **0** | **0** |
| | 5 | 2 | 2 | 2 | **2** | **3** | **0** | **1** | **2** | **2** | **2** | **0** | **0** | **0** | **1** | **1** | **1** |
| | 5 | 3 | 3 | 3 | **3** | **2** | **1** | **0** | * | * | * | * | * | * | * | * | * |
| | 5 | 4 | 4 | 4 | **1** | **2** | **0** | **3** | * | * | * | * | * | * | * | * | * |
| leftover subarray | * | * | * | * | **2** | **1** | **3** | **0** | * | * | * | * | * | * | * | * | * |
| | * | * | * | * | **3** | **0** | **2** | **1** | * | * | * | * | * | * | * | * | * |

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Multiple column expansion, $N > N_{oa}$.
Theorem 3, part 1 of 6.

Given:

$OA(N_{oa};2,k_{oa},v_{oa})$

$n$ Latin ordered designs $LOD(N_{odj};2,k_{odj},v_{odj})$, $v_{odj} \leq v_{oa}$, $1 \leq j \leq n \leq k_{oa}$

$m_j N_{odj} > N_{oa} - v_{oa}v_{odj}$ for one or more $j$, $m_j > 0$.

There is:

$$MCA(N_J;2,k_{oa}-n+\sum_{j=1}^{n} k_{odj}^{m_j},v_1^{1}v_{\mathrm{oa}}^{k_{oa}-n-1}\prod_{j=1}^{n} v_{odj}^{k_{odj}^{m_j}})$$

$$N_J = \operatorname{Max}_{j=1}^{n} m_j N_{odj} + v_{oa}v_{odj}$$

$$v_1 = [N_J / v_{oa}]$$

$N_J$ optimal size when

either $n \leq k_{oa}-2$ or $v_{odj} = v_{oa}$ for some $j$,

and $v_{oa}$ divides $N_J$.

Multiple column expansion, $N > N_{oa}$.
Theorem 3, part 2 of 6. Proof.

Construction:

$N_J - N_{oa}$ rows will be appended to the OA. Consider two cases:

(1) $N_J - N_{oa} \geq v_{oa}$: Choose any column from the OA. The alphabet size of this *incremented column* will be increased to $v_1$.

Partition the appended rows into $v_{oa}$ x $k_{oa}$ *pairing subarrays* and (if $v_{oa}$ does not divide $N_J$) one *leftover subarray* with fewer than $v_{oa}$ rows.

In each pairing subarray fill the incremented column with an unused symbol from $Z_{v_1}$. Symbols in the incremented column of the leftover subarray are left unspecified.

If $n < k_{oa}$-1, there are one or more unexpanded columns. In each unexpanded column in each pairing subarray, assign each of the symbols from $Z_{v_{oa}}$ to one of the rows.

Construction:

Apply the construction of Theorem 1 to any other $n$ columns of the OA, selecting the expansion rows as follows. For each expansion column place one Latin rectangle from the LOD into each of the pairing subarrays. This step can assign up to $(v_1\text{-}v_{oa})v_{odj}$ expansion rows of the $m_j\, N_{odj}$ needed.

(a) $m_j\, N_{odj} > (v_1\text{-}v_{oa})v_{odj}$ : Use any remaining expansion rows to assign the remaining rows from the LOD.

(b) $m_j\, N_{odj} = (v_1\text{-}v_{oa})v_{odj}$ : The assignment of the LOD rows is complete.

(c) $m_j\, N_{odj} < (v_1\text{-}v_{oa})v_{odj}$ : Place one of the previously assigned Latin rectangles into each of the remaining pairing subarrays.

(2) $N_J - N_{oa} < v_{oa}$: The alphabet size will not be increased beyond $v_{oa}$. The appended rows form a leftover subarray. Expand the OA by applying the construction of Theorem 1 to any $n$ columns.

Multiple column expansion, $N > N_{oa}$.
Theorem 3, part 4 of 6. Proof.

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Coverage:

Consider any pair of different columns according to six cases:

(1) 2 columns not expanded from OA: All symbol pairs remain from OA.

(2) 1 not expanded & 1 expanded: As in Theorem 1, all pairs remain from OA.

(3) 2 columns expanded from the same OA column $j$: As in Theorem 1,

  (a) each symbol paired with itself in row from OA, and

  (b) each symbol paired with all other symbols in row from OD.

(4) 2 columns expanded from different OA columns: All symbol pairs remain from OA.

Multiple column expansion, $N > N_{oa}$.
Theorem 3, part 5 of 6. Proof.

Coverage:

(5) 1 incremented column & 1 not expanded:

(a) Pairs with symbols from $Z_{v_{oa}}$ in incremented column remain from OA.

(b) Each additional symbol from $Z_{v_1}$ in incremented column is paired with all $v_{oa}$ symbols in pairing subarray.

(6) 1 incremented column & 1 expanded:

(a) Pairs with symbols from $Z_{v_{oa}}$ in incremented column remain from OA.

(b) Each additional symbol from $Z_{v_1}$ in incremented column is paired with all $v_{odj}$ symbols in column of a Latin rectangle in pairing subarray.

These are the only choices for a pair, so the expanded array is a MCA.

Multiple column expansion, $N > N_{oa}$.
Theorem 3, part 6 of 6. Proof.

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Parameters:

For each expansion column $j$, there are $k_{odj}{}^{m_j}$ resulting columns. There are $k_{oa}$ -$n$ unexpanded columns from the OA with alphabet size $v_1$ or $v_{oa}$. $k = k_{oa} - n + \sum_{j=1}^{n} k_{odj}{}^{m_j}$.

$N = N_J = \mathrm{Max}_{j=1}^{n}\ m_j N_{odj} + v_{oa} v_{odj}$.

$n \leq k_{oa}$ -2 or $v_{odj} = v_{oa}$ for some $j$ => at least one column has alphabet size $v_{oa}$.

$N_J > v_{oa}^2$ => $v_1 = [N_J / v_{oa}] \geq v_{oa}$ => 2 largest alphabet sizes are $v_1$ & $v_{oa}$.

$v_{oa}$ divides $N_J$ => $v_1 = N_J / v_{oa}$, so the size $N = N_J = v_1 v_{oa}$ is optimal.

Thus the size is optimal when

either $n \leq k_{oa}$ -2 or $v_{odj} = v_{oa}$ for some $j$,

and $v_{oa}$ divides $N_J$.

□

$MCA(45;2,131342,9^1 5^{10} 4^{16} 3^{243} 2^{131072})$.
Example 4.

Theorem 3 => Construction for $MCA(45;2,131342,9^1 5^{10} 4^{16} 3^{243} 2^{131072})$.

$v_1 = 9$ (smallest value for optimal array with $v_{oa} = 5$ & $v_{odj} = 5$).

2 OA columns expanded with alphabet size 5 and expansion degree 1 ($N_j = 45$).

1 OA column expanded with alphabet size 4 and expansion degree 2 ($N_j = 44$).

1 OA column expanded with alphabet size 3 and expansion degree 5 ($N_j = 45$).

1 OA column expanded with alphabet size 2 and expansion degree 17 ($N_j = 44$).

Larger allowed values for $v_{odj}$ and $m_j$ enlarge parameter space of column expansion arrays,

increase flexibility of construction.

Multiple column expansion, $N > N_{oa}$.
Condition for optimal size, part 1 of 2.

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Notation convention:

$$N_J = \operatorname{Max}_{j=1}^{n} m_j N_{odj} + v_{oa} v_{odj} = m_J N_{odJ} + v_{oa} v_{odJ}$$

Subscript $J$ denotes values associated with Max $N_j$.

$N_J > N_{oa} \Rightarrow m_J > (N_{oa} - v_{oa} v_{odJ})/N_{odJ}$. $m_J$ has lower bound, but not upper bound.

Necessary condition for optimal size:

$v_{oa}$ divides $N_J \iff v_{oa}$ divides $m_J N_{odJ}$. 2 special cases:

- $v_{oa}$ divides $m_J \Rightarrow$ MCA has optimal size when $m_J$ is a multiple of $v_{oa}$.

  $m_J = i v_{oa} \Rightarrow i > (v_{oa} - v_{odJ})/N_{odJ}$ & $v_1 = v_{odJ} + i N_{odJ}$

- $v_{oa}$ divides $N_{odJ} \Rightarrow$ MCA has optimal size for all values of $m_J$.

  $N_{odJ} = i v_{oa} \Rightarrow i > (v_{oa} - v_{odJ})/m_J$ & $v_1 = v_{odJ} + i m_J$

Multiple column expansion, $N > N_{oa}$.
Condition for optimal size, part 2 of 2.

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When does $v_{oa}$ divide $N_{odJ}$?

$N_{odJ} = iv_{oa}$ =>

$v_{odJ} \leq v_{oa} \leq N_{odJ}$ (and $i$ has an upper bound too: $i \leq v_{odJ}-1$).

Given $v_{odJ} > 2$, there are at least 2 values of $v_{oa}$ for optimal size, at boundaries of

$v_{odJ} \leq v_{oa} \leq N_{odJ}$.

If $v_{oa}$ is a prime or prime power, it will divide $N_{odJ}$ only if $v_{odJ} = v_{oa}$.

Can prove by expressing $N_{odJ}$ as product of prime factors of $v_{odJ}$ and $v_{odJ}-1$, and

observing that $v_{oa} \not> v_{odJ}$.

=> $v_{oa}$ must have distinct prime factors to divide $N_{odJ}$ when $v_{oa} > v_{odJ}$.

Multiple column expansion, $N > N_{oa}$.
Tabulated examples.

Examples with 2 or 3 alphabet sizes (1 alphabet size for expansion)

for small values of expansion degree $m$, for $1 \le n \le k_{oa}$.

Optimal size is indicated when:

- $v_{oa}$ divides $m$,
- $v_{oa}$ divides $N_{od}$,
- and otherwise.

Compare optimal parameters for $v_{oa} = 5$ (typical prime) with those

for $v_{oa} = 6$ and $v_{od} = 3$ or 4,

for $v_{oa} = 10$ and $v_{od} = 5$ or 6.

Note that when $N_{od} > v_{oa}$, $v_1$ can be increased to yield intermediate sized MCAs

without expanding columns. And since $N$ is increased by $v_{oa}$ when $v_1$ is increased by 1,

an optimal sized array yields another.

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 1 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 2 | 2 | 1 | | • | • | 6 | 3 | 1 | 2 | $2^2$ | | |
| | | | | 2 | • | • | • | 8 | 4 | 1 | 2 | $2^3$ | | |
| | | | | 3 | | • | • | 10 | 5 | 1 | 2 | $2^4$ | | |
| 3 | 4 | 2 | 2 | 2 | | | | 10 | 3 | 4-$n$ | 2 | $2^2n$ | | |
| | | | | 3 | • | | • | 12 | 4 | 1 | 3 | 3-$n$ | 2 | $2^3n$ |
| | | | | 4 | | | | 14 | 4 | 1 | 3 | 3-$n$ | 2 | $2^4n$ |
| | | | | 5 | | | | 16 | 5 | 1 | 3 | 3-$n$ | 2 | $2^5n$ |
| | | | | 6 | • | | • | 18 | 6 | 1 | 3 | 3-$n$ | 2 | $2^6n$ |
| | | 3 | 3 | 1 | | • | • | 15 | 5 | 1 | 3 | $3^2$ | | |
| | | | | 2 | | • | • | 21 | 7 | 1 | 3 | $3^3$ | | |
| | | | | 3 | • | • | • | 27 | 9 | 1 | 3 | $3^4$ | | |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 2 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 5 | 2 | 2 | 5 | | | | 18 | 4 | 5-$n$ | 2 | $2^5n$ | | |
| | | | | 6 | | | • | 20 | 5 | 1 | 4 | 4-$n$ | 2 | $2^6n$ |
| | | | | 7 | | | | 22 | 5 | 1 | 4 | 4-$n$ | 2 | $2^7n$ |
| | | | | 8 | • | | • | 24 | 6 | 1 | 4 | 4-$n$ | 2 | $2^8n$ |
| | | 3 | 3 | 1 | | | | 18 | 4 | 5-$n$ | 3 | $3n$ | | |
| | | | | 2 | | | • | 24 | 6 | 1 | 4 | 4-$n$ | 3 | $3^2n$ |
| | | | | 3 | | | | 30 | 7 | 1 | 4 | 4-$n$ | 3 | $3^3n$ |
| | | | | 4 | • | | • | 36 | 9 | 1 | 4 | 4-$n$ | 3 | $3^4n$ |
| | | 4 | 4 | 1 | | • | • | 28 | 7 | 1 | 4 | $4^2$ | | |
| | | | | 2 | | • | • | 40 | 10 | 1 | 4 | $4^3$ | | |
| | | | | 3 | | • | • | 52 | 13 | 1 | 4 | $4^4$ | | |

* for $n \le k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 3 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 6 | 2 | 2 | 8 | | | | 26 | 5 | 6-$n$ | 2 | $2^8n$ | | |
| | | | | 9 | | | | 28 | 5 | 6-$n$ | 2 | $2^9n$ | | |
| | | | | 10 | • | | • | 30 | 6 | 1 | 5 | 5-$n$ | 2 | $2^{10}n$ |
| | | 3 | 3 | 2 | | | | 27 | 5 | 6-$n$ | 3 | $3^2n$ | | |
| | | | | 3 | | | | 33 | 6 | 1 | 5 | 5-$n$ | 3 | $3^3n$ |
| | | | | 4 | | | | 39 | 7 | 1 | 5 | 5-$n$ | 3 | $3^4n$ |
| | | | | 5 | • | | • | 45 | 9 | 1 | 5 | 5-$n$ | 3 | $3^5n$ |
| | | 4 | 4 | 1 | | | | 32 | 6 | 1 | 5 | 5-$n$ | 4 | $4n$ |
| | | | | 2 | | | | 44 | 8 | 1 | 5 | 5-$n$ | 4 | $4^2n$ |
| | | | | 3 | | | | 56 | 11 | 1 | 5 | 5-$n$ | 4 | $4^3n$ |
| | | | | 4 | | | | 68 | 13 | 1 | 5 | 5-$n$ | 4 | $4^4n$ |
| | | | | 5 | • | | • | 80 | 16 | 1 | 5 | 5-$n$ | 4 | $4^5n$ |
| | | 5 | 5 | 1 | | • | • | 45 | 9 | 1 | 5 | $5^2$ | | |
| | | | | 2 | | • | • | 65 | 13 | 1 | 5 | $5^3$ | | |
| | | | | 3 | | • | • | 85 | 17 | 1 | 5 | $5^4$ | | |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 4 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 3 | 2 | 2 | 13 | | | | 38 | 6 | 3-$n$ | 2 | $2^{13}n$ | | |
| | | | | 14 | | | | 40 | 6 | 3-$n$ | 2 | $2^{14}n$ | | |
| | | | | 15 | | | • | 42 | 7 | 1 | 6 | 1 | 2 | $2^{15}$ |
| | | | | 16 | | | | 44 | 7 | 1 | 6 | 1 | 2 | $2^{16}$ |
| | | | | 17 | | | | 46 | 7 | 1 | 6 | 1 | 2 | $2^{17}$ |
| | | | | 18 | • | | • | 48 | 8 | 1 | 6 | 1 | 2 | $2^{18}$ |
| | | 3 | 3 | 4 | | • | • | 42 | 7 | 1 | 6 | 1 | 3 | $3^4$ |
| | | | | 5 | | • | • | 48 | 8 | 1 | 6 | 1 | 3 | $3^5$ |
| | | | | 6 | • | • | • | 54 | 9 | 1 | 6 | 1 | 3 | $3^6$ |
| | | | | 7 | | • | • | 60 | 10 | 1 | 6 | 1 | 3 | $3^7$ |
| | | | | 8 | | • | • | 66 | 11 | 1 | 6 | 1 | 3 | $3^8$ |
| | | | | 9 | | • | • | 72 | 12 | 1 | 6 | 1 | 3 | $3^9$ |
| | | 4 | 4 | 2 | | • | • | 48 | 8 | 1 | 6 | 1 | 4 | $4^2$ |
| | | | | 3 | | • | • | 60 | 10 | 1 | 6 | 1 | 4 | $4^3$ |
| | | | | 4 | | • | • | 72 | 12 | 1 | 6 | 1 | 4 | $4^4$ |
| | | | | 5 | | • | • | 84 | 14 | 1 | 6 | 1 | 4 | $4^5$ |
| | | | | 6 | • | • | • | 96 | 16 | 1 | 6 | 1 | 4 | $4^6$ |
| | | | | 7 | | • | • | 108 | 18 | 1 | 6 | 1 | 4 | $4^7$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 5 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 3 | 5 | 5 | 1 | | | | 50 | 8 | 1 | 6 | 1 | 5 | 5 |
| | | | | 2 | | | | 70 | 11 | 1 | 6 | 1 | 5 | $5^2$ |
| | | | | 3 | | | • | 90 | 15 | 1 | 6 | 1 | 5 | $5^3$ |
| | | | | 4 | | | | 110 | 18 | 1 | 6 | 1 | 5 | $5^4$ |
| | | | | 5 | | | | 130 | 21 | 1 | 6 | 1 | 5 | $5^5$ |
| | | | | 6 | • | | • | 150 | 25 | 1 | 6 | 1 | 5 | $5^6$ |
| | | 6 | 2 | 1 | | • | • | 66 | 11 | 1 | 6 | $2^2$ | | |
| | | | | 2 | | • | • | 96 | 16 | 1 | 6 | $2^3$ | | |
| | | | | 3 | | • | • | 126 | 21 | 1 | 6 | $2^4$ | | |
| 7 | 8 | 2 | 2 | 18 | | | | 50 | 7 | 8-$n$ | 2 | $2^{18}n$ | | |
| | | | | 19 | | | | 52 | 7 | 8-$n$ | 2 | $2^{19}n$ | | |
| | | | | 20 | | | | 54 | 7 | 8-$n$ | 2 | $2^{20}n$ | | |
| | | | | 21 | • | | • | 56 | 8 | 1 | 7 | 7-$n$ | 2 | $2^{21}n$ |
| | | 3 | 3 | 5 | | | | 51 | 7 | 8-$n$ | 3 | $3^5n$ | | |
| | | | | 6 | | | | 57 | 8 | 1 | 7 | 7-$n$ | 3 | $3^6n$ |
| | | | | 7 | • | | • | 63 | 9 | 1 | 7 | 7-$n$ | 3 | $3^7n$ |

* for $n \le k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 6 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 4 | 4 | 2 | | | | 52 | 7 | 8-$n$ | 4 | $4^2n$ | | |
| | | | | 3 | | | | 64 | 9 | 1 | 7 | 7-$n$ | 4 | $4^3n$ |
| | | | | 4 | | | | 76 | 10 | 1 | 7 | 7-$n$ | 4 | $4^4n$ |
| | | | | 5 | | | | 88 | 12 | 1 | 7 | 7-$n$ | 4 | $4^5n$ |
| | | | | 6 | | | | 100 | 14 | 1 | 7 | 7-$n$ | 4 | $4^6n$ |
| | | | | 7 | • | | • | 112 | 16 | 1 | 7 | 7-$n$ | 4 | $4^7n$ |
| | | 5 | 5 | 1 | | | | 55 | 7 | 8-$n$ | 5 | $5n$ | | |
| | | | | 2 | | | | 75 | 10 | 1 | 7 | 7-$n$ | 5 | $5^2n$ |
| | | | | 3 | | | | 95 | 13 | 1 | 7 | 7-$n$ | 5 | $5^3n$ |
| | | | | 4 | | | | 115 | 16 | 1 | 7 | 7-$n$ | 5 | $5^4n$ |
| | | | | 5 | | | | 135 | 19 | 1 | 7 | 7-$n$ | 5 | $5^5n$ |
| | | | | 6 | | | | 155 | 22 | 1 | 7 | 7-$n$ | 5 | $5^6n$ |
| | | | | 7 | • | | • | 175 | 25 | 1 | 7 | 7-$n$ | 5 | $5^7n$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 7 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 6 | 2 | 1 | | | | 72 | 10 | 1 | 7 | $7-n$ | 6 | $2n$ |
| | | | | 2 | | | | 102 | 14 | 1 | 7 | $7-n$ | 6 | $2^2n$ |
| | | | | 3 | | | | 132 | 18 | 1 | 7 | $7-n$ | 6 | $2^3n$ |
| | | | | 4 | | | | 162 | 23 | 1 | 7 | $7-n$ | 6 | $2^4n$ |
| | | | | 5 | | | | 192 | 27 | 1 | 7 | $7-n$ | 6 | $2^5n$ |
| | | | | 6 | | | | 222 | 31 | 1 | 7 | $7-n$ | 6 | $2^6n$ |
| | | | | 7 | • | | • | 252 | 36 | 1 | 7 | $7-n$ | 6 | $2^7n$ |
| | | 7 | 7 | 1 | | • | • | 91 | 13 | 1 | 7 | $7^2$ | | |
| | | | | 2 | | • | • | 133 | 19 | 1 | 7 | $7^3$ | | |
| | | | | 3 | | • | • | 175 | 25 | 1 | 7 | $7^4$ | | |
| 8 | 9 | 2 | 2 | 25 | | | | 66 | 8 | $9-n$ | 2 | $2^{25}n$ | | |
| | | | | 26 | | | | 68 | 8 | $9-n$ | 2 | $2^{26}n$ | | |
| | | | | 27 | | | | 70 | 8 | $9-n$ | 2 | $2^{27}n$ | | |
| | | | | 28 | | | • | 72 | 9 | 1 | 8 | $8-n$ | 2 | $2^{28}n$ |
| | | | | 29 | | | | 74 | 9 | 1 | 8 | $8-n$ | 2 | $2^{29}n$ |
| | | | | 30 | | | | 76 | 9 | 1 | 8 | $8-n$ | 2 | $2^{30}n$ |
| | | | | 31 | | | | 78 | 9 | 1 | 8 | $8-n$ | 2 | $2^{31}n$ |
| | | | | 32 | • | | • | 80 | 10 | 1 | 8 | $8-n$ | 2 | $2^{32}n$ |

* for $n \leq k_{oa}-2$

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 8 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 9 | 3 | 3 | 7 | | | | 66 | 8 | 9-$n$ | 3 | $3^7n$ | | |
| | | | | 8 | • | | • | 72 | 9 | 1 | 8 | 8-$n$ | 3 | $3^8n$ |
| | | | | 9 | | | | 78 | 9 | 1 | 8 | 8-$n$ | 3 | $3^9n$ |
| | | | | 10 | | | | 84 | 10 | 1 | 8 | 8-$n$ | 3 | $3^{10}n$ |
| | | | | 11 | | | | 90 | 11 | 1 | 8 | 8-$n$ | 3 | $3^{11}n$ |
| | | | | 12 | | | • | 96 | 12 | 1 | 8 | 8-$n$ | 3 | $3^{12}n$ |
| | | 4 | 4 | 3 | | | | 68 | 8 | 9-$n$ | 4 | $4^3n$ | | |
| | | | | 4 | | | • | 80 | 10 | 1 | 8 | 8-$n$ | 4 | $4^4n$ |
| | | | | 5 | | | | 92 | 11 | 1 | 8 | 8-$n$ | 4 | $4^5n$ |
| | | | | 6 | | | • | 104 | 13 | 1 | 8 | 8-$n$ | 4 | $4^6n$ |
| | | | | 7 | | | | 116 | 14 | 1 | 8 | 8-$n$ | 4 | $4^7n$ |
| | | | | 8 | • | | • | 128 | 16 | 1 | 8 | 8-$n$ | 4 | $4^8n$ |

* for $n \le k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 9 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 9 | 5 | 5 | 2 | | | ● | 80 | 10 | 1 | 8 | $8-n$ | 5 | $5^2n$ |
| | | | | 3 | | | | 100 | 12 | 1 | 8 | $8-n$ | 5 | $5^3n$ |
| | | | | 4 | | | ● | 120 | 15 | 1 | 8 | $8-n$ | 5 | $5^4n$ |
| | | | | 5 | | | | 140 | 17 | 1 | 8 | $8-n$ | 5 | $5^5n$ |
| | | | | 6 | | | ● | 160 | 20 | 1 | 8 | $8-n$ | 5 | $5^6n$ |
| | | | | 7 | | | | 180 | 22 | 1 | 8 | $8-n$ | 5 | $5^7n$ |
| | | | | 8 | ● | | ● | 200 | 25 | 1 | 8 | $8-n$ | 5 | $5^8n$ |
| | | 6 | 2 | 1 | | | | 78 | 9 | 1 | 8 | $8-n$ | 6 | $2n$ |
| | | | | 2 | | | | 108 | 13 | 1 | 8 | $8-n$ | 6 | $2^2n$ |
| | | | | 3 | | | | 138 | 17 | 1 | 8 | $8-n$ | 6 | $2^3n$ |
| | | | | 4 | | | ● | 168 | 21 | 1 | 8 | $8-n$ | 6 | $2^4n$ |
| | | | | 5 | | | | 198 | 24 | 1 | 8 | $8-n$ | 6 | $2^5n$ |
| | | | | 6 | | | | 228 | 28 | 1 | 8 | $8-n$ | 6 | $2^6n$ |
| | | | | 7 | | | | 258 | 32 | 1 | 8 | $8-n$ | 6 | $2^7n$ |
| | | | | 8 | ● | | ● | 288 | 36 | 1 | 8 | $8-n$ | 6 | $2^8n$ |

* for $n \leq k_{oa}-2$

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 10 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 9 | 7 | 7 | 1 | | | | 98 | 12 | 1 | 8 | 8-$n$ | 7 | $7n$ |
| | | | | 2 | | | | 140 | 17 | 1 | 8 | 8-$n$ | 7 | $7^2n$ |
| | | | | 3 | | | | 182 | 22 | 1 | 8 | 8-$n$ | 7 | $7^3n$ |
| | | | | 4 | | | • | 224 | 28 | 1 | 8 | 8-$n$ | 7 | $7^4n$ |
| | | | | 5 | | | | 266 | 33 | 1 | 8 | 8-$n$ | 7 | $7^5n$ |
| | | | | 6 | | | | 308 | 38 | 1 | 8 | 8-$n$ | 7 | $7^6n$ |
| | | | | 7 | | | | 350 | 43 | 1 | 8 | 8-$n$ | 7 | $7^7n$ |
| | | | | 8 | • | | • | 392 | 49 | 1 | 8 | 8-$n$ | 7 | $7^8n$ |
| | | 8 | 8 | 1 | | • | • | 120 | 15 | 1 | 8 | $8^2$ | | |
| | | | | 2 | | • | • | 176 | 22 | 1 | 8 | $8^3$ | | |
| | | | | 3 | | • | • | 232 | 29 | 1 | 8 | $8^4$ | | |
| 9 | 10 | 2 | 2 | 32 | | | | 82 | 9 | 10-$n$ | 2 | $2^{32}n$ | | |
| | | | | 33 | | | | 84 | 9 | 10-$n$ | 2 | $2^{33}n$ | | |
| | | | | 34 | | | | 86 | 9 | 10-$n$ | 2 | $2^{34}n$ | | |
| | | | | 35 | | | | 88 | 9 | 10-$n$ | 2 | $2^{35}n$ | | |
| | | | | 36 | • | | • | 90 | 10 | 1 | 9 | 9-$n$ | 2 | $2^{36}n$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 11 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 10 | 3 | 3 | 10 | | | | 87 | 9 | 10-$n$ | 3 | $3^{10}n$ | | |
| | | | | 11 | | | | 93 | 10 | 1 | 9 | 9-$n$ | 3 | $3^{11}n$ |
| | | | | 12 | | | • | 99 | 11 | 1 | 9 | 9-$n$ | 3 | $3^{12}n$ |
| | | | | 13 | | | | 105 | 11 | 1 | 9 | 9-$n$ | 3 | $3^{13}n$ |
| | | | | 14 | | | | 111 | 12 | 1 | 9 | 9-$n$ | 3 | $3^{14}n$ |
| | | | | 15 | | | • | 117 | 13 | 1 | 9 | 9-$n$ | 3 | $3^{15}n$ |
| | | | | 16 | | | | 123 | 13 | 1 | 9 | 9-$n$ | 3 | $3^{16}n$ |
| | | | | 17 | | | | 129 | 14 | 1 | 9 | 9-$n$ | 3 | $3^{17}n$ |
| | | | | 18 | • | | • | 135 | 15 | 1 | 9 | 9-$n$ | 3 | $3^{18}n$ |
| | | 4 | 4 | 4 | | | | 84 | 9 | 10-$n$ | 4 | $4^{4}n$ | | |
| | | | | 5 | | | | 96 | 10 | 1 | 9 | 9-$n$ | 4 | $4^{5}n$ |
| | | | | 6 | | | • | 108 | 12 | 1 | 9 | 9-$n$ | 4 | $4^{6}n$ |
| | | | | 7 | | | | 120 | 13 | 1 | 9 | 9-$n$ | 4 | $4^{7}n$ |
| | | | | 8 | | | | 132 | 14 | 1 | 9 | 9-$n$ | 4 | $4^{8}n$ |
| | | | | 9 | • | | • | 144 | 16 | 1 | 9 | 9-$n$ | 4 | $4^{9}n$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 12 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 10 | 5 | 5 | 2 | | | | 85 | 9 | 10-$n$ | 5 | $5^2n$ | | |
| | | | | 3 | | | | 105 | 11 | 1 | 9 | 9-$n$ | 5 | $5^3n$ |
| | | | | 4 | | | | 125 | 13 | 1 | 9 | 9-$n$ | 5 | $5^4n$ |
| | | | | 5 | | | | 145 | 16 | 1 | 9 | 9-$n$ | 5 | $5^5n$ |
| | | | | 6 | | | | 165 | 18 | 1 | 9 | 9-$n$ | 5 | $5^6n$ |
| | | | | 7 | | | | 185 | 20 | 1 | 9 | 9-$n$ | 5 | $5^7n$ |
| | | | | 8 | | | | 205 | 22 | 1 | 9 | 9-$n$ | 5 | $5^8n$ |
| | | | | 9 | • | | • | 225 | 25 | 1 | 9 | 9-$n$ | 5 | $5^9n$ |
| | | 6 | 2 | 1 | | | | 84 | 9 | 10-$n$ | 6 | $2n$ | | |
| | | | | 2 | | | | 114 | 12 | 1 | 9 | 9-$n$ | 6 | $2^2n$ |
| | | | | 3 | | | • | 144 | 16 | 1 | 9 | 9-$n$ | 6 | $2^3n$ |
| | | | | 4 | | | | 174 | 19 | 1 | 9 | 9-$n$ | 6 | $2^4n$ |
| | | | | 5 | | | | 204 | 22 | 1 | 9 | 9-$n$ | 6 | $2^5n$ |
| | | | | 6 | | | • | 234 | 26 | 1 | 9 | 9-$n$ | 6 | $2^6n$ |
| | | | | 7 | | | | 264 | 29 | 1 | 9 | 9-$n$ | 6 | $2^7n$ |
| | | | | 8 | | | | 294 | 32 | 1 | 9 | 9-$n$ | 6 | $2^8n$ |
| | | | | 9 | • | | • | 324 | 36 | 1 | 9 | 9-$n$ | 6 | $2^9n$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 13 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 10 | 7 | 7 | 1 | | | | 105 | 11 | 1 | 9 | 9-$n$ | 7 | 7$n$ |
| | | | | 2 | | | | 147 | 16 | 1 | 9 | 9-$n$ | 7 | $7^2n$ |
| | | | | 3 | | | ● | 189 | 21 | 1 | 9 | 9-$n$ | 7 | $7^3n$ |
| | | | | 4 | | | | 231 | 25 | 1 | 9 | 9-$n$ | 7 | $7^4n$ |
| | | | | 5 | | | | 273 | 30 | 1 | 9 | 9-$n$ | 7 | $7^5n$ |
| | | | | 6 | | | ● | 315 | 35 | 1 | 9 | 9-$n$ | 7 | $7^6n$ |
| | | | | 7 | | | | 357 | 39 | 1 | 9 | 9-$n$ | 7 | $7^7n$ |
| | | | | 8 | | | | 399 | 44 | 1 | 9 | 9-$n$ | 7 | $7^8n$ |
| | | | | 9 | ● | | ● | 441 | 49 | 1 | 9 | 9-$n$ | 7 | $7^9n$ |
| | | 8 | 8 | 1 | | | | 128 | 14 | 1 | 9 | 9-$n$ | 8 | 8$n$ |
| | | | | 2 | | | | 184 | 20 | 1 | 9 | 9-$n$ | 8 | $8^2n$ |
| | | | | 3 | | | | 240 | 26 | 1 | 9 | 9-$n$ | 8 | $8^3n$ |
| | | | | 4 | | | | 296 | 32 | 1 | 9 | 9-$n$ | 8 | $8^4n$ |
| | | | | 5 | | | | 352 | 39 | 1 | 9 | 9-$n$ | 8 | $8^5n$ |
| | | | | 6 | | | | 408 | 45 | 1 | 9 | 9-$n$ | 8 | $8^6n$ |
| | | | | 7 | | | | 464 | 51 | 1 | 9 | 9-$n$ | 8 | $8^7n$ |
| | | | | 8 | | | | 520 | 57 | 1 | 9 | 9-$n$ | 8 | $8^8n$ |
| | | | | 9 | ● | | ● | 576 | 64 | 1 | 9 | 9-$n$ | 8 | $8^9n$ |

* for $n \le k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 14 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 10 | 9 | 9 | 1 | | • | • | 153 | 17 | 1 | 9 | $9^2$ | | |
| | | | | 2 | | • | • | 225 | 25 | 1 | 9 | $9^3$ | | |
| | | | | 3 | | • | • | 297 | 33 | 1 | 9 | $9^4$ | | |
| 10 | 3 | 2 | 2 | 41 | | | | 102 | 10 | 3-$n$ | 2 | $2^{41}n$ | | |
| | | | | 42 | | | | 104 | 10 | 3-$n$ | 2 | $2^{42}n$ | | |
| | | | | 43 | | | | 106 | 10 | 3-$n$ | 2 | $2^{43}n$ | | |
| | | | | 44 | | | | 108 | 10 | 3-$n$ | 2 | $2^{44}n$ | | |
| | | | | 45 | | | • | 110 | 11 | 1 | 10 | 1 | 2 | $2^{45}$ |
| | | | | 46 | | | | 112 | 11 | 1 | 10 | 1 | 2 | $2^{46}$ |
| | | | | 47 | | | | 114 | 11 | 1 | 10 | 1 | 2 | $2^{47}$ |
| | | | | 48 | | | | 116 | 11 | 1 | 10 | 1 | 2 | $2^{48}$ |
| | | | | 49 | | | | 118 | 11 | 1 | 10 | 1 | 2 | $2^{49}$ |
| | | | | 50 | • | | • | 120 | 12 | 1 | 10 | 1 | 2 | $2^{50}$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 15 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 3 | 3 | 3 | 12 | | | | 102 | 10 | 3-$n$ | 3 | $3^{12}n$ | | |
| | | | | 13 | | | | 108 | 10 | 3-$n$ | 3 | $3^{13}n$ | | |
| | | | | 14 | | | | 114 | 11 | 1 | 10 | 1 | 3 | $3^{14}$ |
| | | | | 15 | | | • | 120 | 12 | 1 | 10 | 1 | 3 | $3^{15}$ |
| | | | | 16 | | | | 126 | 12 | 1 | 10 | 1 | 3 | $3^{16}$ |
| | | | | 17 | | | | 132 | 13 | 1 | 10 | 1 | 3 | $3^{17}$ |
| | | | | 18 | | | | 138 | 13 | 1 | 10 | 1 | 3 | $3^{18}$ |
| | | | | 19 | | | | 144 | 14 | 1 | 10 | 1 | 3 | $3^{19}$ |
| | | | | 20 | • | | • | 150 | 15 | 1 | 10 | 1 | 3 | $3^{20}$ |
| | | 4 | 4 | 6 | | | | 112 | 11 | 1 | 10 | 1 | 4 | $4^{6}$ |
| | | | | 7 | | | | 124 | 12 | 1 | 10 | 1 | 4 | $4^{7}$ |
| | | | | 8 | | | | 136 | 13 | 1 | 10 | 1 | 4 | $4^{8}$ |
| | | | | 9 | | | | 148 | 14 | 1 | 10 | 1 | 4 | $4^{9}$ |
| | | | | 10 | • | | • | 160 | 16 | 1 | 10 | 1 | 4 | $4^{10}$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 16 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 3 | 5 | 5 | 3 | | • | • | 110 | 11 | 1 | 10 | 1 | 5 | $5^3$ |
| | | | | 4 | | • | • | 130 | 13 | 1 | 10 | 1 | 5 | $5^4$ |
| | | | | 5 | | • | • | 150 | 15 | 1 | 10 | 1 | 5 | $5^5$ |
| | | | | 6 | | • | • | 170 | 17 | 1 | 10 | 1 | 5 | $5^6$ |
| | | | | 7 | | • | • | 190 | 19 | 1 | 10 | 1 | 5 | $5^7$ |
| | | | | 8 | | • | • | 210 | 21 | 1 | 10 | 1 | 5 | $5^8$ |
| | | | | 9 | | • | • | 230 | 23 | 1 | 10 | 1 | 5 | $5^9$ |
| | | | | 10 | • | • | • | 250 | 25 | 1 | 10 | 1 | 5 | $5^{10}$ |
| | | 6 | 2 | 2 | | • | • | 120 | 12 | 1 | 10 | 1 | 6 | $2^2$ |
| | | | | 3 | | • | • | 150 | 15 | 1 | 10 | 1 | 6 | $2^3$ |
| | | | | 4 | | • | • | 180 | 18 | 1 | 10 | 1 | 6 | $2^4$ |
| | | | | 5 | | • | • | 210 | 21 | 1 | 10 | 1 | 6 | $2^5$ |
| | | | | 6 | | • | • | 240 | 24 | 1 | 10 | 1 | 6 | $2^6$ |
| | | | | 7 | | • | • | 270 | 27 | 1 | 10 | 1 | 6 | $2^7$ |
| | | | | 8 | | • | • | 300 | 30 | 1 | 10 | 1 | 6 | $2^8$ |
| | | | | 9 | | • | • | 330 | 33 | 1 | 10 | 1 | 6 | $2^9$ |
| | | | | 10 | • | • | • | 360 | 36 | 1 | 10 | 1 | 6 | $2^{10}$ |

* for $n \le k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 17 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 3 | 7 | 7 | 1 | | | | 112 | 11 | 1 | 10 | 1 | 7 | 7 |
| | | | | 2 | | | | 154 | 15 | 1 | 10 | 1 | 7 | $7^2$ |
| | | | | 3 | | | | 196 | 19 | 1 | 10 | 1 | 7 | $7^3$ |
| | | | | 4 | | | | 238 | 23 | 1 | 10 | 1 | 7 | $7^4$ |
| | | | | 5 | | | • | 280 | 28 | 1 | 10 | 1 | 7 | $7^5$ |
| | | | | 6 | | | | 322 | 32 | 1 | 10 | 1 | 7 | $7^6$ |
| | | | | 7 | | | | 364 | 36 | 1 | 10 | 1 | 7 | $7^7$ |
| | | | | 8 | | | | 406 | 40 | 1 | 10 | 1 | 7 | $7^8$ |
| | | | | 9 | | | | 448 | 44 | 1 | 10 | 1 | 7 | $7^9$ |
| | | | | 10 | • | | • | 490 | 49 | 1 | 10 | 1 | 7 | $7^{10}$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 18 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 3 | 8 | 8 | 1 | | | | 136 | 13 | 1 | 10 | 1 | 8 | 8 |
| | | | | 2 | | | | 192 | 19 | 1 | 10 | 1 | 8 | $8^2$ |
| | | | | 3 | | | | 248 | 24 | 1 | 10 | 1 | 8 | $8^3$ |
| | | | | 4 | | | | 304 | 30 | 1 | 10 | 1 | 8 | $8^4$ |
| | | | | 5 | | | • | 360 | 36 | 1 | 10 | 1 | 8 | $8^5$ |
| | | | | 6 | | | | 416 | 41 | 1 | 10 | 1 | 8 | $8^6$ |
| | | | | 7 | | | | 472 | 47 | 1 | 10 | 1 | 8 | $8^7$ |
| | | | | 8 | | | | 528 | 52 | 1 | 10 | 1 | 8 | $8^8$ |
| | | | | 9 | | | | 584 | 58 | 1 | 10 | 1 | 8 | $8^9$ |
| | | | | 10 | • | | • | 640 | 64 | 1 | 10 | 1 | 8 | $8^{10}$ |

* for $n \leq k_{oa}$-2

Parameters for mixed covering arrays of size $N > v_{oa}^2$ with 2 or 3 alphabet sizes. Table 2, part 19 of 19.

Testcover.com

| $v_{oa}$ | $k_{oa}$ | $v_{od}$ | $k_{od}$ | $m$ | $v_{oa} \mid m$ | $v_{oa} \mid N_{od}$ | optimal* | $N$ | $v_1$ | $k_1$ | $v_2$ | $k_2$ | $v_3$ | $k_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 3 | 9 | 9 | 1 | | | | 162 | 16 | 1 | 10 | 1 | 9 | 9 |
| | | | | 2 | | | | 234 | 23 | 1 | 10 | 1 | 9 | $9^2$ |
| | | | | 3 | | | | 306 | 30 | 1 | 10 | 1 | 9 | $9^3$ |
| | | | | 4 | | | | 378 | 37 | 1 | 10 | 1 | 9 | $9^4$ |
| | | | | 5 | | | • | 450 | 45 | 1 | 10 | 1 | 9 | $9^5$ |
| | | | | 6 | | | | 522 | 52 | 1 | 10 | 1 | 9 | $9^6$ |
| | | | | 7 | | | | 594 | 59 | 1 | 10 | 1 | 9 | $9^7$ |
| | | | | 8 | | | | 666 | 66 | 1 | 10 | 1 | 9 | $9^8$ |
| | | | | 9 | | | | 738 | 73 | 1 | 10 | 1 | 9 | $9^9$ |
| | | | | 10 | • | | • | 810 | 81 | 1 | 10 | 1 | 9 | $9^{10}$ |
| | | 10 | 2 | 1 | | • | • | 190 | 19 | 1 | 10 | $2^2$ | | |
| | | | | 2 | | • | • | 280 | 28 | 1 | 10 | $2^3$ | | |
| | | | | 3 | | • | • | 370 | 37 | 1 | 10 | $2^4$ | | |

* for $n \leq k_{oa}$-2