

# Rapid Object Pose Estimation from Feature-based Stereo

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## Abstract

This paper addresses the problem of computing the three-dimensional path of a moving rigid object using a calibrated stereoscopic vision setup. The proposed system begins by detecting feature points on the moving object. By tracking these points over time, it produces clouds of 3D points that can be registered, thus giving information about the underlying camera motion. A novel correction scheme is also proposed, that compensates for the accumulated error in the computed positions by the automatic detection of loop back points in the movement of the object. An application to rapid object modelling is presented in which a hand-held object is moved in front of a camera and is reconstructed using silhouette intersection.

## 1 Introduction

One of the main challenges inherent in using images from a large number of viewpoints is the issue of camera pose estimation. For 3D reconstruction to be possible, the location and orientation of the cameras at different capture instant must be accurately known. Several applications can benefit from the knowledge of the position of a camera with respect to some rigid reference frame. Among them are virtual or augmented reality systems, scene reconstruction, object modelling and robotics. In a video sequence in which a camera is moving inside a fixed environment, keeping track of the camera/object respective positions can be challenging. In the case of a moving camera, a workable solution consists in installing calibration targets, precisely registered with respect to a global reference frame. By having them visible inside the scene, it becomes possible to compute the camera position as the camera moves with respect to the global reference frame [5]. Alternately, in an object-based solution, a CAD model of the observed objects can be created and by registering that model to the observations, the spatial relation between the scene and the

camera can be determined [1]. In practice, however, these solutions are not always feasible. It is therefore desirable to develop a method to compute the camera motion in an unprepared scene for which no a priori knowledge is available.

In 3D reconstruction, bundle adjustment is a widely accepted approach [2], and commercial software tools are now available. The technique most often relies on a human operator, who has to supply the matches since there is typically a small number of widely separated views. Bundle adjustment is used for a wide spectrum of applications, such as accident reconstruction, animation and graphics, archaeology, forensics, engineering and architecture. The main drawback of bundle adjustment is its instability. In many situations, the algorithm will fail to converge to an accurate solution. To overcome this problem, it is recommended that the user starts with a small subset of the available pictures, and a small number of feature points that can be seen in many pictures. Once the algorithm succeeded in converging to a first reasonable solution, additional intermediate pictures and more feature points can be added to improve the accuracy. This lack of robustness can be related to the iterative nature of bundle adjustment, which implies initial estimates of the camera positions. If these estimates are very far from the actual solution, the algorithm may fail to converge.

This problem is amplified when one wants to automate the whole process. Matches between narrowly separated views can be found automatically through correlation. Unfortunately, nothing can guarantee that the matches will all be good. Bad matches will definitely affect negatively any structure and motion estimation process. While the use of more widely separated views would help to improve the accuracy of the reconstruction, the matching process would become much more difficult and error prone.

The method proposed in this paper aims at resolving these issues using a calibrated stereoscopic vision setup. This system is observing a rigid object in motion on which feature points are detected. Because they are seen by a stereo setup, these points can be 3D reconstructed when they are matched. By tracking these points over time, the resulting clouds of 3D points can be registered thus giving information about the underlying camera motion. This is the idea that is exploited in this paper to robustly keep track the cameras/object relative motion along a sequence. In addition, a novel correction scheme is proposed, that compensates for the accumulated error in the computed positions, exploiting the automatic detection of loop back points in the movement of the object. An application to rapid object modelling is presented in which a hand-held object is moved in front of a camera and is reconstructed using silhouette intersection.

The path of a binocular or trinocular stereoscopic setup is computed in [4]. In this approach, points are matched at each camera location. In addition points in one view are tracked from image to image. The method uses trilinear tensors and/or fundamental matrix constraints for robust tracking and matching over views. The computed transformations are then cascaded in order to place them in a common coordinate frame. To overcome the problem of error accumulation, it is proposed to add an extra step where the final 3D transformation for all cameras would be computed simultaneously.

In [7], the goal is to compute the registration of two consecutive scene captures along with the extrinsic calibration parameters of the stereo setup and the 3D location of a minimum of four matched and tracked feature points. The essential matrix of the stereo setup is calculated from the eight correspondences given by the four feature points in both captures, and nonlinear methods are used to enforce its constraints. It is decomposed to retrieve the extrinsic calibration parameters up to a scale factor of the translation vector.

At this point, 3D reconstruction can be applied to the feature points, yielding two clouds of a minimum of four 3D points. The registration between the two 3D point clouds can then be calculated. It differs from the proposed method in the fact that they do not compute the extrinsic calibration parameters of the stereo setup prior to the computation of the registration. As a consequence, the matching process cannot be guided by the epipolar constraint. No experimental results along a sequence were shown to display the accumulation of error.

The method in [24] tracks points in each view of a stereo rig. It introduced binocular matching constraints. Camera motion is recovered from the left and right temporal fundamental matrices. Stereo correspondences are then inferred by combining stereo geometry and motion correspondences through a projective mapping. A similar approach is presented in [8], stereoscopic vision and shape-from-motion are combined in an attempt to exploit the strengths of both approaches, i.e. accurate 3D reconstruction for stereo and easy feature tracking for visual motion. The result is a 3D reconstruction of feature points and the camera motion in two separate steps. In this research, the experiments were limited to short sequences where the viewpoints don't change dramatically from the first to the last capture.

Stereo and motion correspondences are computed simultaneously in [23]. They defined a coarse-to-fine algorithm in which local surface parameters and rigid-body motion parameters are iteratively estimated. They were able to extract local range information from a sequence of few stereo images.

In [6], self-calibration and Euclidean reconstruction from a stereo rig are achieved through a stratified approach that proceeds by upgrading a projective reconstruction to affine and to metric reconstruction. Results are shown for images taken before and after a single rigid motion.

A strategy based on active stereo is proposed in [11] for simultaneous localization and mapping in a robotic application. As the robot moves inside the scene, the stereo head is actively moved in order to select the feature measurement that will best improve the current robot position estimation. The solution is based on the definition of measurement and motion models predicted using a Kalman filter.

The rest of this paper is organized as follows. Section 2 reviews the concepts of 3D reconstruction. Section 3 presents the used feature matching and tracking strategy. Section 4 discusses the problem of previously visited locations while Section 5 is concerned with accumulated error correction. Finally, Section 6 presents experimental results and Section 7 is a conclusion.

## 2 Stereo Reconstruction

Three dimensional reconstruction requires the computation of the Euclidian coordinates of image features from the visual data observed in multiple views. Stereoscopic vision involves the use of two cameras for which there is a fixed rigid transformation between them.

### 2.1 Calibration

Stereo calibration aims at computing the projection matrices of the two cameras. When a set of 3D points at precisely known locations is available, the projection matrices can

be obtained straightforwardly. Indeed, a point,  $\mathbf{X}_i$ , along with its corresponding image coordinates  $\mathbf{x}_i$ , satisfy the following relationship:

$$\mathbf{x} = P\mathbf{X} = K[R | T]\mathbf{X}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} \\ p_{10} & p_{11} & p_{12} & p_{13} \\ p_{20} & p_{21} & p_{22} & p_{23} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (1)$$

Eliminating  $\lambda$  and rearranging the expressions yields a pair of homogeneous linear equations in 12 unknowns, the entries of the projection matrix. Putting together the information of  $n$  3D points ( $n \geq 6$ ) gives  $2n$  homogeneous linear equations in 12 unknowns  $p_{00}, p_{01}, \dots, p_{23}$ . This system can be solved up to a scale factor, through SVD. The quality of the computed projection matrix depends on the linearity of the camera model and the accuracy in the measured 3D location of the points. Once the projection matrices are computed for both cameras, they can be decomposed to retrieve their intrinsic and extrinsic calibration parameters [3].

In practice, however, it is more convenient to use a planar configuration object with known metric pattern. By exploiting the homographic constraints that exists between the calibration plane and the corresponding images, it is possible to build a linear system of equations. This is the approach proposed in [17] and [18] where several views of the planar calibration pattern are used to calibrate a single camera. The procedure has been extended to the case of a stereo setting in [20]. The method makes use of the additional constraint provided by the stereo rig configuration. The constraint is also expressed in the form of a homography; the one that links the image of the calibration plane as a function of the rigid transformation between the cameras of the stereo head, i.e. [19]:

$$H = K \left[ R - \frac{Tn^\top}{d} \right] K^{-1} \quad (2)$$

Here  $n$  is the normal of the calibration and  $d$  is the distance from the camera to the plane.

In order to determine the optimal stereo configuration of the cameras, we performed an experiment in which we used three stereo setups with different baselines (0.139 m, 0.416 m and 0.756 m). The angles between the  $Z$ -axes of the two cameras were adjusted in such a way that a given working volume was preserved, resulting in angles of 0.112 rad, 0.463 rad and 1.05 rad respectively. A calibration pattern was used, allowing easy detection of its feature points with sub-pixel resolution. The position of the calibration pattern with respect to the table was measured with a ruler. This procedure provides the ground truth value of the feature points position, with an estimated accuracy of 0.3 mm.

Figure 1 shows the reconstruction error ( $|\vec{x}_{calculated} - \vec{x}_{measured}|$ ) averaged over the 20 feature points of a calibration pattern as a function of the  $Z$  position of the calibration pattern, for three different baselines. It can be observed that the reconstruction error is higher for the stereo setup with the smallest baseline, as expected. No significant difference can be observed by comparing the results of the stereo setups with baselines of 0.416 m and 0.756 m. Since matching is facilitated when the baseline is shorter, we can conclude that there is no need to increase the baseline of our stereo setup above 0.4 m, as it does not seem to provide any significant improvement in reconstruction accuracy and it would make the matching process more difficult.

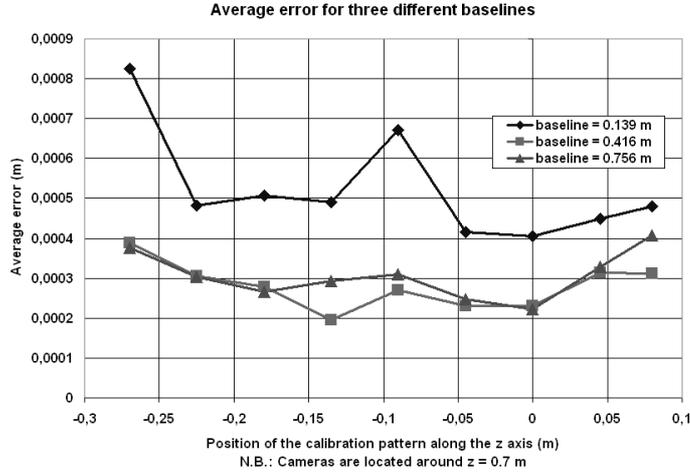


Figure 1: Average reconstruction error, for three different baselines.

## 2.2 3D Point Reconstruction

Once the stereo rig calibrated, and consequently the projection matrices  $P_1$  and  $P_2$  are known, it is possible to compute the 3D position of any point seen by the two cameras. Using the projective relation of Equation (1), the 3D location  $\mathbf{X}$  of a feature point whose image coordinates in the two images are  $\mathbf{x}_1$  and  $\mathbf{x}_2$  can be obtained by solving the following system of 4 linear equations in 3 unknowns:

$$\begin{bmatrix} (p_{00} - xp_{20})_1 & (p_{01} - xp_{21})_1 & (p_{02} - xp_{22})_1 \\ (p_{10} - yp_{20})_1 & (p_{11} - yp_{21})_1 & (p_{12} - yp_{22})_1 \\ (p_{00} - xp_{20})_2 & (p_{01} - xp_{21})_2 & (p_{02} - xp_{22})_2 \\ (p_{10} - yp_{20})_2 & (p_{11} - yp_{21})_2 & (p_{12} - yp_{22})_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (xp_{23} - p_{03})_1 \\ (yp_{23} - p_{13})_1 \\ (xp_{23} - p_{03})_2 \\ (yp_{23} - p_{13})_2 \end{bmatrix} \quad (3)$$

This system can be solved through a least-square method. Even if this approach, involving the minimization of algebraic quantities, works well in practice, a geometric triangulation formulation is often preferred. The method finds the 3D point that minimizes its 3D distance with two non-crossing lines in space. In other words, it returns the middle point of the segment perpendicular to both rays.

Figure 2 shows the geometry of two cameras projecting the images  $\mathbf{x}_1$  and  $\mathbf{x}_2$  of the 3D point  $\mathbf{X}$ . In an ideal situation, the extension of the lines  $O_1K_1^{-1}\mathbf{x}_1$  and  $O_2K_2^{-1}\mathbf{x}_2$  should cross each other in space at the location of the projected 3D point  $\mathbf{X}$ . In reality, the two lines may not cross. The best solution is therefore to search for the point  $\mathbf{X}$  that is the

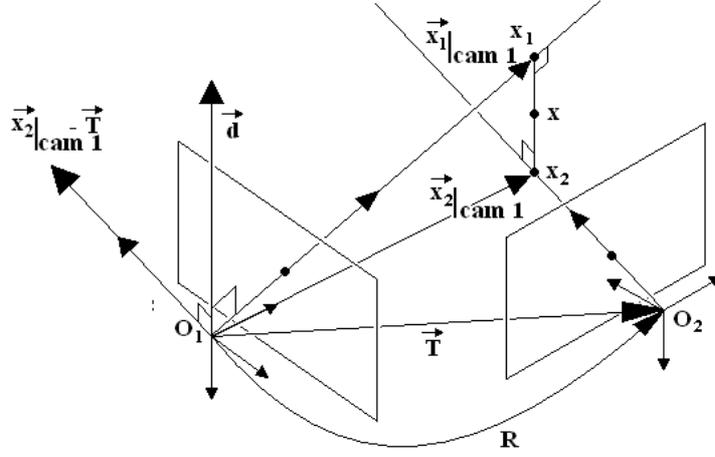


Figure 2: Geometry of the triangulation procedure

middle of the segment perpendicular to both lines. We have, from Figure 2:

$$\mathbf{X}_1 = \frac{1}{\lambda_1} K_1^{-1} \mathbf{x}_1 \quad (4)$$

$$\mathbf{X}_2 = \frac{1}{\lambda_2} K_2^{-1} \mathbf{x}_2 \quad (5)$$

Expressing point  $\mathbf{X}_2$  in the reference frame of the first camera gives:

$$\begin{aligned} \mathbf{X}_2|_{cam1} &= R\mathbf{X}_2 + T \\ &= \frac{1}{\lambda_2} RK_2^{-1} \mathbf{x}_2 + T \end{aligned} \quad (6)$$

Let us now define the vector  $\vec{d}$ , that is proportional to the cross product of  $\mathbf{X}_1|_{cam1}$  and  $(\mathbf{X}_2|_{cam1} - T)$ :

$$\begin{aligned} \vec{d} &\equiv \lambda_1 \lambda_2 \mathbf{X}_1|_{cam1} \times (\mathbf{X}_2|_{cam1} - T) \\ &= K_1^{-1} \mathbf{x}_1 \times RK_2^{-1} \mathbf{x}_2 \end{aligned} \quad (7)$$

The vector  $\vec{d}$  is therefore parallel to the vector  $\overline{\mathbf{X}_1 \mathbf{X}_2}$ . Let us now define three scalars  $a$ ,  $b$  and  $c$  such that the path  $O_1 \mathbf{X}_1 \mathbf{X}_2 O_2 O_1$  forms a closed loop:

$$aK_1^{-1} \mathbf{x}_1 + b\vec{d} + cRK_2^{-1} \mathbf{x}_2 - T = 0 \quad (8)$$

$$aK_1^{-1} \mathbf{x}_1 + b[K_1^{-1} \mathbf{x}_1 \times RK_2^{-1} \mathbf{x}_2] + cRK_2^{-1} \mathbf{x}_2 = T \quad (9)$$

Equation (9) provides three linear equations in three unknowns,  $a$ ,  $b$  and  $c$ . Once this system is solved for a given match  $(\mathbf{x}_1, \mathbf{x}_2)$ , the location of the point  $\mathbf{X}$  can be calculated:

$$\mathbf{X}|_{cam1} = aK_1^{-1} \mathbf{x}_1 + \frac{1}{2} b [K_1^{-1} \mathbf{x}_1 \times RK_2^{-1} \mathbf{x}_2] \quad (10)$$

### 3 Feature-based Stereokineopsis

In order to keep track of the position of an object that moves in front of a set of cameras, we used a *match-and-track* paradigm. At one instant, feature points are detected and matched across views. The matched points are then independently tracked in each view until a new matching process is initiated. Proceeding this way, we benefit from both the accuracy of the reconstruction provided by stereo-matching and the reliability and efficiency of image tracking. Robustness of the process to the unavoidable presence of outliers is ensured by the 3D registration procedure that is applied between each stereo matching phase. Note that matching is typically applied every  $X$  frames, while tracking is performed at full frame rate. In fact, the matching rate is determined by the frequency at which a given application requires new object position data. The following subsections detail each of these steps. Another important difficulty related to sequential pose estimation approaches concerns the error accumulation problem; this aspect is addressed in sections 4 and 5.

#### 3.1 Tracking

Feature point tracking is achieved using the Intel OpenCV implementation of the Lucas-Kanade tracker [14]. It is an accurate and robust tracker that can run at several frames per second. In order to reduce the computational load, the tracker uses a pyramid of resolutions in the computation of the displacement vectors.

When Harris corners are used, the tracker performs reliably over quite long sequences. However, it is unavoidable to have some false tracks occurring. Occlusion boundary, for instance, are particularly problematic as they tend to produce moving corners on the image. Consequently, even when starting with an exact match set, the independent tracking of the features in each view will most probably cause the introduction of some false matches; therefore the 3D registration process has to be robust to outliers.

#### 3.2 Matching

Since the epipolar geometry is available through a preliminary calibration phase it is used to guide the matching of features. Thus, only points from the second image that lie close to the epipolar line of a point in the first image are considered as possible matches.

Feature comparison is done using variance normalized correlation (VNC), which is designed to produce reliable results over a wide range of viewing conditions. VNC is defined for a candidate match  $(\mathbf{x}_1, \mathbf{x}_2)$  as:

$$VNC(\mathbf{x}_1, \mathbf{x}_2) = \frac{\sum_{\mathbf{k}_1, \mathbf{k}_2} [I_1(\mathbf{k}_1) - \overline{I_1(\mathbf{x}_1)}][I_2(\mathbf{k}_2) - \overline{I_2(\mathbf{x}_2)}]}{N \sqrt{\sigma_{I_1}^2(\mathbf{x}_1) \sigma_{I_2}^2(\mathbf{x}_2)}} \quad (11)$$

where the sum is taken over the points  $\mathbf{k}_1$  and  $\mathbf{k}_2$  in the neighborhoods of  $\mathbf{x}_1$  and  $\mathbf{x}_2$  and where  $\overline{I(\mathbf{x})}$  and  $\sigma_I^2(\mathbf{x})$  are respectively the mean and the variance of the pixel intensities over the neighborhoods.

The point pairs found through correlation along the epipolar lines are not necessarily accurate correspondences. This is why additional matching constraints must be applied. In particular, the uniqueness and symmetry constraints have been shown to be simple and

advantageous [13]. Uniqueness requires that only the best match in the second image be kept, for a given point in the first image. Symmetry requires that the point in the first image also be the best match for the other point.

Finally, in order to prune additional mismatches that might still be present, the disparity gradient is used, as in [25]. The disparity gradient is a measure of the compatibility of matched points. For two pairs  $(\mathbf{x}_1, \mathbf{x}_2)$  and  $(\mathbf{y}_1, \mathbf{y}_2)$ , having disparities  $d(\mathbf{x}_1, \mathbf{x}_2)$  and  $d(\mathbf{y}_1, \mathbf{y}_2)$  respectively, the cyclopean separation,  $d_{cs}(\mathbf{x}_1, \mathbf{x}_2; \mathbf{y}_1, \mathbf{y}_2)$ , is the vector joining the midpoints of the line segments  $\overline{\mathbf{x}_1\mathbf{x}_2}$  and  $\overline{\mathbf{y}_1\mathbf{y}_2}$ , and, their disparity gradient is defined as:

$$\Delta d(\mathbf{x}_1\mathbf{x}_2; \mathbf{y}_1\mathbf{y}_2) = \frac{|d(\mathbf{x}_1, \mathbf{x}_2) - d(\mathbf{y}_1, \mathbf{y}_2)|}{|d_{cs}(\mathbf{x}_1, \mathbf{x}_2; \mathbf{y}_1, \mathbf{y}_2)|} \quad (12)$$

This compatibility measure is used in a constraint that accepts pairs that share a disparity gradients below some threshold value, with at least 2 of their 3 closest neighbors. This eliminates false matches as long as they are not surrounded by other similar false matches.

### 3.3 Robust Registration

After having found matches and tracked the corresponding points in both sequences,  $3D$  points can be reconstructed. Based on the matches at a given instant  $M$  and their tracked correspondents at a latter instant  $N$ , the resulting two clouds of  $3D$  points can be registered to find the rigid motion of the object (or, reciprocally, the rigid motion of the stereo setup, when the reference frame is attached to the object). Unfortunately, in identifying the motion complying with the observations, one cannot simply use the complete data set, since the false matches and the tracking errors will corrupt the result. Instead, it is necessary to introduce a random sample consensus (RANSAC) algorithm [9] in order to filter out the corrupted pairs of  $3D$  points.

A minimum of 3 pairs of non-collinear  $3D$  points are necessary to compute an unambiguous  $3D$  registration. As a consequence, the first step of the algorithm will consist in finding a triplet of  $3D$  matches.

In order to make sure that a randomly selected triplet of  $3D$  matches does not constitute a degenerate case (i.e. is not in a collinear configuration), two conditions are imposed:

1. The distance between any two points of the trio must be greater than a given minimum;
2. The area defined by the three points must be greater than a given minimum.

The first item alone is not sufficient since three collinear points that are located far apart would satisfy it, while the second item alone would allow a triplet constituted of two points close from each other with a third point far away, such that the area of the triangle is sufficient.

Once corresponding triplets have been identified as being non-collinear, the rotation and the translation that best describe the rigid movement of the points can be computed. This now constitutes a candidate registration  $(R_{N/M}, T_{N/M})$ .

Given such a candidate registration, a count of the number of supporting matches can be obtained as follows. For each  $3D$  match, if the distance between  $\mathbf{X}_N$  and  $R_{N/M}\mathbf{X}_M + T_{N/M}$  is less than a maximum distance, then this match is said to agree with or support

the candidate registration. This procedure is repeated several times with the number of trials set so that the probability of success is above a desired value. The candidate registration having the highest number of supporting matches is declared the best candidate registration.

Finally, all the matches that support the best candidate registration are used to compute the final output registration through least-square fitting of the two sets of points [10]:

$$Q_{N/M} = \begin{bmatrix} R_{N/M} & T_{N/M} \\ 0^T & 1 \end{bmatrix} \quad (13)$$

From the computed homogeneous transformation  $Q_{N/M}$ , the new world coordinates of the cameras can be computed as:

$$Q_N = Q_{N/M} Q_M \quad (14)$$

The main problem associated with this technique resides in the accumulation of errors, due to the fact that every new position is computed from the previous one. Since it is assumed that no special target points that could allow recalibration are available on the object, the only information that can be used here is the knowledge of the approximate camera positions that will allow us to identify points of view that were previously captured. This is the information that will be used here to correct for the drift, each time the cameras pass by a location where they have been before.

## 4 Detection of Previously Visited Locations

The goal of this procedure is to take a sequence of camera positions and identify those that are close to their previous positions in an earlier image capture. Whenever such a loop back situation is detected, a connection between the earlier and later views becomes possible. Once this is done a registration between the two positions and correction of the accumulated error can be undertaken.

Because the tracking algorithm described in Section 3.1 is used to match the two extreme views of a detected loop, it is necessary that these views be separated by a relatively short baseline; this requirement can be expressed by the following two conditions:

- The  $Z$ -axes of the two views must be nearly parallel;
- The distance between the center of projection of the views must be sufficiently small.

At first sight, these conditions might appear to be insufficient for loop back detection to work properly as this test is not completely rotationally invariant. Indeed, it is also necessary that the  $Y$ - (or the  $X$ -) axes be nearly parallel for correct neighborhood matching. Nevertheless, we can relax this constraint since our knowledge of the approximate camera positions will allow us to de-rotate the images around their  $Z$ -axes in such a way that they become adequately aligned.

The distance between the center of projection of the views is directly calculated from the length of the vector going from one center to the other. In order to calculate the maximum distance we can tolerate, we must take into consideration the fact that the two views may be collinear along their parallel  $Z$ -axes (i.e. one view may be in front of the other), resulting in a scale difference between the two images.

The angle between the  $Z$ -axes of two views can be computed through a scalar product of unit vectors parallel to the  $Z$ -axes of the two cameras, as expressed in the world reference frame:

$$\hat{k}_M = Q_M \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (15)$$

$$\hat{k}_N = Q_N \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (16)$$

$$\cos(\theta) = \hat{k}_M \cdot \hat{k}_N \quad (17)$$

In a sequence, the minimal angle (or distance) with respect to a given frame may not happen at the same frame for the left and the right camera. When trying to identify the best capture to be matched with an earlier capture, we must find a compromise between the two cameras.

Whenever a view is detected as being close to a previously captured view, the drift of the later view can be compensated. Of course, it is assumed that the earlier the view, the better the accuracy, since its location has been computed from a smaller number of cascaded transformations. This is discussed in Section 5.

#### 4.1 Identification of the Rotation Angle Around the $Z$ -Axis

As discussed previously, two views are similar if their  $Z$ -axes are nearly parallel; they can however have a wide angular difference around their  $Z$ -axes. Since the tracking algorithm is not rotation invariant, this situation could prevent the identification of correspondences. We can overcome this difficulty by making use of the knowledge we have of the approximate positions of the camera; that is we can determine the rotation that must be applied to the images of the later view, such that it is as aligned as possible with the earlier view.

In a first approach, we will aim at minimizing the angle between the  $Y$ -axes of two views by applying a rotation around the  $Z$ -axis of the second view. Let us state the result:

Let  $r_{ij}$  be the element  $(i, j)$  of the rotation matrix linking the view  $N$  with the view  $M$ ,  $R_{N/M}$ . If  $r_{10} \sin(\arctan(-\frac{r_{10}}{r_{11}})) < r_{11} \cos(\arctan(-\frac{r_{10}}{r_{11}}))$ , then:

$$\alpha_Y = \arctan\left(-\frac{r_{10}}{r_{11}}\right) \quad (18)$$

else:

$$\alpha_Y = \arctan\left(-\frac{r_{10}}{r_{11}}\right) + \pi \quad (19)$$

*Proof:* The rotation component of a reference system built with a pure rotation  $\alpha$  around the  $Z$ -axis of the second reference system is:

$$R_{N/M} R_{(\alpha, 0, 0)} = \begin{bmatrix} r_{00} \cos(\alpha) + r_{01} \sin(\alpha) & -r_{00} \sin(\alpha) + r_{01} \cos(\alpha) & r_{02} \\ r_{10} \cos(\alpha) + r_{11} \sin(\alpha) & -r_{10} \sin(\alpha) + r_{11} \cos(\alpha) & r_{12} \\ r_{20} \cos(\alpha) + r_{21} \sin(\alpha) & -r_{20} \sin(\alpha) + r_{21} \cos(\alpha) & r_{22} \end{bmatrix} \quad (20)$$

A unit vector, oriented along the  $Y$ -axis of the camera  $N$  will be expressed, in the reference frame of camera 1:

$$\hat{j}_N|_M = R_{N/M}R_{(\alpha,0,0)} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (21)$$

$$\stackrel{(20)}{=} \begin{bmatrix} -r_{00} \sin(\alpha) + r_{01} \cos(\alpha) \\ -r_{10} \sin(\alpha) + r_{11} \cos(\alpha) \\ -r_{20} \sin(\alpha) + r_{21} \cos(\alpha) \end{bmatrix} \quad (22)$$

We aim at maximizing the scalar product between the  $Y$ -axes of the two cameras:

$$\begin{aligned} \hat{j}_N|_M \cdot \hat{j}_M|_M &= \hat{j}_N|_M \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ &= -r_{10} \sin(\alpha) + r_{11} \cos(\alpha) \end{aligned} \quad (23)$$

To maximize the scalar product (23), we pose its first derivative with respect to  $\alpha$  equal to 0 and we pose its second derivative negative:

$$\begin{aligned} \frac{\partial}{\partial \alpha} (\hat{j}_N|_M \cdot \hat{j}_M|_M) &= \frac{\partial}{\partial \alpha} (-r_{10} \sin(\alpha) + r_{11} \cos(\alpha)) \\ &= -r_{10} \cos(\alpha) - r_{11} \sin(\alpha) = 0 \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{\partial^2}{\partial \alpha^2} (\hat{j}_N|_M \cdot \hat{j}_M|_M) &= \frac{\partial}{\partial \alpha} (-r_{10} \cos(\alpha) - r_{11} \sin(\alpha)) \\ &= r_{10} \sin(\alpha) - r_{11} \cos(\alpha) < 0 \end{aligned} \quad (25)$$

Together, constraints (24) and (25) yield to (18) and (19).

Alternatively, one can aim at minimizing the angle between the  $X$ -axes of the two cameras by applying a rotation  $\alpha_X$  around the  $Z$ -axis of the later view. It can be shown that, in the case where the  $Z$ -axes are perfectly aligned, the two angles  $\alpha_Y$  and  $\alpha_X$  are equal (the two reference frames can be made to coincide). In the general case where the  $Z$ -axes are not perfectly parallel, the optimal angles  $\alpha_Y$  and  $\alpha_X$  will not be equal. The optimal angle  $\alpha_X$  that will minimize the angle between the  $X$ -axes is given by the following relations: If  $-r_{00} \cos(\arctan(\frac{r_{01}}{r_{00}})) < r_{01} \sin(\arctan(\frac{r_{01}}{r_{00}}))$ , then:

$$\alpha_X = \arctan\left(\frac{r_{01}}{r_{00}}\right) \quad (26)$$

else:

$$\alpha_X = \arctan\left(\frac{r_{01}}{r_{00}}\right) + \pi \quad (27)$$

The proof is similar to the one given for  $\alpha_Y$ .

Since there is no *a priori* reason to believe that it is more important to align the  $X$ -axes nor the  $Y$ -axes, we will use a rotation angle that is the average value of  $\alpha_X$  and  $\alpha_Y$ . The center of the rotation that must be applied to the image is the principal point of the camera. Figure 3 shows an example where the described image rectification scheme is applied on an image pair in order to make it similar to a previously captured pair. This result is further discussed in Section 6.

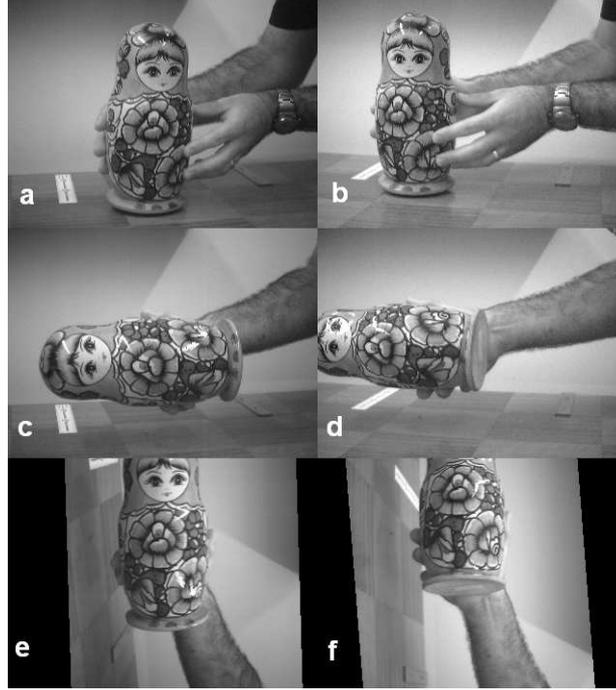


Figure 3: (a) Initial left image; (b) Initial right image; (c) Left image after 17 registrations; (d) Right image after 17 registrations; (e) Optimally rotated left image; (f) Optimally rotated right image. These two transformed images can now be matched with images (a) and (b).

## 5 Accumulated Error Correction

Let us assume the view  $N$  has been identified as being close to the earlier view  $M$ . Then it is possible to compute a correction matrix that can be premultiplied to the initially computed location of the view  $N$ .

$$Q'_{N(\text{corrected})} \equiv Q_{\text{correction},N} Q_N \quad (28)$$

The problem we would like to address now is how to correct the intermediate views. We assume there is a high level of confidence in the knowledge of the location of the view  $M$  (and therefore in the corrected location of view  $N$ ). The goal is therefore to correct the intermediate views using  $Q_{\text{correction},N}$ .

Let us assume the drift in the calculated location of the views was uniformly distributed over all the registration steps. Furthermore, let us assume the individual registration steps along with the error in the registration had small rotation components. Let us model the uniform error in the following way, rewriting (14) with the introduction of  $Q_{\text{error}}$ , the unit error transformation matrix that happened at every registration:

$$Q_n = Q_{\text{error}} Q_{n,n-1} Q_{n-1}$$

$$= \left( \prod_{i=n-1}^M Q_{error} Q_i \right) Q_M \quad (29)$$

with  $(M < n < N)$ . Under the assumption of small rotation components of  $\{Q_i\}$  and  $Q_{error}$ , we can commute the matrices in such a way that we gather the error matrices to the left and take them out of the product, that is:

$$Q_n = Q_{error}^{n-M} \left( \prod_{i=n-1}^M Q_i \right) Q_M \quad (30)$$

The correction matrix is assumed to annihilate the error of the view  $N$ . Therefore

$$Q_{correction,N} = (Q_{error}^{N-M})^{-1} = Q_{error}^{M-N} \quad (31)$$

$$Q_{error} = Q_{correction,N}^{\frac{1}{M-N}} \quad (32)$$

The correction matrix at view  $n$  will have to annihilate the error at view  $n$ , i.e.

$$\begin{aligned} Q_{correction,n} &= (Q_{error}^{n-M})^{-1} = Q_{error}^{M-n} \\ &\stackrel{(32)}{=} Q_{correction,N}^{\frac{M-n}{M-N}} \\ &= Q_{correction,N}^{\frac{n-M}{N-M}} \end{aligned} \quad (33)$$

Equation (33) gives the correction matrix that must be premultiplied to the calculated location of a camera at view  $n$ , given the correction matrix at view  $N$ , under the assumption of uniform distribution of the error along the registration steps, and under the assumption of small rotation components of both the registration matrices and the error transformation matrix.

It should be noted that the uniform distribution of the correction can be extended to the case of non-uniform distribution of the error. One can have some information about which registration steps contributed most to the overall error, according to some *suspicion level*. In this scheme, the ratio  $\frac{n-M}{N-M}$  in equation (33) would be replaced by some factor  $\alpha$ . This factor would increase monotonously between 0 (for view  $M$ ) and 1 (for view  $N$ ) according to the suspicion level of each registration step. For instance, the number of 3D pairs of points that were used in the robust registration procedure could be used as a measure of the suspicion level (a high number of 3D pairs corresponding to a low suspicion level). In all cases, the correction matrix takes care of the average component of the error.

## 6 Experimental validation of the Error Correction Scheme

In a first experiment, the movement of a Russian headstock was recorded by the stereo setup. Four images of this sequence have been presented in Figure 3.

The projection matrices of the cameras at each position have been computed by the matching, tracking and 3D registration of reconstructed points. The estimated angles between the  $Z$ -axes and distances between the centers of projection, with respect to the

first capture, have been plotted in Figure 4. The minimum angle and distance happens at Capture 19 for the left camera. For the right camera, the minimum distance happens at Capture 15, while the minimum angle happens at Capture 16. For the sake of the illustration, we will correct the error at Capture 18, but it probably could have been done for Captures 16 to 19.

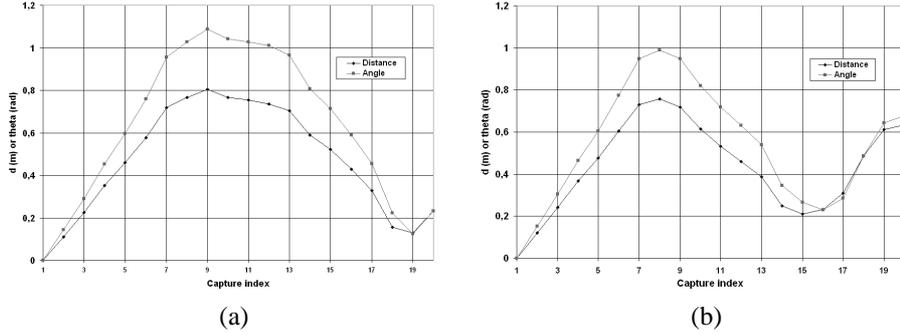


Figure 4: (a) Angle and distance of the left camera, with respect to the first capture (b) Angle and distance of the right camera, with respect to the first capture.

The rotation angles around the  $Z$ -axes of the left and right cameras were estimated to be  $-1.54$  rad and  $-1.49$  rad respectively. Figure 3 shows the first pair of views of a sequence, the  $18^{th}$  pair of views and the rotated  $18^{th}$  images such that tracking is possible with the first images. The error was corrected at view 18 through tracking of matched points from the initial views to the rotated  $18^{th}$  views. The error correction matrices were then uniformly distributed along the sequence. Figure 5 shows the *Russian Headstock* sequence, augmented with an attached reference frame projected on each image after reconnection of the camera positions. The natural movement of the augmented reference frame confirms the validity of the corrected projection matrices.

The second set of experiments tries to evaluate the quality of the 3D information obtained. To this end, a bundle adjustment program was used. Bundle adjustment is an iterative method of computing the camera pose and the 3D location of feature points, given a set of matches from different cameras and the intrinsic calibration parameters of the cameras. The problem is to minimize the sum of the Euclidian squared distance between the reprojection of the 3D points and their corresponding image points[26],[27] by varying the cameras and the 3D points locations:

$$\min[\sum_i \sum_j \|\mathbf{x}_i^j - P_i \mathbf{X}_i\|^2] \quad (34)$$

where  $P_i$  are the projection matrices,  $\mathbf{x}_i^j$  is the image of  $j$ -th point at view  $i$ .

Bundle adjustment is difficult to automate since it is very sensitive to false matches and initial camera pose estimates. But, when it converges correctly, bundle adjustment gives an optimal solution, i.e. the 3D configuration that best explains the observations. For this reason, the bundle adjustment solution will be used here to generate the ‘ground truth’ in our experiments in order to validate our error correction scheme.

Most commercial implementations of bundle adjustment rely on manual selection of



Figure 5: Russian headstock sequence recorded by the right camera augmented with an attached reference frame (after recorection).

matches. PhotoModeler<sup>1</sup> is a commercial software that implements bundle adjustment from a set of manually identified matches in a set of images; it has been used to process a subset of the *Duck* sequence (see Figure 6). Fourteen images were manually matched with 68 feature points. The returned camera positions are displayed in Figure 7, along with the 3D location of the selected feature points. Because a good match set was used and the bundle software was run with judiciously chosen optimization parameters, we were able to obtain a small reprojection error. This solution can therefore be safely considered to be accurate.

The camera path forms two loops:  $360^\circ$  in the  $X - Y$  plane and a half-turn under the duck. These loops allow us to apply our error correction scheme. Following the procedure defined in Section 4, frames 1 and 40 were detected as the two extreme views of a looping sequence (and similarly for 17 and 64). These were then matched and their 3D point sets registered.

Figure 8(a) and (b) shows the disagreement (in the position and the orientation of the cameras) between PhotoModeler and the proposed method, without error correction. The position disagreement is the distance between the computed centers of projection. The orientation disagreement is the angle between the  $Z$ -axes of the camera reference frames. As expected, the magnitude of the disagreement increases with the number of registrations, as the proposed method accumulates error. The PhotoModeler project had matches between the first and the last image, allowing for a closed loop configuration and

<sup>1</sup>EOS Systems Technology ([www.photomodeler.com](http://www.photomodeler.com))

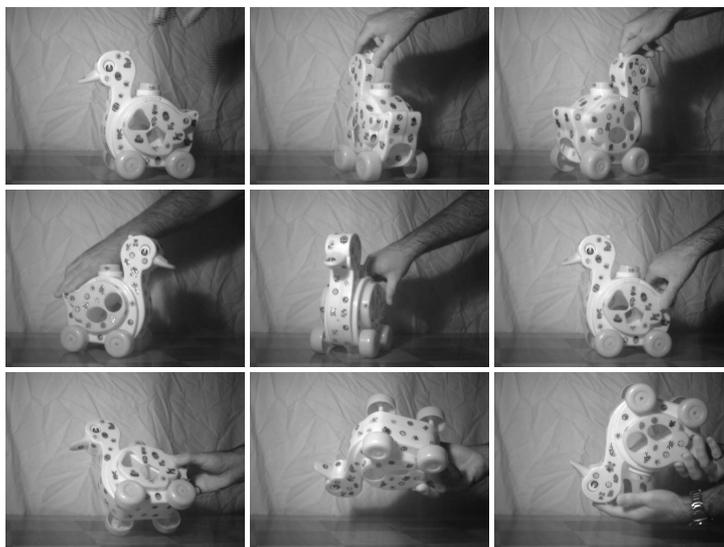


Figure 6: Some images of the Duck sequence recorded by the left camera (frames 1, 11, 17, 24, 32, 40, 49, 59, 64).

thus preventing error accumulation.

It is worth noticing the fact that, as opposed to the proposed method, bundle adjustment does not grant any special status to the first capture. It can be adjusted, like every other camera position. In contrast, the proposed method gives a higher level of confidence in the earlier captures. The discrepancy between the two methods at the first capture is most probably related to errors in the bundle adjustment solution.

Figure 8(c) and (d) shows the disagreement between PhotoModeler and the proposed method after the two passes of error correction through uniform distribution of the correction matrix. It can be seen that the disagreement magnitude increases a lot more slowly with the number of registrations, as compared with Figure 8(a) and (b), indicating that the error correction provided an improvement in the projection matrices.

The computed locations of the cameras can be used to build a volumetric representation of the object, through shape-from-silhouette [12]. Figure 9 shows the model obtained by silhouette intersection of 82 images. The model contains approximately 12600 voxels, each having dimensions of  $5 \text{ mm} \times 5 \text{ mm} \times 5 \text{ mm}$ . The presence of the hand in the images did not pose a problem here since view registration is robustly computed. Indeed, matches on the hand surface were filtered out, as their reconstructions were not moving rigidly with respect to the surface of the object. As can be seen, our error correction scheme gave object pose information of sufficient accuracy to obtain a model of relatively good accuracy. Rapid object modelling is therefore achieved in a very convenient way by simply moving an object in front a stereo setup in a totally unconstrained manner.



Figure 7: Positions of the left camera in the Duck sequence, as computed by PhotoModeler.

## 7 Conclusion

In this paper, we addressed the problem of 3D registration of a rigid object moving in front of two cameras, which is equivalent to the problem of camera pose estimation. We used a calibrated stereoscopic vision setup to track the camera positions along sequences of a moving rigid object. We proposed a robust 3D registration procedure that exploits the rigidity of the scene to automatically filter out the reconstructed points originating from false matches and errors in feature tracking. An error correction scheme was introduced, which takes advantage of loops in the movement of the cameras to compensate for the accumulated error. Through experimental results, we showed the validity of the obtained projection matrices and that their accuracy was sufficient for tasks such as model building or scene augmentation.

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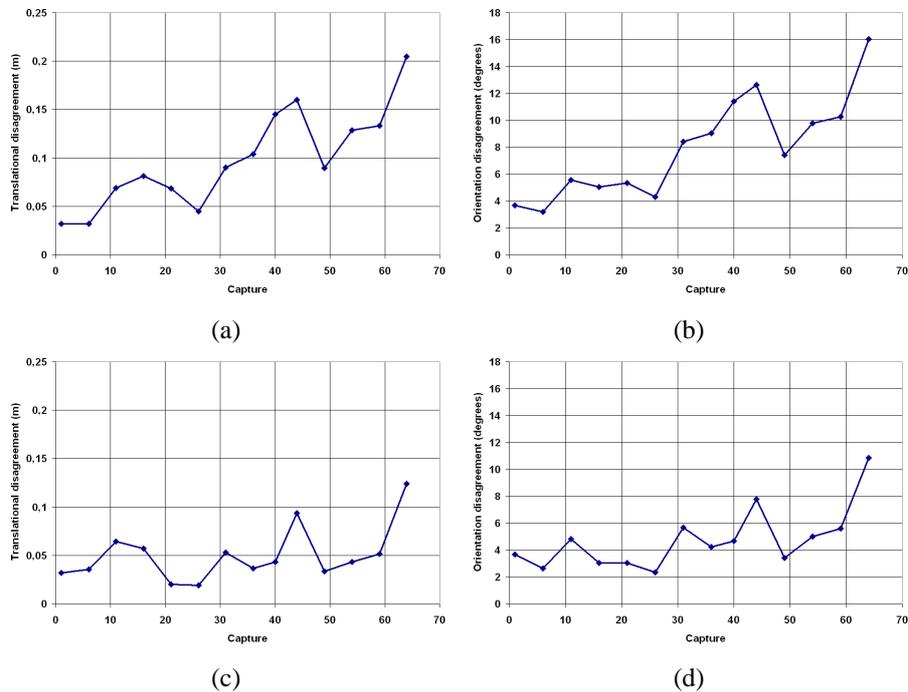


Figure 8: (a) Position disagreement between bundle adjustment and the proposed method, without error correction. (b) Z-axis orientation disagreement between bundle adjustment and the proposed method, without error correction. (c) Position disagreement between bundle adjustment and the proposed method, after error correction. (d) Z-axis orientation disagreement between bundle adjustment and the proposed method, after error correction.

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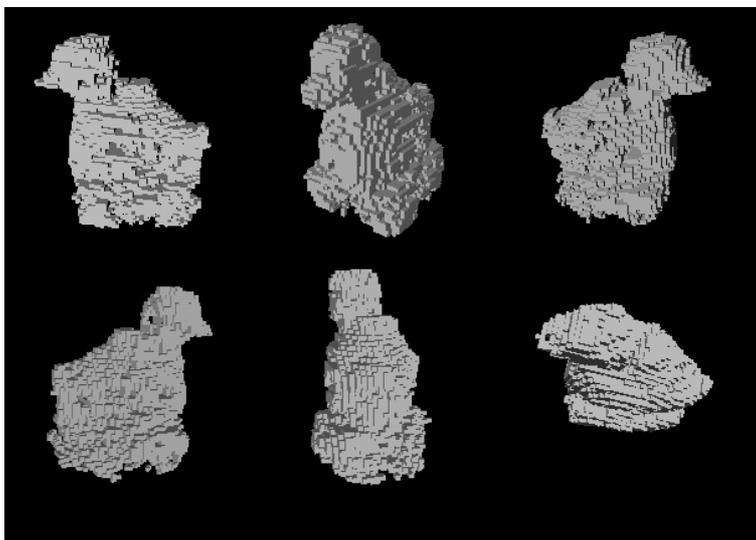


Figure 9: A few views of the Duck model.

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