Continuously Tunable Photonic Microwave Frequency Multiplication by Use of an Unbalanced Temporal Pulse Shaping System

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***Abstract*—A novel approach to achieving microwave frequency multiplication with a continuously tunable multiplication factor using an unbalanced temporal pulse shaping (TPS) system is proposed and demonstrated. The proposed TPS system consists of a Mach–Zehnder modulator and two dispersive elements having opposite dispersion values. Since the values of dispersion are not identical in magnitude, the entire system can be considered as a typical balanced TPS system for a real-time Fourier transformation followed by a residual dispersive element for a second real-time Fourier transformation. The microwave frequency multiplication factor is determined by the dispersion values of the two dispersive elements and can be continuously tunable by changing the values. Numerical simulations and experiments are performed. Microwave frequency multiplication with a factor as high as 13.2 is experimentally demonstrated.**

***Index Terms*—Frequency multiplication, microwave photonics, real-time Fourier transform, temporal pulse shaping (TPS).**

I. Introduction

MICROWAVE signal synthesis in the optical domain has been a topic of interest for a few decades and numerous techniques have been proposed [1]–[3]. Among the different microwave signals, high-frequency pulsed microwave signals have found important applications such as in radar systems [2] and microwave tomography [3]. One method to generate a high-frequency microwave pulse is to multiply the carrier frequency of a low-frequency microwave drive signal. For example, frequency division or multiplication of a pulsed microwave signal has been reported through dispersively stretching or compression of a highly chirped optical pulse that is modulated by a microwave drive signal [4]. The main difficulty associated with the approach in [4] is that the maximum microwave frequency achievable is limited due to the dispersion-induced power penalty using optical double-sideband (DSB) modulation, which is similar to the power penalty in a DSB-modulation-based analog fiber link [5]. Although the use of optical single-sideband (SSB) modulation can eliminate this penalty [6], it may increase the complexity of the system. Recently, Azana *et al.* demonstrated an approach to achieving frequency multiplication of a pulsed microwave signal based on a general temporal self-imaging effect [7]. Since the focused image of the microwave drive signal can only be obtained under specific dispersion conditions (so-called integer Talbot conditions), the multiplication factor can be tuned only at specific values.

In this letter, we propose and experimentally demonstrate an optical approach to achieving continuously tunable frequency multiplication for a pulsed microwave signal. The technique is implemented using an unbalanced temporal pulse shaping (TPS) system that incorporates two dispersive elements (DEs) having opposite dispersion. Since the values of dispersion are not identical in magnitude, the entire system is considered as a typical TPS system with a pair of complementary DEs [8] for real-time Fourier transformation (FT) followed by a residual DE with its dispersion being the offset of the dispersion of the two DEs, to achieve a second real-time FT [9]. The proposed system has a similar setup to that reported in [4], but a different modulation scheme is employed—double-sideband modulation with suppressed carrier (DSB-SC), which enables the elimination of the dispersion-induced power penalty [4]. In addition, a novel and simple theoretical model is developed to describe the operation. Our analysis shows that the frequency multiplication factor is only determined by the dispersion values of the two DEs. In addition, since the use of the DSB-SC modulation scheme eliminates the requirement for the specific dispersion conditions [7], the frequency multiplication factor can be continuously tunable by changing the dispersion values.

II. Principle

The proposed unbalanced TPS system is illustrated in Fig. 1. It consists of a mode-locked laser (MLL), two DEs, and a Mach–Zehnder modulator (MZM). Here we assume that the values of the third-order dispersion (TOD) of the two DEs are small and negligible, and only the group velocity dispersion (GVD) is considered. The DEs can then be characterized by the transfer functions given by $H_i(\omega) = \exp(-j\ddot{\Phi}_i\omega^2/2)$, $(i = 1,2)$, where $\ddot{\Phi}_1$ and $\ddot{\Phi}_2$ (ps^2) are the dispersion of the two DEs. In the proposed unbalanced TPS system, the dispersion values should satisfy $\ddot{\Phi}_1\ddot{\Phi}_2 < 0$, and $\left|\ddot{\Phi}_1\right| \neq \left|\ddot{\Phi}_2\right|$. Therefore, the entire unbalanced TPS system can be modeled as a typical TPS system with a pair of complementary DEs, followed by a residual DE with a transfer function of $H_\Delta(\omega) = \exp(-j\Delta\ddot{\Phi}\omega^2/2)$, where $\Delta\ddot{\Phi} = \ddot{\Phi}_1 + \ddot{\Phi}_2$ is defined as the residual dispersion.

In our proposed approach, the MZM is dc-biased at the minimum transmission point to suppress the optical carrier. If the

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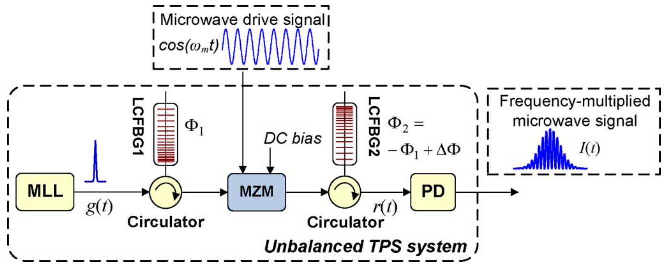

Fig. 1. Unbalanced TPS system for microwave frequency multiplication.

modulating signal applied to the MZM is a sinusoidal signal, under the small-signal-modulation condition, the intensity modulation function can be simplified to

$$e_{\mathrm{IM}}(t) = 2 \times J_1(\beta)\cos(\omega_m t) \tag{1}$$

where $J_1(\beta)$ is the first-order Bessel function of the first kind, β is the phase modulation index, and ω_m is the angular frequency of the microwave drive signal applied to the MZM.

It is known that if the first DE has an adequate dispersion, i.e., $\ddot{\Phi}_1 \gg \tau_0^2$, τ_0 is the pulsewidth of the input optical pulse $g(t)$, the output signal of the typical TPS system $s(t)$ is given by [8]

$$s(t) \propto g(t) * E_{\mathrm{IM}}(\omega)|_{\omega=t/\ddot{\Phi}_1} = J_1(\beta)[g(t-T_1)+g(t+T_1)] \tag{2}$$

where $E_{\mathrm{IM}}(\omega)$ is the Fourier transform of $e_{\mathrm{IM}}(t)$, $*$ denotes the convolution operation, and $T_1 = \left|\omega_m \ddot{\Phi}_1\right|/2\pi$. Therefore, two time-delayed replicas of the input pulse are generated at the output of the typical TPS system, which correspond to the two optical sidebands at the output of the DSB-SC modulator.

The electrical field at the output of the entire unbalanced TPS system $r(t)$ is obtained by propagating $s(t)$ through the residual DE. If $\left|\Delta\ddot{\Phi}\right| \geq {T_1}^2/2$ is satisfied, then $r(t)$ can be approximated by the real-time FT of $s(t)$ in the residual DE [9]

$$\begin{aligned} r(t) &\approx \exp\left[\frac{jt^2}{(2\Delta\ddot{\Phi})}\right] S(\omega)|_{\omega=2\pi\times t/\Delta\ddot{\Phi}} \\ &= \exp\left[\frac{jt^2}{(2\Delta\ddot{\Phi})}\right] J_1(\beta) G\left(2\pi\frac{t}{\Delta\ddot{\Phi}}\right)\cos\left(2\pi\frac{tT_1}{\Delta\ddot{\Phi}}\right) \end{aligned} \tag{3}$$

where $G(\omega)$ is the Fourier transform of the input pulse $g(t)$.

The current at the output of the PD is proportional to the intensity of the input electrical field, which is given by

$$I(t) \propto |r(t)|^2 = K\exp\left(-\frac{t^2}{\tau^2}\right)\left[1+\cos(2\pi\frac{2T_1}{\Delta\ddot{\Phi}}t)\right] \tag{4}$$

where $K = {J_1}^2(\beta)\pi{\tau_0}^2/2$ is a time-independent constant, and $\tau = \sqrt{2}\Delta\ddot{\Phi}/\tau_0$ is the output pulsewidth. A frequency-multiplied microwave signal is generated, which is a pulsed microwave signal with a Gaussian envelope. The new carrier frequency is

$$\omega_{\mathrm{RF}} = 2\pi\left|\frac{2T_1}{\Delta\ddot{\Phi}}\right| = \omega_m\left|\frac{2\ddot{\Phi}_1}{\Delta\ddot{\Phi}}\right|. \tag{5}$$

From (5), we can conclude that the frequency multiplication factor $M = \omega_{\mathrm{RF}}/\omega_m = 2\left|\ddot{\Phi}_1/\Delta\ddot{\Phi}\right|$ is determined by both the stretching dispersion $\ddot{\Phi}_1$ and the residual dispersion $\Delta\ddot{\Phi}$. The key significance of the approach is that the multiplication factor can be continuously tunable by changing the dispersion values of the DEs, while only discrete multiplication factors can be achieved in [7]. In fact, the dispersion of a linearly chirped fiber Bragg grating (LCFBG) can be continuously tunable in a range of 10 nm or more by using a simply supported beam tuning technique [10]. In addition, the proposed approach offers an additional two-fold improvement in frequency multiplication thanks to the use of the DSB-SC modulation.

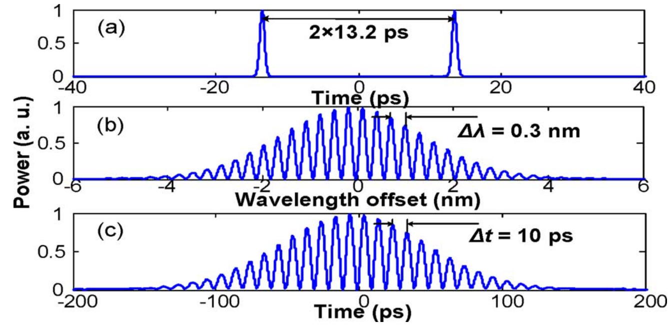

Fig. 2. Simulation results. (a) Signal at the output of the typical TPS system. (b) The optical spectrum of the signal in (a). (c) The frequency-multiplied microwave signal at the output of the entire system.

III. Results

The proposed approach is first evaluated by simulations. We assume the sinusoidal modulating signal has a frequency of $f_m = 5$ GHz. The phase modulation index β is 0.35 rad. The dispersion values are selected as $\ddot{\Phi}_1 = 2640$ ps^2 and $\ddot{\Phi}_2 = -2904$ ps^2. Fig. 2(a) shows the signal at the output of the typical TPS system. Two pulses separated by a time shift of $2T_1$ are observed, as predicted by (2). The optical spectrum of the two output pulses is shown in Fig. 2(b). Fig. 2(c) shows the microwave signal at the output of the entire system. A pulsed microwave signal with 20-fold frequency multiplication is achieved, which matches well with the theoretical predictions given by (4) and (5). In addition, by comparing the results in Fig. 2(b) and (c), the real-time FT in the residual DE is confirmed.

To verify the capability of the proposed system in eliminating the dispersion-induced power penalty [4], an additional simulation is performed, in which the input RF signal at three different frequencies of 10, 12, or 16 GHz for DSB modulation and DSB-SC modulation is considered. As can be seen from Fig. 3, the power penalty, which occurs in DSB modulation at 12 and 16 GHz, does not exist in DSB-SC modulation.

A proof-of-concept experiment based on the setup shown in Fig. 1 is then performed. The ultrashort optical pulse from the MLL has a pulsewidth of 550 fs and a 3-dB bandwidth of 8 nm. A 6.1-km dispersion-compensating fiber with a dispersion of $\ddot{\Phi}_1 = 7624$ ps^2, and single-mode fibers with different dispersion values are used as the first DE and the second DE, respectively. The generated microwave waveforms are measured both in the time and frequency domains, with the results shown in Fig. 4. Experimental parameters and results are summarized in Table I. Tunable frequency multiplication is verified. The experimental results agree well with the theoretical results given by (5). Note that the generated frequencies are relatively low, which are limited due to the low operation bandwidth of the oscilloscope.

IV. Discussion and Conclusion

Note that a dispersion condition $\left|\Delta\ddot{\Phi}\right| \geq {T_1}^2/2$ must be satisfied for the residual DE to perform the real-time FT. On the other hand, the microwave drive signal must be much faster

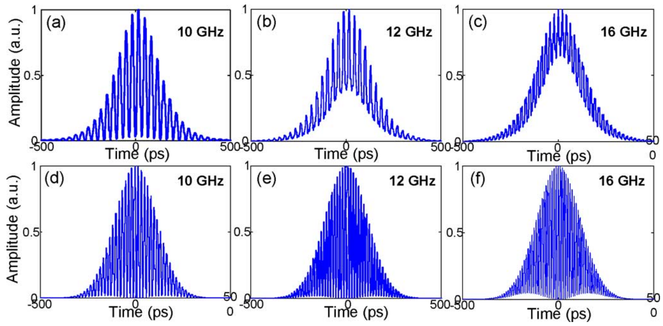

Fig. 3. Simulated output microwave pulses based on DSB modulation [(a), (b), (c)] and DSB-SC modulation [(d), (e), (f)].

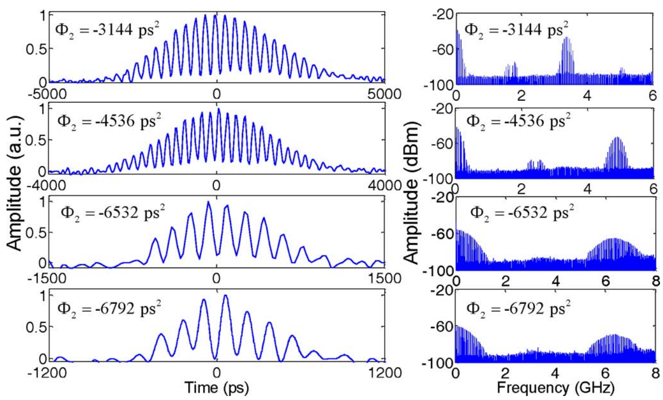

Fig. 4. Experimental results measured both in the time and frequency domains.

TABLE I
EXPERIMENTAL PARAMETERS AND RESULTS

Φ_2 (ps^2)	f_m (GHz)	f_{RF} (GHz)	M (measured /theoretical)	τ (ps)
-3144	1	3.38	3.38/3.40	2450
-4536	1	4.91	4.91/4.94	1700
-6532	0.45	6.31	14.02/13.96	725
-6792	0.35	6.34	18.11/18.32	550

than the dispersed optical pulse stretched by the first DE [8], i.e., $\omega_m \geq (2\pi)^2 N/(\Delta\omega_{\mathrm{opt}}\ddot{\Phi}_1)$, where $\Delta\omega_{\mathrm{opt}}$ is the input optical bandwidth, and N is the minimum number of cycles in the modulated pulse. With these two constraints, we have

$$M = \left|\frac{2\ddot{\Phi}_1}{\Delta\ddot{\Phi}}\right| \leq \frac{4\Delta\omega_{\mathrm{opt}}}{(N\omega_m)}$$
$$\omega_{\mathrm{RF}} = M\omega_m \leq \frac{4\Delta\omega_{\mathrm{opt}}}{N}. \tag{6}$$

Theoretically, the highest frequency achievable is only limited by the input optical bandwidth. The frequency multiplication is practically limited by the bandwidth of the PD.

It is also worth noting that decreasing the residual dispersion may increase the frequency multiplication factor, but would reduce the output pulse duration, as shown in Fig. 4. Therefore, a tradeoff exists. To provide more design guidelines, more simulations are performed. Fig. 5 shows the frequency multiplication factor and the output pulse duration as a function of the residual dispersion for a given stretching dispersion value (5280 ps^2), a modulation frequency (1 GHz), and an input optical pulsewidth. By employing an input optical pulse with a shorter pulsewidth, we can achieve a higher multiplication factor while maintaining a suitable output pulse duration.

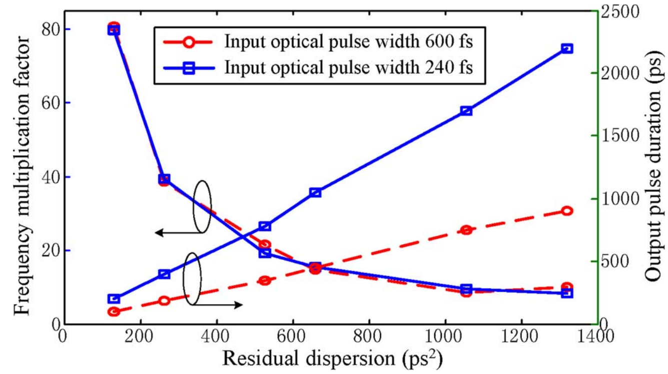

Fig. 5. Calculated freuquency multiplication factor and output pulse duration as a function of the residual dispersion.

In our analysis, only the GVD are considered. This treatment is valid for fibers with a moderate length ($<$70 km). For a longer dispersive fiber, the TOD has to be considered. As a result, a chirped microwave pulse would be generated due to the higher order dispersion-induced nonlinear real-time FT [2].

In conclusion, a novel approach to achieving microwave frequency multiplication with a continuously tunable multiplication factor based on optical pulse shaping in an unbalanced TPS system has been proposed and demonstrated. The entire system is interpreted as a combination of a typical TPS system for a real-time FT followed by a residual DE for a second real-time FT. The approach provides a potential solution to generate high-frequency microwave pulses for applications such as radar and microwave tomography.

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