All-Optical Short-Time Fourier Transform Based on a Temporal Pulse-Shaping System Incorporating an Array of Cascaded Linearly Chirped Fiber Bragg Gratings

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***Abstract*—An all-optical approach to implementing short-time Fourier transform (STFT) of a high-speed and broadband electrical signal is proposed and demonstrated for the first time to our knowledge. The STFT is implemented based on a temporal pulse shaping system incorporating an array of cascaded linearly chirped fiber Bragg gratings (LCFBGs). An electrical signal to be analyzed is applied to a Mach–Zehnder modulator (MZM) to modulate the optical spectrum of a time-stretched optical pulse from a mode-locked laser (MLL). Each individual LCFBG in the LCFBG array functions as a bandpass filter to filter a specific range of the spectrum, which is equivalent to applying a window function to the corresponding section of the temporal signal, and at the same time, as a dispersive element to implement real-time Fourier transform. A theoretical analysis is performed which is verified by a numerical simulation and a proof-of-concept experiment. The key feature of this technique is that STFT can be implemented in real time which can find applications in analyzing an electrical signal with a bandwidth up to several hundreds of gigahertz.**

***Index Terms*—All-optical signal processing, linearly chirped fiber Bragg gratings (LCFBGs), short-time Fourier transform (STFT), temporal pulse shaping.**

I. INTRODUCTION

SHORT-TIME Fourier transform (STFT) is an important technique for the time-frequency domain characterization of electrical signals, which can find many important applications, such as in speech and image processing, radar signal processing, and quantum physics [1], [2]. STFT is usually implemented in the electrical domain using digital electronics, but the processing speed is slow due to the limited sampling rate of the state-of-the-art digital circuits. Thanks to the inherent high speed and large bandwidth offered by optics, STFT can be implemented in the optical domain with a much higher speed and greater bandwidth.

A time-stretched STFT with an improved signal processing bandwidth was proposed in [3]. In this technique, the electrical signal is first time stretched in the optical domain by using a dispersive element. The stretched electrical signal is then short-time Fourier transformed in the electrical domain using digital electronics. A linearly chirped fiber Bragg grating (LCFBG) can be used to implement real-time Fourier transform of an ultrafast optical signal [4]. The limitation of using a single LCFBG is that the optical signal in the time domain cannot be divided into many sections to implement Fourier transform of the signal locally in time. In addition, since the waveform at the output of a temporal pulse shaping (TPS) system is a Fourier-transformed version of the modulation signal, a TPS system has been used to perform Fourier transform of a microwave signal [5] and to implement arbitrary waveform generation [6]. But the concept of STFT using a TPS system has not been previously investigated.

In this letter, we propose and demonstrate a novel technique to implement all-optically STFT based on a TPS system incorporating an array of LCFBGs. An electrical signal to be analyzed is applied to a Mach–Zehnder modulator (MZM) to modulate the optical spectrum of a time-stretched optical pulse from a mode-locked laser (MLL). Each individual LCFBG in the array of LCFBGs functions as a bandpass filter to filter a specific range of the spectrum, which is equivalent to applying a window function to the corresponding section of the temporal signal, and at the same time, as a dispersive element to perform real-time Fourier transform. A theoretical analysis is performed which is verified by a numerical simulation and a proof-of-concept experiment. The short time Fourier transform of an electrical signal consisting of two cascaded electrical signals with a carrier frequency of 1.5 GHz and 3 GHz is experimentally demonstrated.

II. PRINCIPLE

Fig. 1 shows the TPS system incorporating four cascaded LCFBGs for the all-optical implementation of STFT. A transform-limited Gaussian pulse is generated by a mode-locked laser (MLL), which can be expressed as $a(t) = \exp(-t^2/\tau_0^2)$, where τ_0 is the half pulse width at $1/e$ maximum, and its Fourier transform is given by $A(\omega) = \sqrt{\pi}\tau_0 \exp(-\tau_0^2\omega^2/4)$, where ω denotes the angular frequency. As shown in Fig. 1(a), a dispersive element (DE), connected before a Mach–Zehnder modulator (MZM), is used to stretch the ultrashort optical pulse. It is assumed that the DE has a broad enough bandwidth to cover the entire spectrum of the optical pulse. The ultrashort optical pulse is temporally stretched and spectrally dispersed by passing through the DE. At the output of the DE, the optical signal is given $P(\omega) = A(\omega) \times \exp(j\ddot{\Phi}_1\omega^2/2)$, where $\ddot{\Phi}_1$ is

Manuscript received March 26, 2011; revised June 12, 2011; accepted July 16, 2011. Date of publication July 22, 2011; date of current version September 21, 2011. This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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Digital Object Identifier 10.1109/LPT.2011.2162624

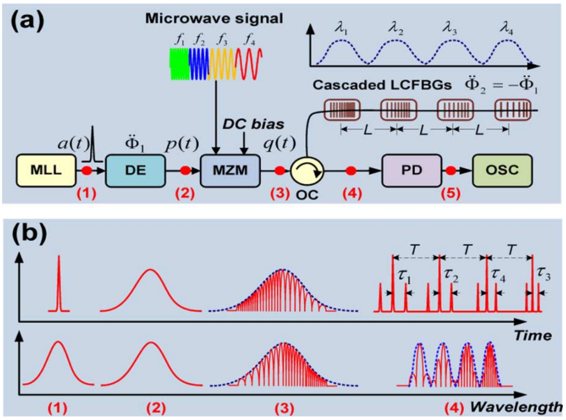

Fig. 1. (a) Schematic for an all-optical STFT based on a TPS system incorporating an array of cascaded LCFBGs. (b) The signals at different locations of the system in the frequency and the temporal domains. LCFBG: Linearly chirped fiber Bragg grating. MLL: Mode-locked laser. DE: Dispersive element. MZM: Mach–Zehnder modulator. OC: Optical circulator. PD: Photodetector. OSC: Oscilloscope.

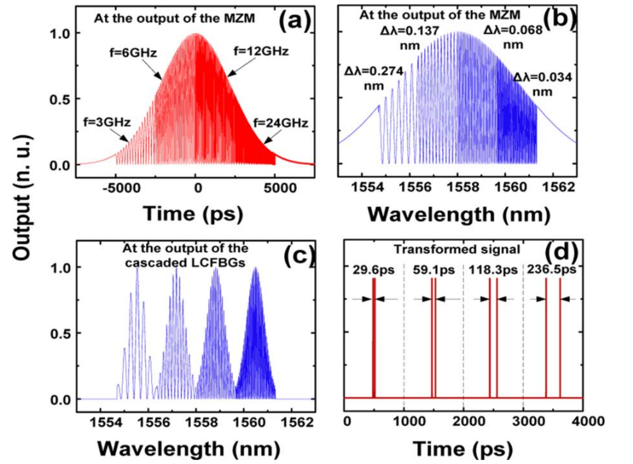

Fig. 2. Simulation results. (a) The optical signal at the output of the MZM in the time-domain and (b) in the wavelength-domain; (c) optical spectrum of the output signal from the array of cascaded LCFBGs and (d) the transformed signal measured by an OSC.

the group velocity dispersion (GVD) of the DE. The stretched optical pulse is then directed into the MZM and is modulated by an electrical signal $x(t)$. The optical signal at the output of the MZM is given by $q(t) = p(t) \times m(t)$ or

$$Q(\omega) = \frac{1}{2\pi} P(\omega) * M(\omega) = \frac{1}{2\pi} A(\omega) \times \exp\left(j\frac{\ddot{\Phi}_1}{2}\omega^2\right) * M(\omega), \quad (1)$$

where $M(\omega)$ is the Fourier transform of $m(t)$ which is the modulation function of the MZM when an electrical signal $x(t)$ is used to modulate the stretched optical pulse. When the time-to-frequency mapping condition given by $\ddot{\Phi}_1 \gg 8\pi/\Delta\omega_m^2$ is satisfied [7], we have

$$\exp\left(j\ddot{\Phi}_1\omega^2/2\right) * M(\omega) \propto \exp\left(j\ddot{\Phi}_1\omega^2/2\right) \times m(t)|_{t=\ddot{\Phi}_1\omega},$$

or

$$Q(\omega) \propto A(\omega) \times \exp\left(j\frac{\ddot{\Phi}_1}{2}\omega^2\right) \times m(t)|_{t=\ddot{\Phi}_1\omega}, \quad (2)$$

where $\Delta\omega_m$ is the bandwidth of the electrical signal. It can be seen from (2) that the amplitude of the optical spectrum of the input ultrashort pulse is modulated by a scaled version of the modulation signal $m(t)$ at the MZM, as shown in Fig. 1(b). The modulated signal is then directed into the array of cascaded LCFBGs. The transfer function of the ith LCFBG is given by $S_i(\omega) = G_i(\omega)\exp(j\ddot{\Phi}_i\omega^2/2)$, where $\ddot{\Phi}_i = -\ddot{\Phi}_1$ is the GVD of the LCFBG, and $G_i(\omega)$ is the magnitude response of the ith LCFBG. The output signal from the ith LCFBG in the time domain is

$$y_i(t) \propto \int_{\omega_i-\delta\omega/2}^{\omega_i+\delta\omega/2} m(\ddot{\Phi}_1\omega) \times G_i'(\omega) \times \exp(-j\omega t)d\omega \quad (3)$$

where $G_i'(\omega) = A(\omega)\times G(\omega)$ is the product between the optical spectrum of the ultrashort pulse and the magnitude response $G_i(\omega)$ of the ith LCFBG, $\delta\omega$ is the bandwidth of the LCFBG (all the LCFBGs have the same bandwidth), and ω_i represents the central frequency. It can be seen from (3) that a scaled version of the modulation signal is broken up into frames by a series of window function $G_i'(\omega)$ and each frame is Fourier transformed separately. The output signal is detected at a photodetector (PD) and measured by an oscilloscope (OSC). Since the cascaded LCFBGs are spatially separated with a longitudinal spacing L, any two adjacent transformed output signals have a time spacing of $T = 2n_{\text{eff}}L/c$, where n_{eff} is the effective refractive index of optical fiber, c is the speed of light in vacuum. Finally, the spectrum of the modulation signal locally in time can be retrieved from the measured signal.

Since joint time-frequency representation has an intrinsic limitation, the product of the resolutions in time Δt and in frequency $\Delta\omega$ is limited by the uncertainty principle given by $\Delta t\Delta\omega \geq 1/2$. A Gaussian apodization is employed thanks to a minimum time-bandwidth product achieved by a Gaussian function. In addition, the processing bandwidth of the STFT is determined by the condition for time-to-frequency mapping. When the dispersion value of the dispersive element is 100 ps^2/rad, an electrical signal with a frequency lower than 501.3 GHz can be processed. It is worth noting that the bandwidth of all the devices involved in the TPS system will also limit the operation bandwidth of the proposed system.

III. Numerical Simulation

A numerical simulation is then performed. An electrical signal with four different frequencies at 3 GHz, 6 GHz, 12 GHz and 24 GHz in four consecutive time slots is employed to modulate a time-stretched optical pulse. An optical pulse with a time-width of 500 fs is stretched by a dispersive element with a dispersion value of 784.24 ps^2/rad. The MZM is biased to operate in the double-sideband with suppressed-carrier (DSB-SC) modulation mode [8]. The optical signal at the output of the MZM and its corresponding spectrum are shown in Fig. 2(a) and (b).

The optical signal at the output of the MZM is then sent to the array of cascaded LCFBGs. To ensure the filtered spectrum at the output of each of the LCFBG has a Gaussian profile, the amplitude response of each of the LCFBGs is specially designed based on the discrete layer peeling method [9]. It can be

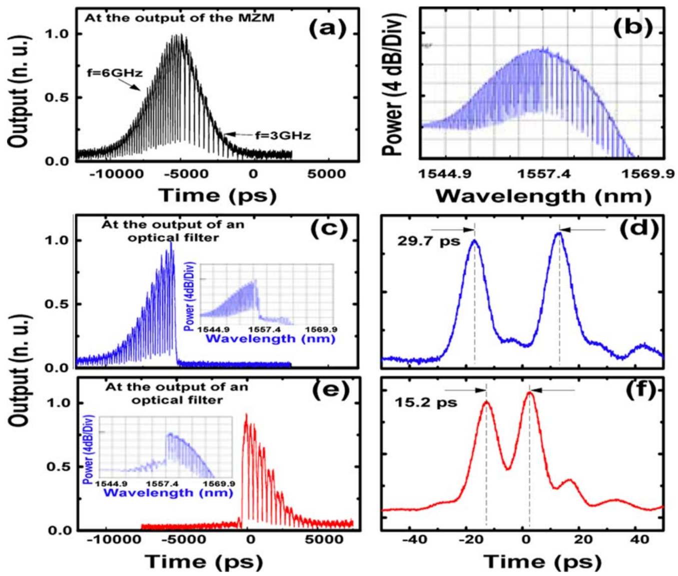

Fig. 3. Experimental results. (a) The temporal waveform and (b) its optical spectrum of the modulated pulse at the output of the MZM. (c) The temporal waveform at the output of a rectangular optical filter with a central wavelength of 1551.22 nm, and (d) the corresponding spectrum of the signal. (e) The temporal waveform at the output of a rectangular optical filter with a central wavelength of 1563.58 nm, and (f) the corresponding spectrum of the signal. The insets in (c) and (e) show the optical spectra of the signal.

seen from Fig. 2(c), the optical spectrum at the output of each LCFBG is Gaussian apodized with an identical peak power. At the same time, the Gaussian apodized spectrum is inversely Fourier transformed by the same LCFBG due to the dispersion of the LCFBG. The short-time Fourier-transformed signal is shown in Fig. 2(d). Since DSB-SC modulation is employed, the temporal spacing τ_i between the two peaks can be calculated by $\tau_i = 2\pi\ddot{\Phi}_1 \times 2f_i$ based on (3), where f_i is the frequency of the electrical signal at the ith window. The cascaded LCFBGs are spatially distributed with a longitudinal spacing of 10.34 cm, which corresponds to a time separation of 1000 ps.

IV. Proof-of-Concept Experiment and Discussion

A proof-of-concept experiment is then implemented to further verify the proposed technique. An MLL generating an ultrashort pulse train with a repetition rate of 48.6 MHz and a pulse width of 550 fs is directed into a length of dispersion compensating fiber (DCF) having a value of dispersion of 784.24 ps^2/rad, to stretch the ultrashort pulse from the MLL, as shown in Fig. 1(a). The pulse from the MLL has a full-width at half-maximum (FWHM) of 8 nm with a central wavelength of 1557.4 nm. Since a phase mask that can be used to fabricate the cascaded LCFBGs with a dispersion value of 784.24 ps^2/rad and a bandwidth of about 20 nm is not available, the cascaded LCFBGs are equivalently implemented using a tunable optical filter (TOF). Since the TOF is dispersion free in the passband, a single-mode fiber (SMF) with a value of dispersion of -784.24 ps^2/rad is connected after the TOF; thus the joint function of the TOF and the SMF is equivalent to a LCFBG.

As shown in Fig. 3(a), the stretched pulse is modulated by two consecutive electrical signals with a frequency of 1.5 GHz and 3 GHz at the MZM. The MZM is biased to operate at the DSB-SC modulation mode. Fig. 3(b) shows the spectrum of the optical signal at the output of the MZM. The spectrum has a shape that is a scaled version of the input electrical signal, which verifies the theoretical analysis given by (2). By tuning the central wavelength of the optical filter to 1551.22 nm, which corresponds to applying a temporal window to the 3-GHz signal, as shown in Fig. 3(c). The Fourier transformed signal is measured at the output of the SMF by using a sampling OSC (Agilent 86116A), as shown in Fig. 3(d). The time spacing between the two peaks is 29.7 ps, corresponding to a microwave frequency of the electrical signal of 3.02 GHz. By tuning the central wavelength of the optical filter to 1563.58 nm, the 1.5-GHz signal is Fourier transformed, with the result show in Figs. 3(e) and (f). The time spacing between the two peaks is 15.2 ps, corresponding to a microwave frequency of the electrical signal of 1.54 GHz.

It can be seen from Fig. 3(f) that the two peaks are partly overlapped due to the low frequency of the input electrical signal. When the frequency of the input signal is further decreased, the two peaks cannot be distinguished due to the overlap. By increasing the dispersion of the dispersive element, the temporal spacing between the two peaks can be increased. However, when the dispersion becomes too large, any two adjacent optical pulses at the output of the first dispersive element after time stretching will overlap. So, there exists a trade-off between pulse repetition rate and the minimum bandwidth of the signals to be measured. Therefore, it can be concluded that the minimum resolvable frequency of the input electrical signal is determined by the dispersion of the dispersive element and the bandwidth of the PD and the OSC.

V. Conclusion

An all-optical approach to implementing STFT of a high-speed and broadband electrical signal was proposed and demonstrated for the first time to our knowledge. A theoretical analysis was performed which was verified by a numerical simulation and a proof-of-concept experiment. The key feature of this technique is that STFT can be implemented in real time which can find applications in the characterization of an electrical signal with a bandwidth up to several hundreds of GHz.

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